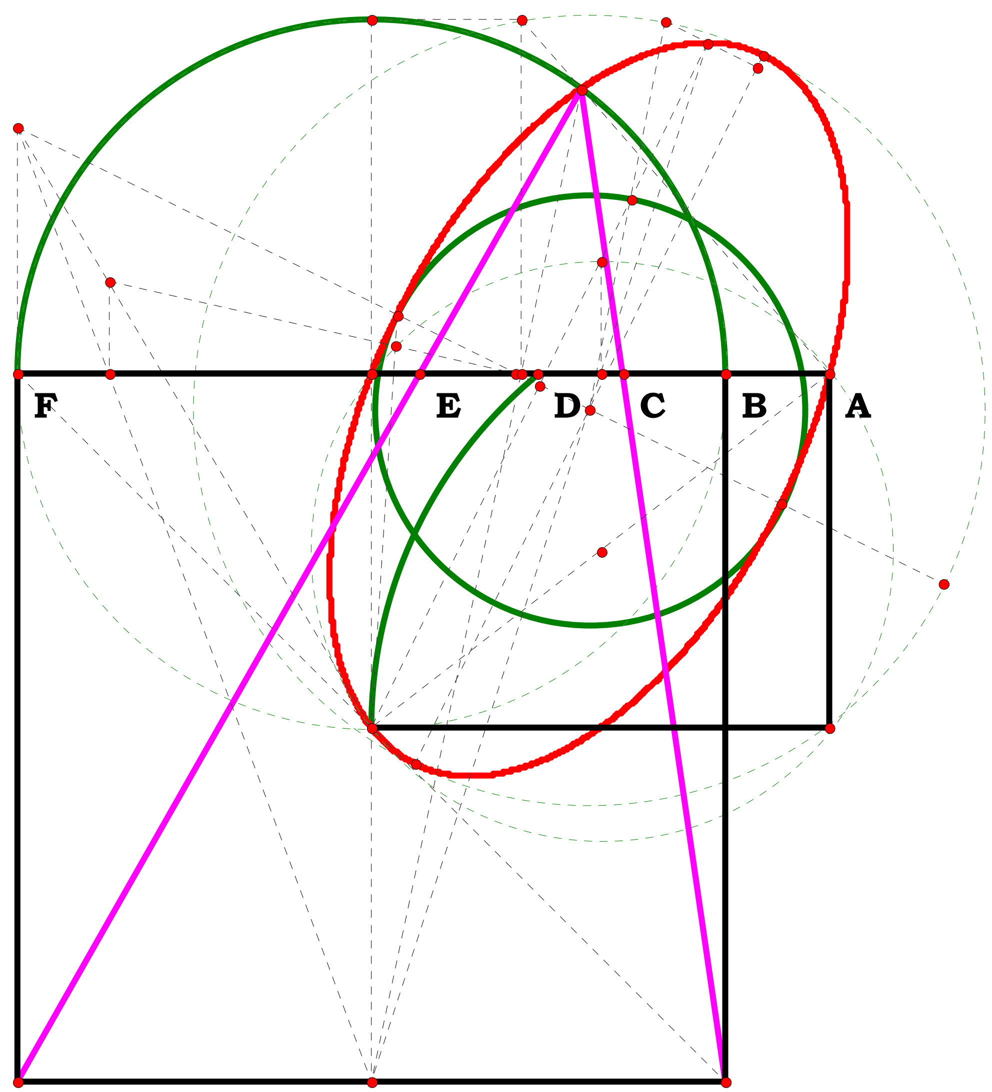
The Delian Quest.

2015 rev.



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On the Principles of Dialectic.

I am not going to make a long and tedious demonstration here, I will be presenting a more or less concise outline.

Dialectic is used for the maintenance and promotion of the functionality of the human mind in order for it to do its own work. The fact that the mind has a biologically determine job to perform is not an assumption. Nor is it an assumption that other life forms in the universe have guided the development of the mind of man so that one day the mind can perform this biologically determined job. One of the reasons one teaches someone to perform a task is because the teacher is not going to do that task for someone. The concept taught through religions, that a teacher is going to do man's job for him, is the product of a dysfunctional mind and it is a self-referential fallacy which defeats the purpose and efficacy of education. Another fallacy is that philosophy is the product of an individual mind.

The mind is responsible for the behavior of the body within which that mind resides. When that behavior is the product of language, language principles not currently recognized nor taught, it is called *will*. Will is, therefore, a linguistic product of a functional mind. Thus behavior is divided, itself, into two categories which can simply be indexed as "good" and "evil". Will is obtained as a learned process. No one is born free, nor is freedom something bestowed on an individual by another. Freedom is the unobstructed ability to do one's own work. One of the greatest obstructions to human freedom is the lack of education required to learn to perform one's biologically determined job; another is the personal interference from dysfunctional behavior of other individuals in one's environment; and a third is the physical inability of the mind, itself, to attain to that performance. Mankind is currently proto-linguistic but this does not negate the fact that these obstructions must be dealt with commensurately with the specific disability. This small essay and demonstration is educational.

Dialectic is functional as a craft. Like any craft, there are two functional parts. These parts are the material one works with and the processes upon that material in order to render a specific product. This craft can be recorded, retained in memory, as a *Two-Element Metaphysics*. Due to the principles of language, this *Two-Element Metaphysics* can be and have been put into a large number of metaphors such as the *Two-Witnesses of God*, or the *Two-Stone Tablets of Law* and even *The Theory of Forms*. As the mind is wholly linguistic by function, one would be committing a linguistic error and thus behavioral errors to conclude that these terms are either "*religious*" or "*philosophical*"; the grouping function of the mind is factual. Metaphor is used in the exercise of linguistic functionality.

As the mind is wholly linguistic by function, both elements of the crafting processes of the mind must be comprehended in terms of language. Thus language is factually divided into *logic* and *analogic*. analogic being the material and logic the form applied to that material. Again, one can view this pair of functions in terms of metaphor as the *Two Witnesses of God*, or as *The Two Tablets of Law*, primitively this conceptual pair are derived from the definition of a thing and this definition is exampled in living biology. As this pair maintain an image of a thing, language can be effected starting with either of them.

Definition: *A thing* is any *material* in some *form*.

Here we see a natural division in things themselves which establishes a crafting paradigm which is universal. The name of a thing is equal to the names of that things elements. One of those elements, form, is a container for the other, material. Even the word dialectic is constructed as a recognition of this division in language which is commensurate with biology.

It is living biology which is the aim of that product called human will, and learning human will entails the comprehension of all human behavior along with the ability to distinguish *good* and *evil* behavior. In short, good and evil is in respect to the product of the mind, human behavior. In this regard I will lay down a concise out line of human behaviors with which the mind functions to regulate in order to maintain and promote life.

In short, the mind is responsible for a product of human behavior that maintains and promotes the life of the body. It performs its job wholly through the artifice of language. Language itself is produced as standards in human behavior to maintain and promote life. Therefore, language is standards of human behavior that maintains and promotes life and is called Law. Law therefore is not the product of any individual or group of individuals, organizations, or so called governing bodies, Law is a derivative of the Law of Identity for the specific purpose of maintaining a particular image which can be put in a concise metaphor as I AM THAT I AM. It means that our life is obtained through the images provided by perception, or in other words by the functions of the seven life support systems of our own body.

The crafting systems of a biological organism can be comprehended as life support systems. Life support systems can be comprehended in terms of environmental acquisition systems of a living organism.

Environmental Acquisition Systems.

Life support systems which are obvious and which abstract from the environment.

Definition: An environmental acquisition system of a living organism is that system of an organism which must acquire from the environment an *element* from some *thing* and process that element which it has acquired for a product that maintains and promotes the life of that organism.

Those Systems that Acquire Material.

1) The Digestive-System.
2) The Manipulative-System.
3) The Respiratory-System.

Those Systems that Acquire Form.

4) The Ocular-System.
5) The Vestibular-System.
6) The Procreative-System.
7) The Judgmental-System.

That system, which can be called our *self*, is that system which is responsible for the behavior of the remaining systems through a function called judgment. Judgment determines behavior. Judgment is effected wholly through the artifice of language. Thus, one can list our responsibilities for the rendition of judgment according to the division exhibited in the definition of a thing.

The Self.

1) The Judgmental-System.

The human mind is wholly linguistic by function. Its product is behavior. The first order of behavior is language itself. Language is effected as standards of behavior. There are two branches of language:

a) **Analogic**. The application of forms to standard given material.
Examples are provided by:

1) The Digestive-System.
2) The Manipulative-System.
3) The Respiratory-System.

b) **Logic**. The application of materials to standard given forms.
Examples are provided by:

1) The Ocular-System.
2) The Vestibular-System.
3) The Procreative-System.

From the paradigm expressed in the definition of a thing and exampled in living biology, we learn the fundamentals of both logic and analogic to govern the behaviors specified by the function of our environmental acquisition

systems. We learn to do our own work by example. Refusing these examples only indicates the degree of mental dysfunction.

The Law of Identity.

The Law, or principle, of Identity is an expression of perception. Its specific expressions such as, *we learn by experience*, and *seeing is believing* are among them as well as *relation to self is inadmissible* and;

A equals A.

It also means that we, a biological life form, are not different from ourselves. Language is a biological function. Every functional part of a living organism functions in order to maintain and promote the life of that organism.

> "Everything which has a function exists for its function." On the Heavens, by Aristotle, W. D. Ross.

Therefore, the first place that one starts with, in the study of any language, is the desire to maintain and promote the relationship between language and survival. This desire to do our own work is our most fundamental ally to becoming functional.

The Paradigm.

Every environmental acquisition system of a living organism, functions by what it can acquire from the environment. It acquires abstractions from things and these abstractions are called a things *elements*. As the mind is responsible for judgment in relation to these things in the environment, the paradigm of judgment starts with the definition of a thing itself. This definition is a product of abstraction by a functional mind and establishes the *unit*, the *foundation*, the *first principle* of language and judgment. If one is not capable of making this abstraction, one cannot do their own work.

Definition: A thing is any material in any form.

In regard to the definition of a thing, neither *form*, nor *material* are, in of themselves, things. The part is not the whole.

From the definition of a thing, we acquire, commensurate with our own biology, two, and only two, primitive branches of language; logic commensurate with form and analogic commensurate with material. Also, by the definition of a thing, one of these languages must play the part of form while the other plays

the part of material difference. Also, by the definition of a thing, both branches of reasoning are paired which can only say one and the same thing. Also by definition, neither branch of reasoning is a branch of reasoning if the complementary language is not present—i.e., functionally resident in the mind.

Logics play the part of form. As such logics are seen as containers for the memory of experiences. One can also say that logics are indexing systems, or grouping systems, or again scripting systems for information retrieval and manipulation. The material indexed or contained is the material difference of the memory of perceptions. Memory can be said to be retained fragments of experience or perceptions. These experiences can be either perceptible or intelligible.

Analogics play the part of the material difference. Analogics are best learned through well ordered behaviors which include crafts. One such craft is called geometry.

Logics have the form as a given and the material for those forms must be supplied, while analogics have the material as a given and the form to those materials must be applied.

> 'A part' may be a part either of the form (i.e. of the essence), or of the compound of the form and the matter, or of the matter itself. Metaphysics by Aristotle.

Repairing this translation for accuracy of statement:

'A part' is ether *form*, or the *material* in that *form* of a compound or composite called *a thing.*

> "Therefore one kind of substance can be defined and formulated, i.e. the composite kind, whether it be perceptible or intelligible; but the primary parts of which this consists cannot be defined, since a definitory formula predicates some thing of some thing, and therefore, one part of the definition must play the part of matter and the other that of form." Metaphysics by Aristotle. W. D. Ross.

Things are thus defined in terms of that things *parts*, or *elements*, but neither of these parts can be defined as they are not things; one can only name them.

The method that Plato used to make his reader aware of Aristotle's second statement was as a demonstration in psychotherapy by dialog.

> "SOCRATES: Let me give you, then, a dream in return for a dream:—Methought that I too had a dream, and I heard in my dream that the primeval letters or elements out of which you and I and all other things are compounded, have no reason or explanation; you can only name them, but no predicate can be either affirmed or denied of them, for in the one case existence, in the other non-existence is already implied, neither of which must be added, if you mean to speak of this or

> that thing by itself alone. It should not be called itself, or that, or each, or alone, or this, or the like; for these go about everywhere and are applied to all things, but are distinct from them; whereas, if the first elements could be described, and had a definition of their own, they would be spoken of apart from all else. But none of these primeval elements can be defined; they can only be named, for they have nothing but a name, and the things which are compounded of them, as they are complex, are expressed by a combination of names, for the combination of names is the essence of a definition. Thus, then, the elements or letters are only objects of perception, and cannot be defined or known; but the syllables or combinations of them are known and expressed, and are apprehended by true opinion. When, therefore, any one forms the true opinion of anything without rational explanation, you may say that his mind is truly exercised, but has no knowledge; for he who cannot give and receive a reason for a thing, has no knowledge of that thing; but when he adds rational explanation, then, he is perfected in knowledge and may be all that I have been denying of him. Was that the form in which the dream appeared to you?" *Theætetus*, by Plato, Jowett.

One may be confused as to why Socrates stated that you and I and all things are composed of letters—Plato was very aware that analog information was indeed a language. As the mind is wholly linguistic by function, it can only conceive all information via language itself. What can be determined is that many presentations to teach mankind this concept historically failed. That this would be so is recorded in the Judeo-Christian Scripture. There is no amount of teaching which can change the physical development of the mind itself.

By recursion of the paradigm for a *container* or *form*, *definition* then demonstrates the equality between the name of a thing which contains as a compound of the elements of form and matter which construct that thing by listing names of those elements. Thus we get the *Theory of Forms*, and why Plato was fixated on *definition*. Truth, at the foundation of language is achieved by maintaining the biological image as a convention of names, this is the whole of the correct process of both virtual construction and deconstruction.

Definition is a linguistic convention preserving the identity between the name of a thing and the names of the elements which comprise that thing. It is effected by maintaining then the equality between the name of a thing and the names of that things forms and the names of the materials contained in those forms.

Definition is a standard of individual and social behavior that is critical to the mind's ability to do its own work. The importance of definition, as an image, cannot be overstressed. A mind, or a collection of minds, incapable of this standard of behavior cannot be said to be linguistic, at best it can only be said to be proto-linguistic and fundamentally savage.

This is why the *Theory of Forms* is not a theory at all, but a factual observation concerning the foundation of language.

The fact that all of language resolves to this convention of names also determines what a proof is, and what it functions to do, to resolve a group of words back to the original naming convention which is based on perception. It also determines why we parse statements and parse them to isolate the individual assertions and denials contained in a sentence. Parsing is aimed at examining a statement for the compliance with the original naming convention also. Parsing is therefore part of the proofing process. This convention of names also determines that truth is the compliance with this convention. Naming a thing does not change a thing, but naming is part of the virtualization of experience.

As every environmental acquisition system of a living organism crafts from things to make other things, the paradigm of a thing itself will be used recursively, as a unit of conceptualization in the development of our understanding of language. In the short of it, the psychology of a linguistic species is founded upon a *Two-Element Metaphysics*, or in terms of the product of human behavior, *Two Tablets of Law*, or in other words, *definition* which is functional, by biological fact, only through perception. We learn, like anything else in reality, by experience.

Some important facts to take away from the paradigm. The form is not, nor ever can be the material difference, nor can the material difference ever be the form. Whether or not we use the paradigm as the basis for a name or for an entire system of reasoning, this one fact remains, it cannot exist as a thing in of itself. It takes both, the container and the contained, the form and the material to make any thing and everything—even in the realm of language. One may see this as a simple concept, however a proto-linguistic mind cannot comprehend it enough to effect behavior in accordance with it. A proto-linguistic mind cannot think in accordance with definition.

Within a proto-linguistic species there will always be individuals doing their best to establish another animal, real or imagined, as the standard for human behavior as a means of asserting what they themselves simply desire. They really have no other option than to rely on someone else to set their standards of behavior. This means that in terms of psychology, the psychological foundation of a proto-linguistic species rests upon the self-referential fallacy, or again, ignorance. This is the source of all savage behavior. It also indicates that all governing bodies today are either potentially or actually savage by nature.

Logic.

There is a metaphor for the fact that language, i.e. Law is based on two branches. The most widely recognized is the *Two Stone Tablets of Law*, which would one day be revealed to man again. Another is in *Set Theory*, that a set can be constructed either through definition which is commensurate with form

or by enumeration which is commensurate with material. Or again as the container and the contained.

Logic is based on form; this is why Plato's so called *Theory of Forms* can also be called the *Theory of Definition*; this is why, in some dialogs, the question was ask if the topic were a thing or not. There is no amount of discourse which can define an element of a thing. The elements of things can only be known by direct perception, perceptible or intelligible, to which an agreed upon name is given to that abstraction. All we can do is name an abstraction, and all language functions to maintain that correspondence.

Definition: A thing is any material in any form.

Categories of Names.

From the definition of a thing, we find that we have three, and only three primitive categories of names. We can name a thing directly and we can name a thing, as Plato pointed out, by a combination of the names for a thing's elements, those elements being a thing's material and the forms within which those materials reside.

Parts of Speech.

These three categories define the parts of speech. Just like an algebraic equation, they are added together as building blocks for more complex statements. *Parts of Speech* begins with the recognition of two fundamental naming conventions. The parts of speech not only include the naming convention for things, but the convention of names for the operations upon those names. For example, in assertion and denial, the words "is" and "is not" are operands. Assertion and denial are operations performed with names. Thus one does not fault the names for an incorrect operation, one faults the operator. Operands are directed towards the operator, or that system which is manipulating the conventions used in naming.

Naming Conventions.

From the definition of a thing, we find that we have two distinct naming conventions for logic. We can name a thing directly, which is called the *Subject Naming Convention*, and we can name a thing as a combination of the names of a thing's materials and the names of the forms which contain those materials. This second convention is called the *Predicate Naming Convention*.

The convention of naming things, itself has two categories commensurate with form and material. Some names are standards while others are named *in situ*. Groups of names have been established for assignment *in situ*. Many of those are based on the physical act of pointing to an object from which the

abstraction is to be made. These pointers also can point to the perceptible and the intelligible.

One of the mistakes made in logic is using pointers which cannot, do not, point to a shared abstractable, and again perceptible or intelligible.

Assertion and Denial.

Going back to the naming convention itself, assertion and denial turn out to be operands, the first operand in regard to the naming convention, establishing that convention itself. When used in context it becomes a proposition. One can proof this proposition by referring back to the original convention of names, or again the definition.

Assertion is commensurate with the paradigm of form, meaning no difference. Denial is commensurate with the paradigm of material, meaning some difference. The explicit coordinate system of reference, i.e. environmental acquisition system, for comparison for either of these is not in the words.

When we assign names to abstractions, we are establishing a standard behavioral system. Our commitment to doing our own work can be measured by the effort we expend in maintaining that convention as a one-to-one correspondence, or again truth. What is trying to be achieved is a one-to-one correspondence between an abstractable and a name. When we maintain that correspondence is what is meant by truth.

1) Truth is the state of being true between two or more things.
2) True is a lack of difference between two or more things.

Language is functional an intelligible equality. To a simple mind, it is imagined to be mystical.

That state of functionality for a mind called truth is then measured in terms of the maintenance and preservation of the correspondence between a name and the abstraction named, or again by the maintenance and preservation of the naming convention; or again, definition. This is why it was written that we shall know the truth and the truth shall set us free. It is a simple biological fact.

When we name we are not naming our abstractions. We are naming the source of abstraction. As we are not the standard for a name, nor is it possible to standardize the abstracting systems, standardization is afforded by things in the environment itself. These are sometimes established through bureaus of standards. When man becomes functional, this bureau of standards will imply the entire ecosystem of man, including his own behavior, man is destined to have dominion over the earth, a dominion aimed at maintaining and promoting a life supporting ecosystem.

One of the most common fallacies in trying to negate the efficacy of words is by claiming that the name of a thing is referencing a particular persons abstraction, or again, “man is the measure of all things.” It is a simple minded argument which has apparently mastered many so called intellectuals and it is this fallacy which denotes a savage proto-linguistic species.

When we make an assertion we are saying that two names by their respective conventions of names are equal by some means of comparison. When we make a denial, we are saying that two names are not equal and again by some means of comparison. The means of comparison is often implied, i.e. fundamental to an equation is often an ellipsis for the system of comparison itself. It is often taken for granted that as a multi-sensing organism that one can locate the system used by the names given itself to complete the meaning of an assertion or denial. This aids in economy, but detracts from those without what is called “common sense.” Sometimes those without “common sense” can be corrected by affording them the opportunity to participate in the naming convention; this means pointing them to something commonly sensed from which they may make an abstraction; one can afford someone the opportunity, but not the ability. Higher level of processing involves metaphor. As fundamental to language is the ability to abstract the similar concept from many examples, these functions are tested through metaphor. Metaphor tests the range of one’s ability to depend upon definition.

Locating the senses systems used to make either an assertion or denial is a requirement of the indexing system of the mind.

Unit Sentences.

Combining the naming conventions with assertion and denial, provide us with unit sentences.

Subject is or is not Subject.

Subject is or is not Predicates.

Predicate equals Predicate.

Thus not every sentence has both subject and predicates. Notice that in these statements the use of both singular and plural for the *Predicate Naming Convention.*

Parsing.

Parsing a statement means to break that statement down to all of its expressed assertions and denials.

As every thing is crafted by bringing together material and form, we craft knowledge of things through combinations of a things predicates. The content of predicates come from direct perception; in regard to metaphor, one must not forget that the mind is a environmental acquisition system in its own right and therefore its predicates are, more often than not, intelligibles. We parse a statement in order to proof it, at least to the limit of our ability. One should never expect that the analog domain is equivalent in individuals concerning any statement.

Proofing.

Proofing involves the products of parsing for determining both the authors compliance with the original naming convention and testing one's facility in using the indexing system. An assertion or denial is true if it complies with the original naming convention, false if it is not. Proofing does not exclude indexing based on the principles of the naming convention at all, which means that proofing also tests one's ability to following the indexing system through what is called metaphor.

It is not language when the original naming convention is not complied with. It is not comprehensible to a reader when they cannot follow the indexing systems in play. In other words, every proof actually proves the author of the words for that authors linguistic integrity while simultaneously proves the linguistic skill of the reader.

Currently what is called popular education leaves the student product with a false sense security by believing that they understand even common grammar.

Often one does not have the analog references to effect a proof, in this case, the parsing products indicate which analog information one has to acquire. This limitation is why proofing is part and parcel of formal presentation which provide both branches of reasoning in parallel.

As all we can do is assign names to the elements via abstraction, all that proof can do is check that the naming convention has, indeed, been complied with. In order to parse correctly, one has to have a level of ability in using the indexing system itself along with sufficient analog memory required by the naming convention to perform that task.

Maintenance and Preservation of the Naming Convention.

Again, commensurate with the paradigm of a thing, there are two methods of maintaining and preserving the naming convention.

Form: Definition:

Definition is equating the name of a thing to the names of that things materials and the names of the forms which contain those materials. As the mind is aimed at standards of behavior, a standard definition is then the preservation of the naming convention through preserving the assertions that equate the *Subject Naming Convention* to the *Predicate Naming Convention* in regard to the names of any perceptible and of any intelligible thing.

As mankind is currently proto-linguistic, this process is left to writers of dictionaries who still do not know the foundation of language, nor the difference between a definition and a description. Writers of dictionaries also think it is an improvement in logic that names "grow" by changing their indexing references. Anyone in a right frame of mind does not think it is a growth of language to scramble and obfuscate the indexes.

Material: Description:

Descriptions are directions for locating a thing from which an abstraction may be made. Locating a thing can even entail directions for constructing that thing. Descriptions also involve direct experience being pointed out by someone else. By definition the acquisition of analog memory is required for linguistic functionality.

Throughout history many have confused a description with a definition, and thought they were arguing over definition, when in fact they were arguing over descriptions. The only thing provable by this process is that these persons could not even attain to the first principle of language itself, a convention of names.

The Perceptible and the Intelligible.

The perceptible are things directly observable within the environment. The intelligible are things constructed within the mind on the same paradigm of any thing. Intelligibles are constructed by a complex combination of perceptibles. Given the time and effort, every intelligible can be parsed back to primary perceptions; thus, even intelligibles have for their origin, perceptible reality. These intelligibles can be proofed by a linguistic process demonstrated, for example, in the works of Euclid such as the *Elements*.

What I have outlined here are basic elements for logic in general. This outline also exhibits a fact in regard to mankind's current place in the process of becoming functional as there has yet to be written a correct book of elementary grammar.

Parsing demonstrates, in logic, that there is only one error possible, that of assertion and denial in reference to the original naming convention upon which a logic resides.

All of logic resolves to yea and nay, in reference only to the original naming convention itself. There never has been anything in logic which would suggest that as we start a logic by naming abstractions, that the results could be anything more than maintaining that original process. In other words, maintaining a simple human behavior. If one cannot do the simple, one certainly cannot do the complex. As names neither adds to, nor subtracts from, that which is named, proofing only insures that we, the user of the indexing system have kept our word. If we cannot give and keep our word, how is it that one can claim that we are linguistic?

Analogic.

Analogics are based on material difference. What is added to make something is simply the application of form. Thus, human industry, human will, is the product of analogic.

It is on this wise that the complex customs developed by Confucius was aimed at linguistic functionality. The tie between language and human behavior has always been the goal of a true philosopher, a true prophet, and even true philosophy and true religion—all of them true craftsmen.

> "Tsze-lu said, "The ruler of Wei has been waiting for you, in order with you to administer the government. What will you consider the first thing to be done?"
>
> The Master replied, "What is necessary is to rectify names."
>
> "So! indeed!" said Tsze-lu. "You are wide of the mark! Why must there be such a rectification?"
>
> The Master said, "How uncultivated you are, Yu! A superior man, in regard to what he does not know, shows a cautious reserve.
>
> "If names be not correct, language is not in accordance with the truth of things. If language be not in accordance with the truth of things, affairs cannot be carried on to success.
>
> "When affairs cannot be carried on to success, proprieties and music do not flourish. When proprieties and music do not flourish, punishments will not be properly awarded. When punishments are not properly awarded, the people do not know how to move hand or foot.
>
> "Therefore, a superior man considers it necessary that the names he uses may be spoken appropriately, and also that what he speaks may be carried out appropriately. What the superior man requires is just that, in his words there may be nothing incorrect." *Analects* by Confucius

In regard to analogic as a written language, we have what is called Geometry. Along with the logic of Algebra, one has a formal pairing used in elementary teaching of dialectic. As all that one can do is apply form to material in an analog language, the word geometry can be used metaphorically

for standards of all behaviors of man. The difference in analog language is that concepts are presented as part of the language itself. As the mind of most cannot comply with even the principles of logic, analog presents a greater challenge. One can write an equation analogically that simply will not register at all to a simple mind, they cannot abstract the concepts of what they are perceiving. This is why mankind has not even suspected that lucid dreaming is an analog language.

It is a very naive and simple mind that imagines that one can have many contradicting geometries as a written language, as if form, which is not a difference, is a difference. This only denotes how dysfunctional the mind of man is.

Some of the greatest mistakes in the analogic of geometry is confusing the process of writing the language with the language itself and confusing the writing materials with the grammar itself. The foundation for all languages is simply form and material, and, for linguistic purposes, standards in producing them in the language. Those who formulate non-Euclidean Geometries have been, and are too simple to even notice that at every turn, they violated the principles of language itself. The known parallel between the logic of mathematics and the analog figures in geometry was insufficient for them to realize that if one imagines the corruption of geometry, then because of the parallel, they eradicated any recourse to mathematics. One can, however, congratulate them on their consistency in not seeing contradiction after contradiction. A proto-linguistic mind cannot function in accordance with the principles of language.

A Duplicate Ratio 062092

Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.

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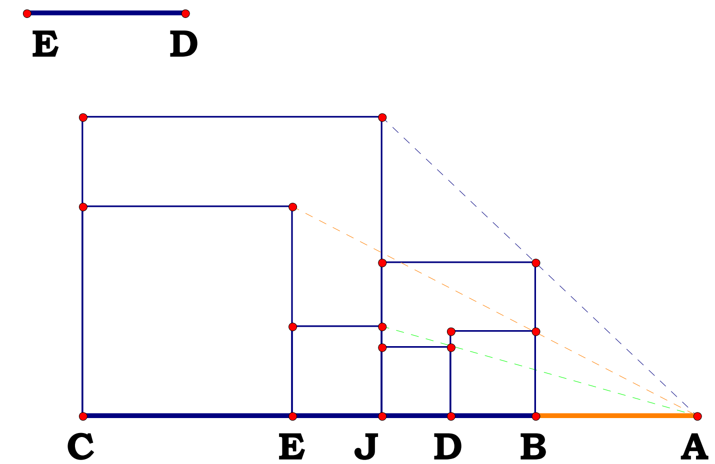
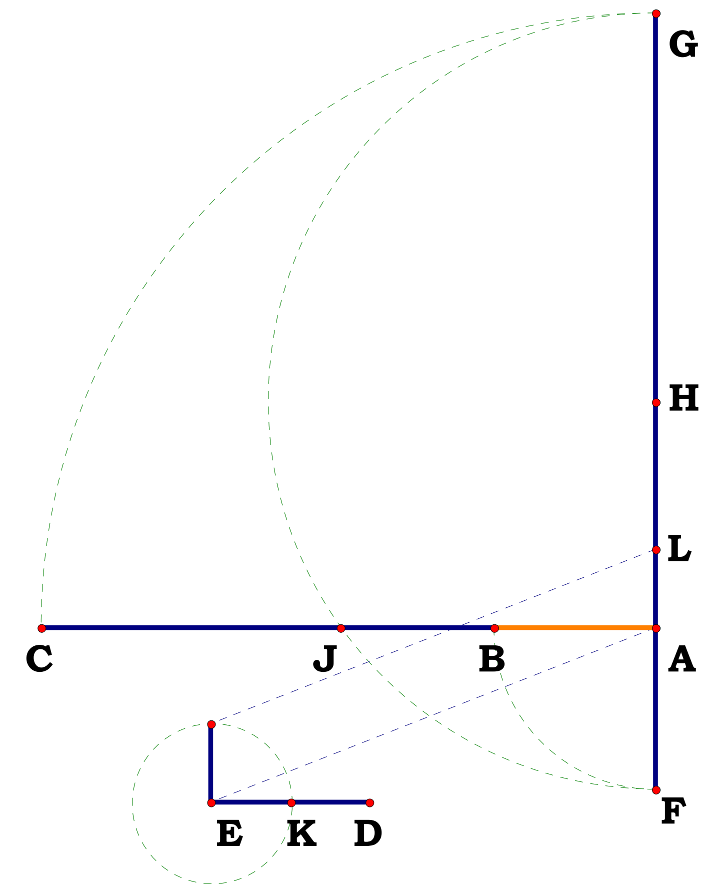
$N_1 := 5.99017$

$N_2 := 2.09550$

$BC := N_1 \qquad DE := N_2$





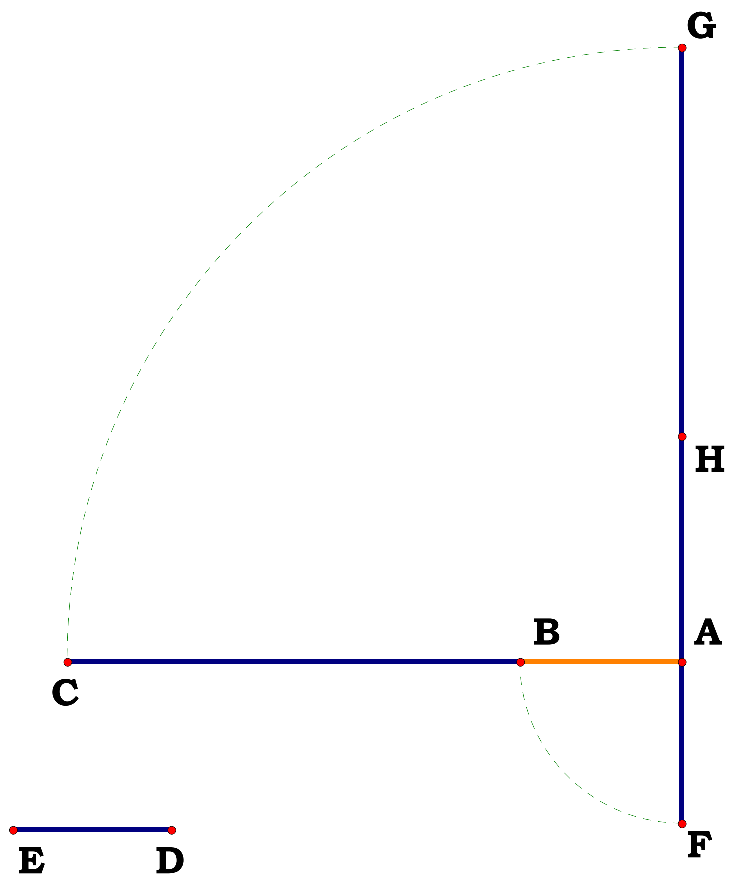
$AC := AB + BC$

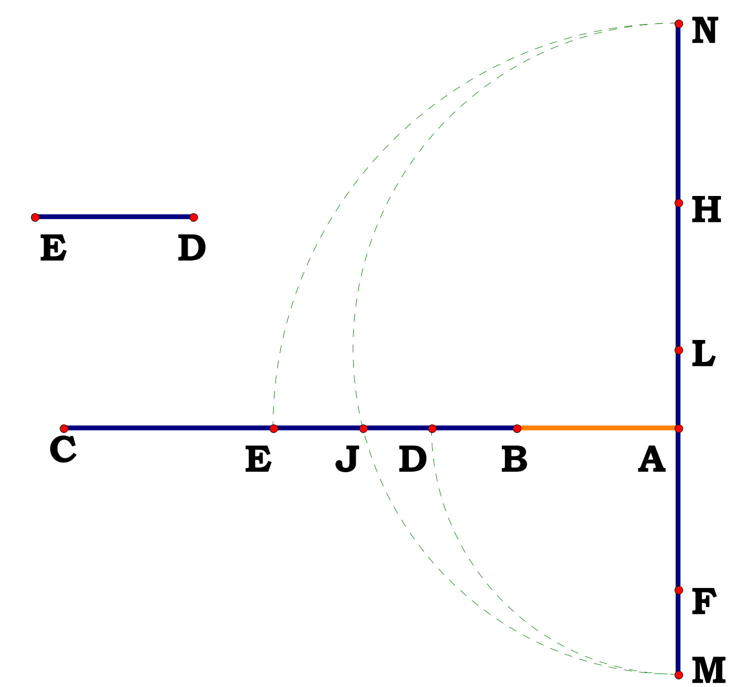
$AF := AB \qquad AG := AC \qquad AJ := \sqrt{AF \cdot AG}$

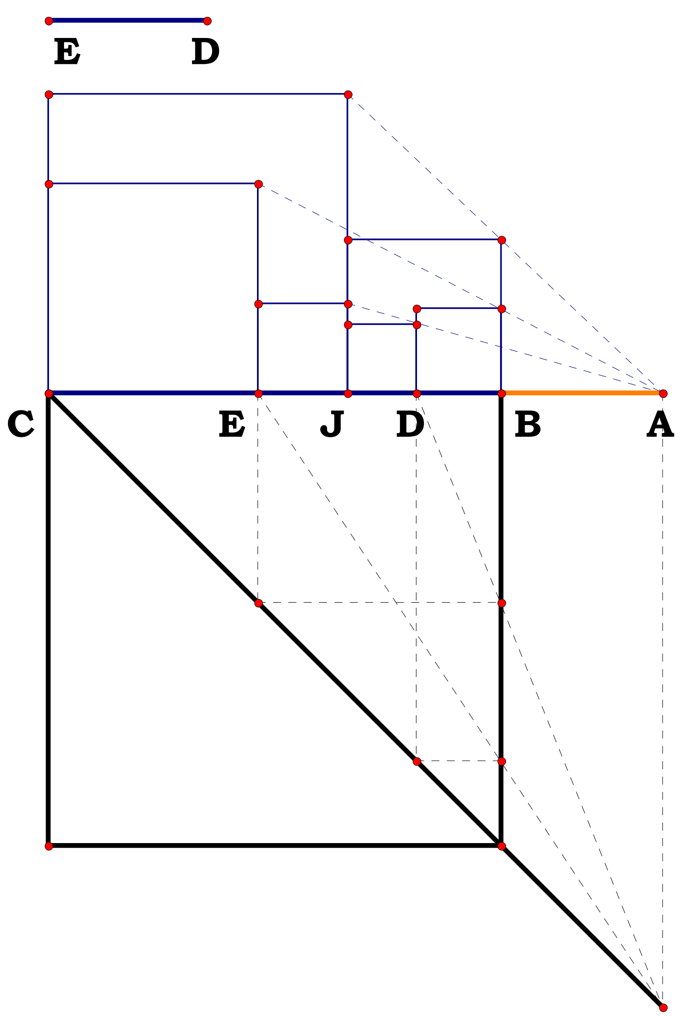
$AL := \dfrac{DE}{2} \qquad FL := AF + AL \qquad JL := \sqrt{AJ^2 + AL^2}$

$AD := JL - AL \qquad AE := JL + AL$

$\dfrac{AB}{AD} - \dfrac{AE}{AC} = 0 \qquad \dfrac{AD}{AJ} - \dfrac{AJ}{AE} = 0 \qquad \dfrac{AB}{AJ} - \dfrac{AJ}{AC} = 0$







Definitions

$$AC - \left(N_1 + 1\right) = 0$$

$$AF - 1 = 0 \qquad AG - \left(N_1 + 1\right) = 0 \qquad AJ - \sqrt{\left(N_1 + 1\right)} = 0$$

$$AL - \frac{N_2}{2} = 0 \qquad FL - \frac{N_2 + 2}{2} = 0 \qquad JL - \sqrt{\frac{N_2^{\,2}}{4} + N_1 + 1} = 0$$

$$AD - \frac{\sqrt{N_2^{\,2} + 4 \cdot N_1 + 4} - N_2}{2} = 0 \qquad AE - \frac{\sqrt{N_2^{\,2} + 4 \cdot N_1 + 4} + N_2}{2} = 0$$

$$-\frac{2}{N_2 - \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}} - \frac{N_2 + \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}}{2 \cdot \left(N_1 + 1\right)} = 0 \qquad \frac{AB}{AD} - \frac{AE}{AC} = 0$$

$$-\frac{N_2 - \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}}{2 \cdot \sqrt{N_1 + 1}} - \frac{2 \cdot \sqrt{N_1 + 1}}{N_2 + \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}} = 0 \qquad \frac{AD}{AJ} - \frac{AJ}{AE} = 0$$

$$\frac{1}{\sqrt{\left(N_1 + 1\right)}} - \left(N_1 + 1\right)^{\frac{-1}{2}} = 0 \qquad \frac{AB}{AJ} - \frac{AJ}{AC} = 0$$

$$\sqrt{N_1 + 1} - AJ = 0$$

081292 Rusty Cube of a Sphere

Given AB, how close is BJ to the cube root of AB taken as a sphere?

$$N_1 := 4 \qquad CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N_1{}^3\right)^{\frac{1}{3}}$$

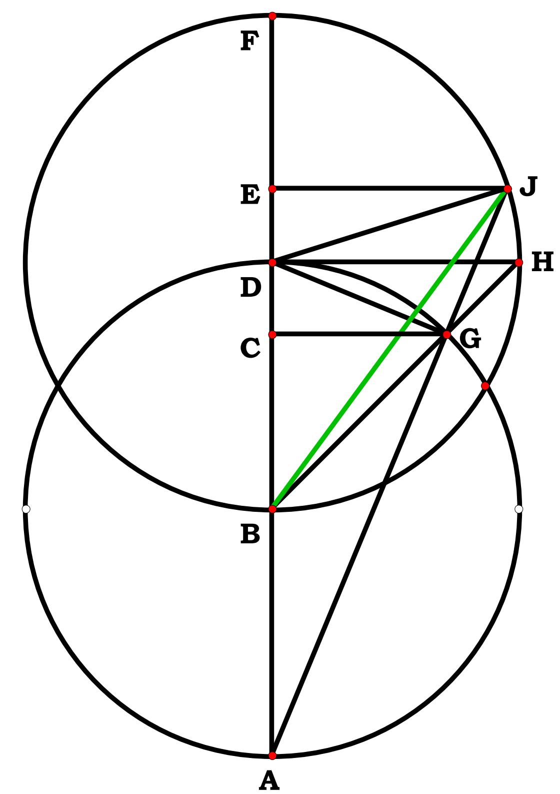
$$AB := N_1 \qquad BH := \sqrt{2\cdot AB^2} \qquad CG := \frac{AB^2}{BH}$$

$$AG := \sqrt{CG^2 + (AB + CG)^2} \qquad DG := CG\cdot\frac{2AB}{AG}$$

$$GJ := \sqrt{AB^2 - DG^2} \qquad AE := \frac{(AB + CG)\cdot(AG + GJ)}{AG}$$

$$EJ := \frac{CG\cdot AE}{AB + CG} \qquad BJ := \sqrt{EJ^2 + (AE - AB)^2}$$

$$\frac{BJ}{\left(\frac{4}{3}\cdot\pi\cdot N_1{}^3\right)^{\frac{1}{3}}} = 1.000943 \qquad BJ - \left(\frac{4}{3}\cdot\pi\cdot N_1{}^3\right)^{\frac{1}{3}} = 0.00608$$

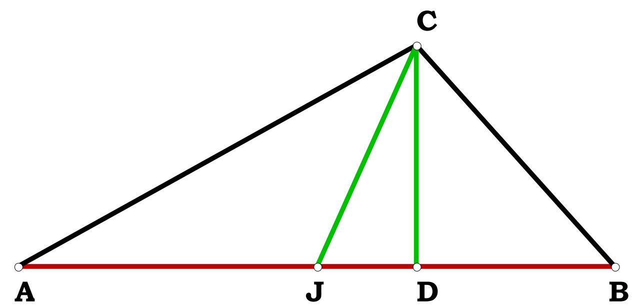


Definition

$$BJ - N_1\cdot\sqrt{\sqrt{2} + 2^{\frac{1}{4}}} = 0$$

Pythagoras Revisited 010893

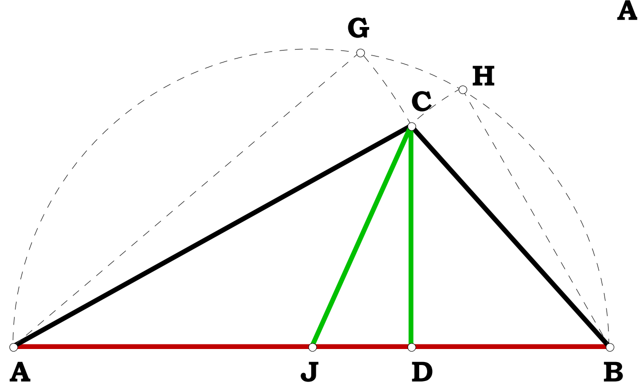
Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.



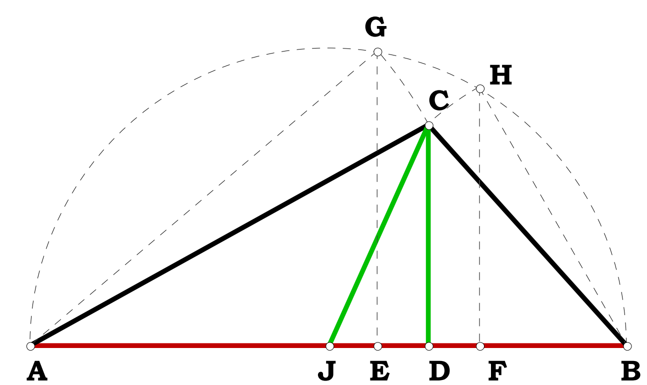
$AB := 7.89517$ $\quad S_1 := AB$

$AC := 6.02581$ $\quad S_2 := AC$

$BC := 3.92697$ $\quad S_3 := BC$



$$AE := \frac{S_2^{\,2}}{S_1} \qquad BF := \frac{S_3^{\,2}}{S_1} \qquad EF := AB - (AE + BF) \qquad DE := \frac{EF}{2} \qquad AD := AE + DE$$



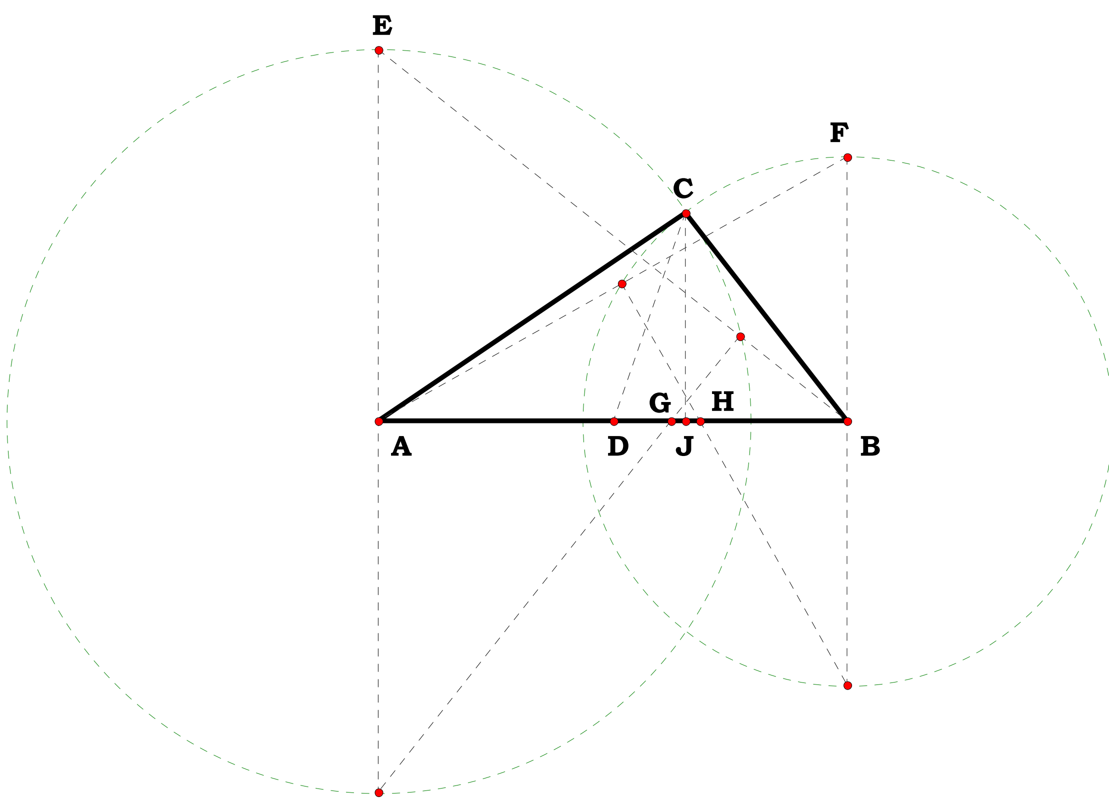
$$BD := AB - AD \qquad CD := \sqrt{S_2^{\,2} - AD^2} \qquad AJ := \frac{S_1}{2} \qquad DJ := AD - AJ \qquad CJ := \sqrt{CD^2 + DJ^2}$$

$$EF - \frac{S_1^{\,2} - S_2^{\,2} - S_3^{\,2}}{S_1} = 0 \qquad DE - \frac{S_1^{\,2} - S_2^{\,2} - S_3^{\,2}}{2 \cdot S_1} = 0 \qquad AD - \frac{S_1^{\,2} + S_2^{\,2} - S_3^{\,2}}{2 \cdot S_1} = 0 \qquad BD - \frac{S_1^{\,2} - S_2^{\,2} + S_3^{\,2}}{2 \cdot S_1} = 0$$

$$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0 \qquad DJ - \frac{\sqrt{\left(S_2^{\,2} - S_3^{\,2}\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot S_2^{\,2} - S_1^{\,2} + 2 \cdot S_3^{\,2}}}{2} = 0$$

08092015 Pythagoras Revisited Again!

One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.



$$AB := 6.20183 \quad AC := 4.89358 \quad BC := 9.20468$$

$$GA := \frac{AC^2}{AB} \quad HB := \frac{BC^2}{AB} \quad GH := AB - (GA + HB) \quad JA := GA + \frac{GH}{2}$$

$$JB := HB + \frac{GH}{2} \quad CJ := \sqrt{AC^2 - JA^2} \quad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$$

$$JA - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \qquad JB - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

$$CJ - \frac{\sqrt{(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)}}{2 \cdot AB} = 0$$

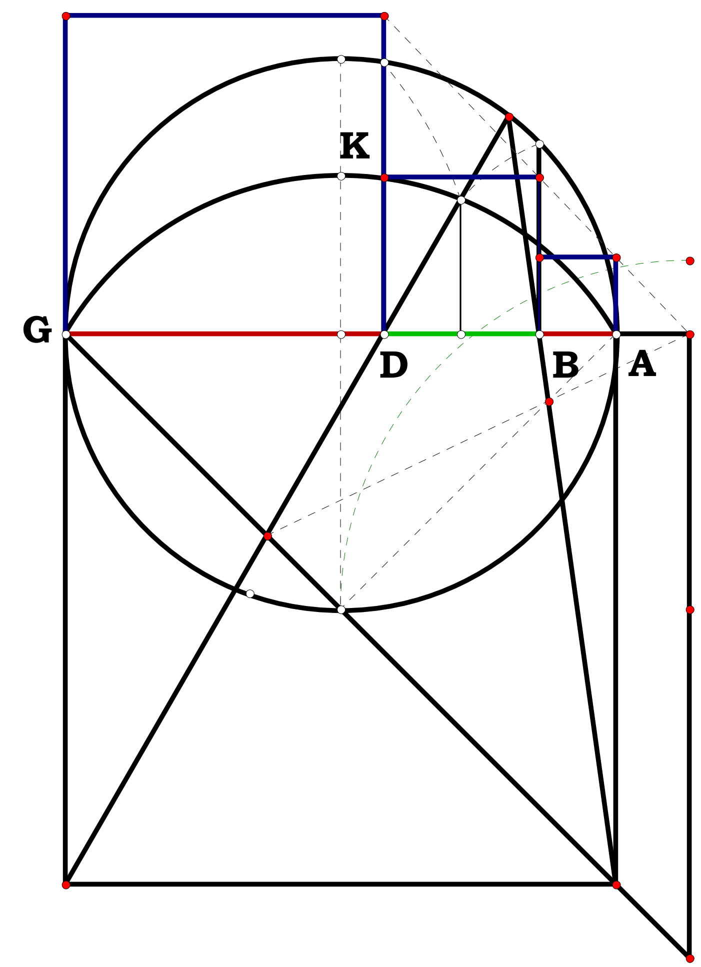
Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.

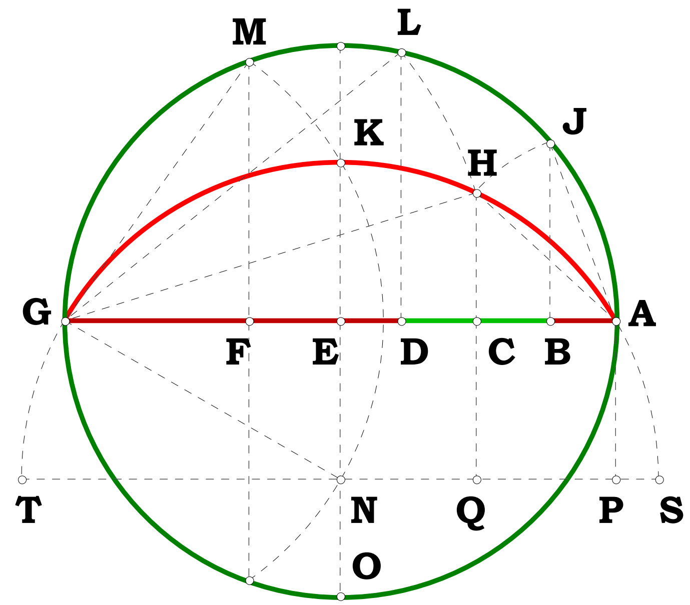
$$AJ := \sqrt{AC^2 - CJ^2} \quad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \quad BJ := \sqrt{BC^2 - CJ^2} \quad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2 \cdot AB} = 0$$

060393 Exploring The Curve AK

The curve AK is derived from the cube root figure as demonstrated.

Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?
Divide a segment twice such that the mean segment is the root of the extreems.





Givens: $N_1 := 3$ $\quad N_2 := 4$ $\quad AG := N_1$ $\quad AC := \frac{AG}{N_2}$

$GF := \frac{AG}{3}$ $\quad FM := \sqrt{GF \cdot (AG - GF)}$

$GM := \sqrt{GF^2 + FM^2}$ $\quad ST := 2 \cdot GM$ $\quad EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$

$PS := \frac{ST - AG}{2}$ $\quad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$

$CH := HQ - EN$ $\quad AH := \sqrt{AC^2 + CH^2}$ $\quad GH := \sqrt{(AG - AC)^2 + CH^2}$

$AB := \frac{AH^2}{AG}$ $\quad DG := \frac{GH^2}{AG}$ $\quad BD := AG - (AB + DG)$

$BD - \sqrt{AB \cdot DG} = 0$ $\quad AB = 0.349$ $\quad BD = 0.803$ $\quad DG = 1.849$

Definitions:

$AC - \frac{N_1}{N_2} = 0$ $\quad GF - \frac{N_1}{3} = 0$ $\quad FM - \frac{\sqrt{2} \cdot N_1}{3} = 0$ $\quad GM - \frac{\sqrt{3} \cdot N_1}{3} = 0$ $\quad ST - \frac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0$ $\quad EN - \frac{N_1}{\sqrt{12}} = 0$

$PS - \frac{N_1 \cdot (2 \cdot \sqrt{3} - 3)}{6} = 0$ $\quad HQ - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12}}{N_2 \cdot \sqrt{12}} = 0$ $\quad CH - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0$ $\quad AH - \frac{N_1 \cdot \sqrt{N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6}}{\sqrt{6 \cdot N_2}} = 0$

$GH - \frac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0$ $\quad AB - \frac{N_1 \cdot \left(N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6\right)}{6 \cdot N_2} = 0$ $\quad DG - \frac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{6 \cdot N_2} = 0$

$BD - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0$ $\quad BD^2 - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0$ $\quad AB \cdot DG - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0$

AG = 2.86667 in.
AC = 0.74179 in.
GF = 0.95556 in.
FM = 1.35136 in.
GM = 1.65507 in.
ST = 3.31014 in.
EN = 0.82754 in.
PS = 0.22174 in.
HQ = 1.50367 in.
CH = 0.67614 in.
AH = 1.00370 in.
GH = 2.22986 in.
AB = 0.35142 in.
BD = 0.78074 in.
DG = 1.73451 in.

$\frac{AG}{AC} = 3.86452$

$N_1 = 2.86667$ in.

$N_2 = 3.86452$

$\frac{AG}{N_2} = 0.74179$ in.

$AC - \frac{N_1}{N_2} = 0.00000$ in.

$GF - \frac{N_1}{3} = 0.00000$ in.

$FM - \frac{\sqrt{2} \cdot N_1}{3} = 0.00000$ in.

$GM - \frac{\sqrt{3} \cdot N_1}{3} = 0.00000$ in.

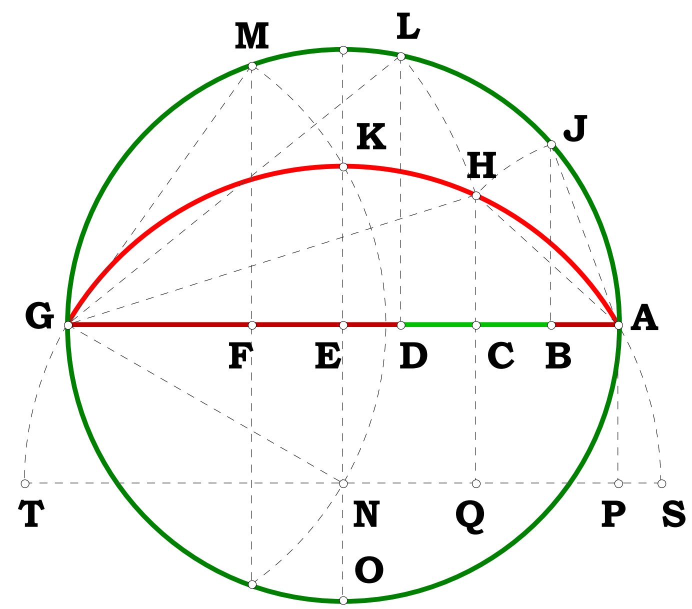
$ST - \frac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0.00000$ in.

$EN - \frac{N_1}{\sqrt{12}} = 0.00000$ in.

$PS - \frac{N_1 \cdot (2 \cdot \sqrt{3} - 3)}{6} = 0.00000$ in.

$HQ - \frac{N_1 \cdot \sqrt{(N_2^2 + 12 \cdot N_2) - 12}}{N_2 \cdot \sqrt{12}} = 0.00000$ in.

$CH - \frac{N_1 \cdot \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0.00000$ in.



$AH - \frac{N_1 \cdot \sqrt{\left(N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12}\right) + 6}}{\sqrt{6 \cdot N_2}} = 0.00000$ in.

$GH - \frac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0.00000$ in.

$AB - \frac{N_1 \cdot \left(\left(N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12}\right) + 6\right)}{6 \cdot N_2} = 0.00000$ in.

$DG - \frac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - 6\right)}{6 \cdot N_2} = 0.00000$ in.

$BD - \frac{N_1 \cdot \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0.00000$ in.

060793 Two Triangles with a Common Side.

Given two triangles with a common side, find the difference between their free vertices when the triangles do not intersect and when they do intersect.

Let the two triangles ABD and ACD be given.

$AD := 2.17506 \quad AB := 3.14654 \quad AC := 1.74732$

$BD := 2.61333 \quad CD := 1.38168$

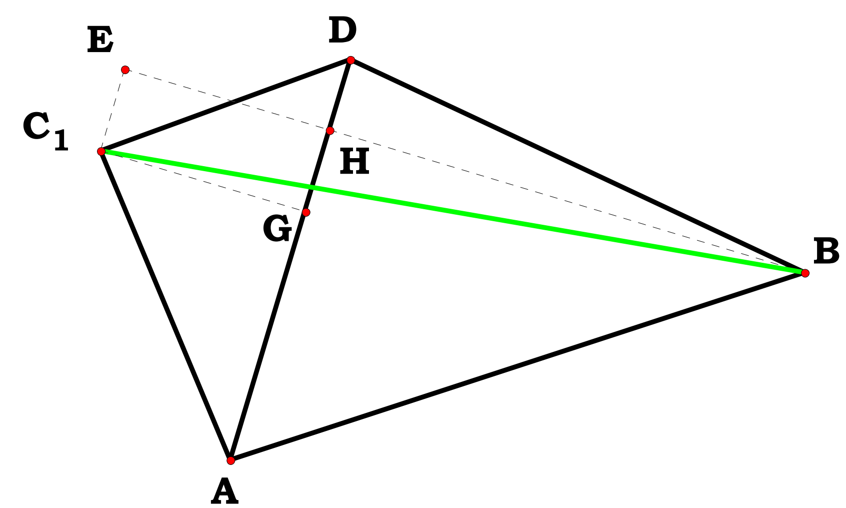
$$CG := \frac{\sqrt{(AD+CD+AC)\cdot(-AD+CD+AC)\cdot(AD-CD+AC)(AD+CD-AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD+AB+BD)\cdot(-AD+AB+BD)\cdot(AD-AB+BD)(AD+AB-BD)}}{2\cdot AD}$$

$$AG := \frac{AD^2+AC^2-CD^2}{2\cdot AD} \qquad AH := \frac{AD^2+AB^2-BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

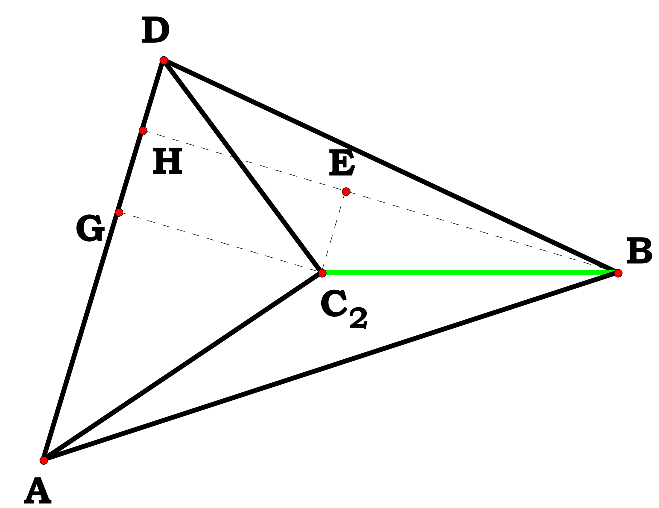
Non-intersecting:



$$BC_1 := \sqrt{GH^2+(CG+BH)^2}$$

$$BC_2 := \sqrt{GH^2+(BH-CG)^2}$$

Intersecting:



$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(AB+AD-BD)\cdot(AB-AD+BD)\cdot(AD-AB+BD)\cdot(AB+AD+BD)}\cdot\sqrt{(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}\ \ldots + -AD^4-AB^2\cdot AC^2+AB^2\cdot AD^2+AC^2\cdot AD^2+AC^2\cdot BD^2+AD^2\cdot BD^2+AB^2\cdot CD^2+AD^2\cdot CD^2-BD^2\cdot CD^2}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2+AC^2\cdot BD^2+AD^2\cdot BD^2+AB^2\cdot CD^2+AD^2\cdot CD^2-BD^2\cdot CD^2+AB^2\cdot AD^2-AD^4-AB^2\cdot AC^2\ \ldots + -\sqrt{(AB+AD-BD)\cdot(AB-AD+BD)\cdot(AD-AB+BD)\cdot(AB+AD+BD)}\cdot\sqrt{(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}}}{2\cdot AD} = 0$$

060993 Rectangular Roots.

Given any value N_1, any other value, N_2, greater than twice the square root of N_1 can be divided such that the resulting pair of values equals N_1.

Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.

$\mathbf{AB := 11}$

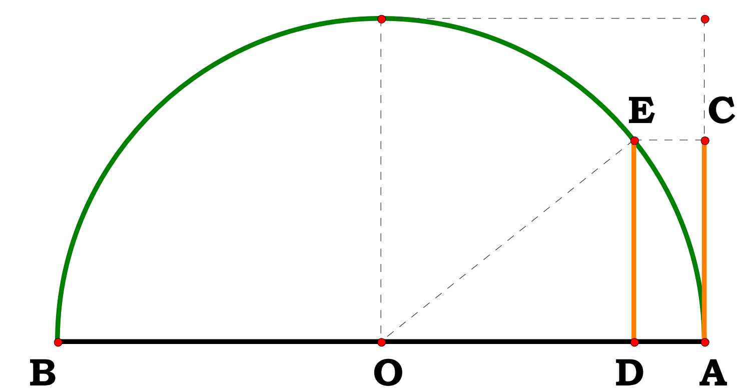
$\mathbf{AC := 5}$

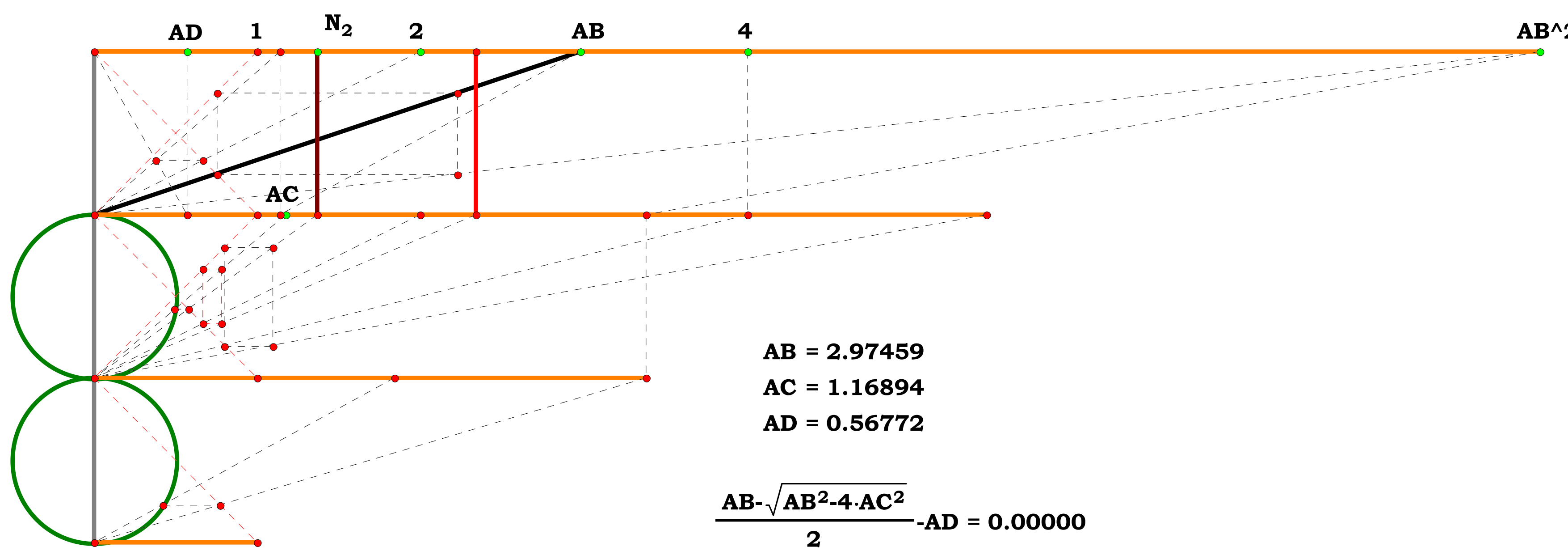
$$DE := AC \quad EO := \frac{AB}{2} \quad DO := \sqrt{EO^2 - DE^2} \quad BD := EO + DO$$

$$AD := AB - BD \quad AD \cdot BD - AC^2 = 0$$

$$AD - \frac{AB - \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \qquad AD = 3.209$$

$$BD - \frac{AB + \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \qquad BD = 7.791 \qquad AD + BD = 11$$



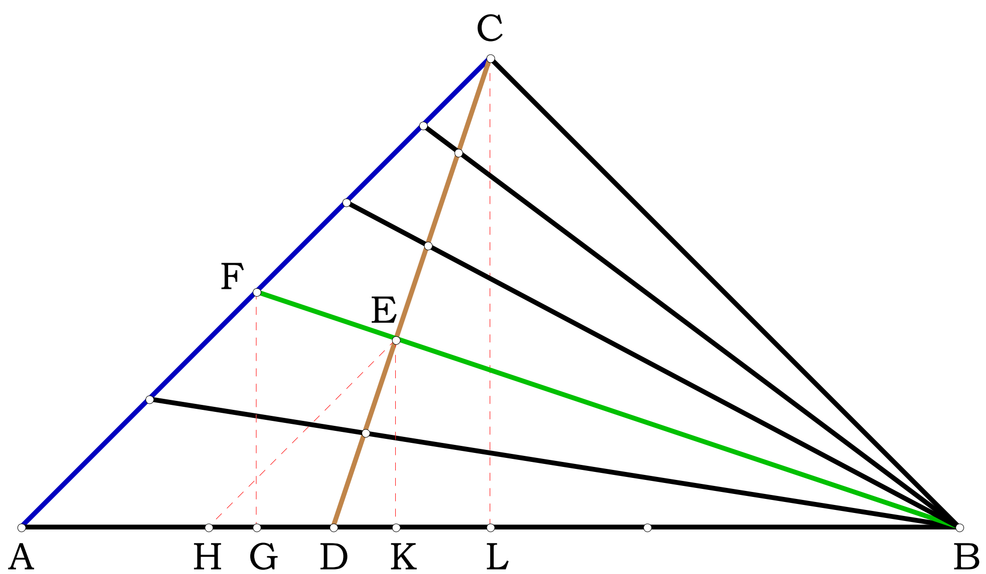


AB = 2.97459
AC = 1.16894
AD = 0.56772

$$\frac{AB-\sqrt{AB^2-4 \cdot AC^2}}{2}-AD = 0.00000$$

930621 Pyramid of Ratios I

Divide AB by N_1 then divide CD by N_2, what are BF/EF and AC/AF?



$$\mathbf{AB} := 4.47022 \quad \mathbf{N_1} := 3.82131 \quad \mathbf{N_2} := 2.62629 \quad \mathbf{AD} := \frac{\mathbf{AB}}{\mathbf{N_1}} \quad \mathbf{AL} := \frac{\mathbf{AB}}{2}$$

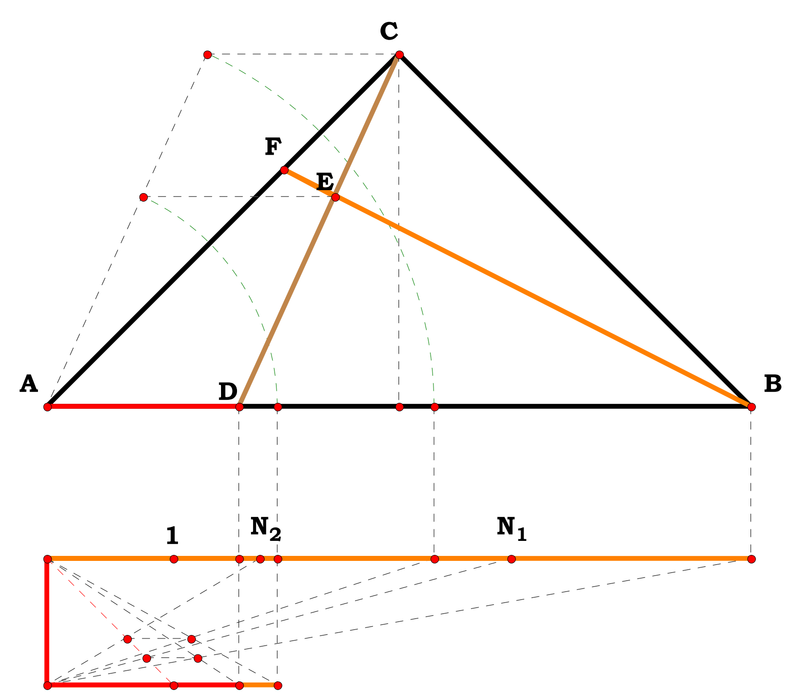
$$\mathbf{AC} := \sqrt{2\cdot \mathbf{AL}^2} \quad \mathbf{CL} := \mathbf{AL} \quad \mathbf{DL} := \mathbf{AL} - \mathbf{AD} \quad \mathbf{CD} := \sqrt{\mathbf{DL}^2 + \mathbf{CL}^2}$$

$$\mathbf{DE} := \frac{\mathbf{CD}}{\mathbf{N_2}} \quad \mathbf{EK} := \frac{\mathbf{CL}\cdot\mathbf{DE}}{\mathbf{CD}} \quad \mathbf{DK} := \frac{\mathbf{DL}\cdot\mathbf{EK}}{\mathbf{CL}} \quad \mathbf{AK} := \mathbf{AD} + \mathbf{DK}$$

$$\mathbf{BK} := \mathbf{AB} - \mathbf{AK} \quad \mathbf{BE} := \sqrt{\mathbf{BK}^2 + \mathbf{EK}^2} \quad \mathbf{HK} := \frac{\mathbf{AL}\cdot\mathbf{EK}}{\mathbf{CL}} \quad \mathbf{BH} := \mathbf{BK} + \mathbf{HK}$$

$$\mathbf{EH} := \frac{\mathbf{AC}\cdot\mathbf{EK}}{\mathbf{CL}} \quad \mathbf{AF} := \frac{\mathbf{EH}\cdot\mathbf{AB}}{\mathbf{BH}} \quad \mathbf{BF} := \frac{\mathbf{BE}\cdot\mathbf{AB}}{\mathbf{BH}} \quad \mathbf{EF} := \mathbf{BF} - \mathbf{BE}$$

$$\frac{\mathbf{BF}}{\mathbf{EF}} = 6.171 \quad \frac{\mathbf{AC}}{\mathbf{AF}} = 2.201 \quad \frac{\mathbf{AC}}{\mathbf{AF}} - \frac{\mathbf{N_1}\cdot\mathbf{N_2} - \mathbf{N_2} + 1}{\mathbf{N_1}} = 0 \quad \frac{\mathbf{BF}}{\mathbf{EF}} - \frac{\mathbf{N_1}\cdot\mathbf{N_2}}{\mathbf{N_2} - 1} = 0$$



$$\frac{\mathbf{BF}}{\mathbf{EF}} = 9.08799 \quad \frac{\mathbf{N_1}\cdot\mathbf{N_2}}{\mathbf{N_2}\text{-}1} = 9.08799$$

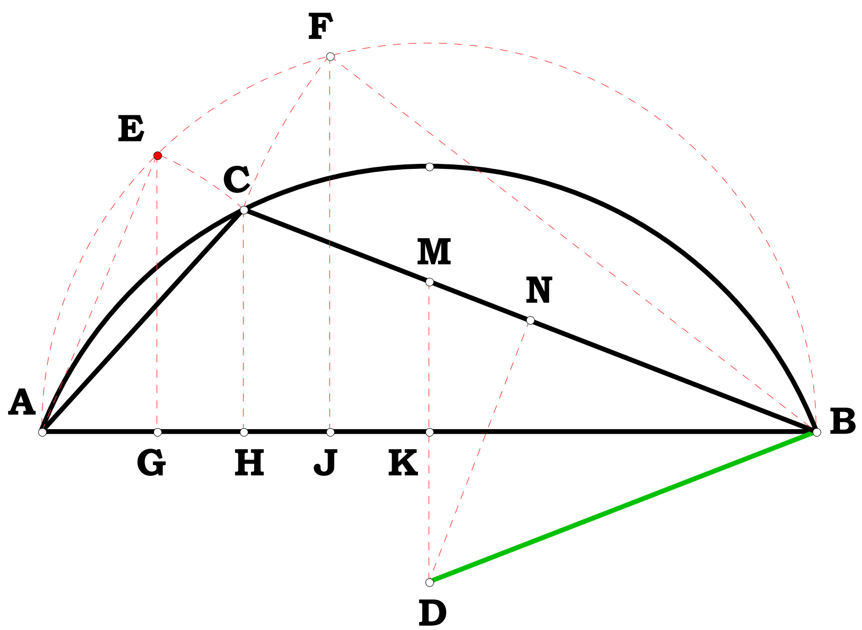
$$\frac{\mathbf{AC}}{\mathbf{AF}} = 1.49057 \quad \frac{(\mathbf{N_1}\cdot\mathbf{N_2}\text{-}\mathbf{N_2})+1}{\mathbf{N_1}} = 1.49057$$

$$\frac{\mathbf{BF}}{\mathbf{EF}} \text{-} \frac{\mathbf{N_1}\cdot\mathbf{N_2}}{\mathbf{N_2}\text{-}1} = 0.00000 \quad \frac{\mathbf{AC}}{\mathbf{AF}} \text{-} \frac{(\mathbf{N_1}\cdot\mathbf{N_2}\text{-}\mathbf{N_2})+1}{\mathbf{N_1}} = 0.00000$$

062793 Describe A Circle About a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

$\Delta := (AB + AC > BC)\cdot(AB + BC > AC)\cdot(BC + AC > AB)$ $\quad NOT(X) := X = \quad \delta := 0..2 \qquad AB \equiv 3 \quad AC \equiv 4 \quad BC \equiv 6$



$BK := \frac{AB}{2} \quad AE := AC \quad BF := BC \quad AG := \frac{AE^2}{AB} \quad BJ := \frac{BF^2}{AB} \quad GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2} \quad BH := BJ + HJ \quad CH := \sqrt{BC^2 - BH^2} \quad BN := \frac{BC}{2} \quad BM := \frac{BC \cdot BK}{BH} \quad MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH} \quad BD := \sqrt{BN^2 + DN^2}$

$$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix} \quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix}$$

radius := if $(\Delta, BD, 0)$ $\qquad$ imaginary_radius := if $(NOT(\Delta), BD, 0)$

radius = 3.375

imaginary_radius = 0

$\Delta = 1$

The construction is independent of the side one starts with.

$$R_\delta := \frac{S_{1_\delta} \cdot S_{2_\delta} \cdot S_{3_\delta}}{\sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}$$

The name of the Radius in terms of the givens.

$R^T = (3.375 \quad 3.375 \quad 3.375)$

The equation is a statement in regard to the relationship between each side of a triangle.

$$\frac{XO}{01} = 1.35962 \qquad \frac{WO}{01} = 1.35962 \qquad \frac{ZO}{01} = 1.35962$$

$X = 1.35962$

$M = 1.35962$

$X\text{-}M = 0.00000$

$XO = 1.18110$ in.

$WO = 1.18110$ in.

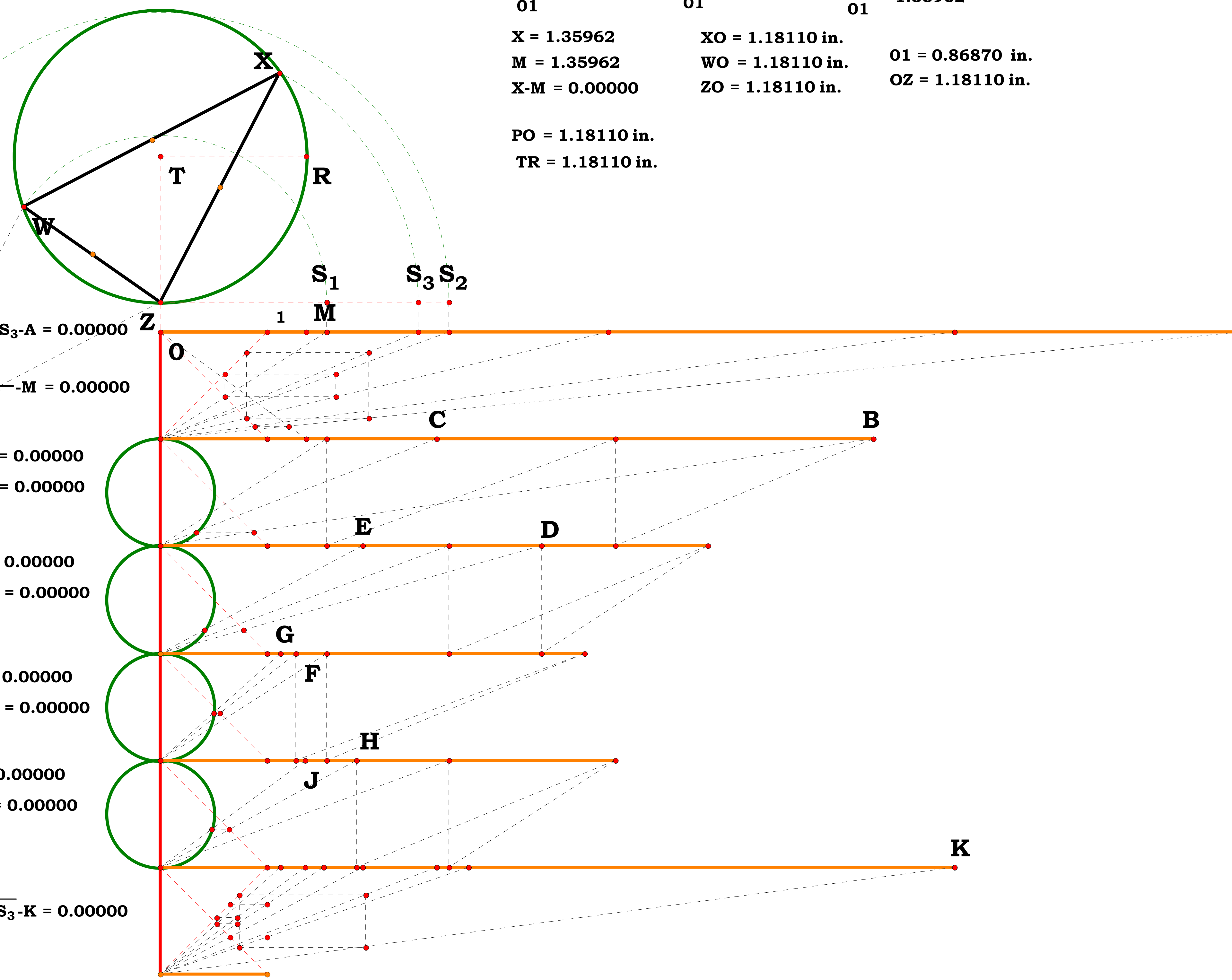
$ZO = 1.18110$ in.

$01 = 0.86870$ in.

$OZ = 1.18110$ in.

$PO = 1.18110$ in.

$TR = 1.18110$ in.



$S_1 = 1.55042$

$S_2 = 2.69740$

$S_3 = 2.41211$

$A = 10.08768$ $\quad S_1{\cdot}S_2{\cdot}S_3\text{-}A = 0.00000$

$$\frac{S_1{\cdot}S_2{\cdot}S_3}{\sqrt{S_1+S_2+S_3}\cdot\sqrt{(S_2+S_3)\text{-}S_1}\cdot\sqrt{(S_1+S_3)\text{-}S_2}\cdot\sqrt{(S_1+S_2)\text{-}S_3}}\text{-}M = 0.00000$$

$B = 6.65993$ $\quad (S_1+S_2+S_3)\text{-}B = 0.00000$

$C = 2.58068$ $\quad \sqrt{S_1+S_2+S_3}\text{-}C = 0.00000$

$D = 3.55909$ $\quad (S_2+S_3)\text{-}S_1\text{-}D = 0.00000$

$E = 1.88655$ $\quad \sqrt{(S_2+S_3)\text{-}S_1}\text{-}E = 0.00000$

$F = 1.26512$ $\quad (S_1+S_3)\text{-}S_2\text{-}F = 0.00000$

$G = 1.12478$ $\quad \sqrt{(S_1+S_3)\text{-}S_2}\text{-}G = 0.00000$

$H = 1.83571$ $\quad (S_1+S_2)\text{-}S_3\text{-}H = 0.00000$

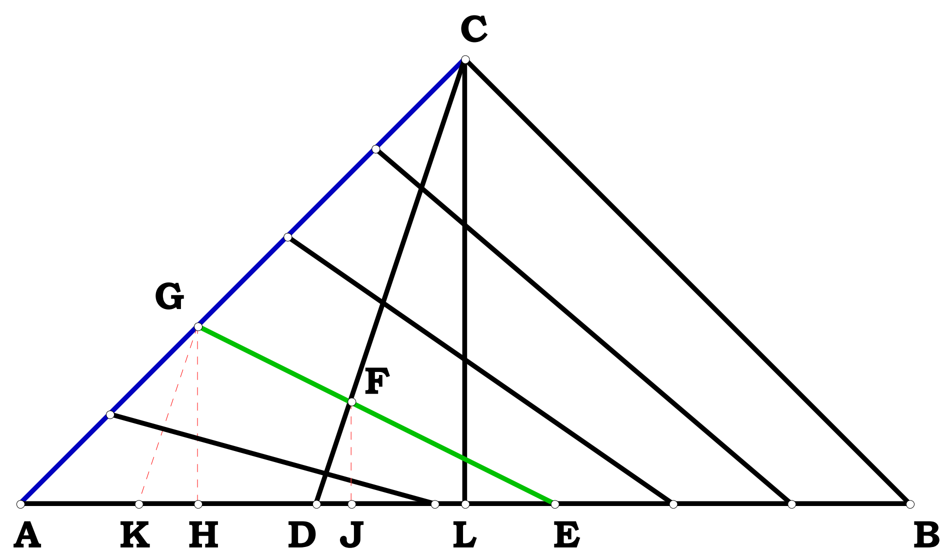
$J = 1.35489$ $\quad \sqrt{(S_1+S_2)\text{-}S_3}\text{-}J = 0.00000$

$K = 7.41948$

$$\sqrt{S_1+S_2+S_3}\cdot\sqrt{(S_2+S_3)\text{-}S_1}\cdot\sqrt{(S_1+S_3)\text{-}S_2}\cdot\sqrt{(S_1+S_2)\text{-}S_3}\text{-}K = 0.00000$$

071593 Pyramid of Ratios II

AB is divided by N_1 and AC and BD is divided by N_2, what are EG/FG and CD/DF?



$AB := 1 \quad N_1 := 3 \quad N_2 := 5 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD \quad AL := \frac{AB}{2}$

$DE := \frac{BD}{N_2} \quad AG := \frac{AC}{N_2} \quad AE := AD + DE \quad AH := \sqrt{\frac{AG^2}{2}} \quad GH := AH \quad EH := AE - AH$

$EG := \sqrt{EH^2 + GH^2} \quad DL := AL - AD \quad CL := AL \quad HK := \frac{DL \cdot AH}{AL} \quad EK := EH + HK$

$DJ := \frac{HK \cdot DE}{EK} \quad EF := \frac{EG \cdot DE}{EK} \quad FG := EG - EF \quad FJ := \frac{GH \cdot DE}{EK}$

$CD := \sqrt{CL^2 + DL^2} \quad DF := \sqrt{DJ^2 + FJ^2}$

$\frac{EG}{FG} = 1.5 \quad \frac{CD}{DF} = 15 \quad \frac{N_2 + N_1 - 2}{N_2 - 1} = 1.5 \quad \frac{N_2{}^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} = 15$

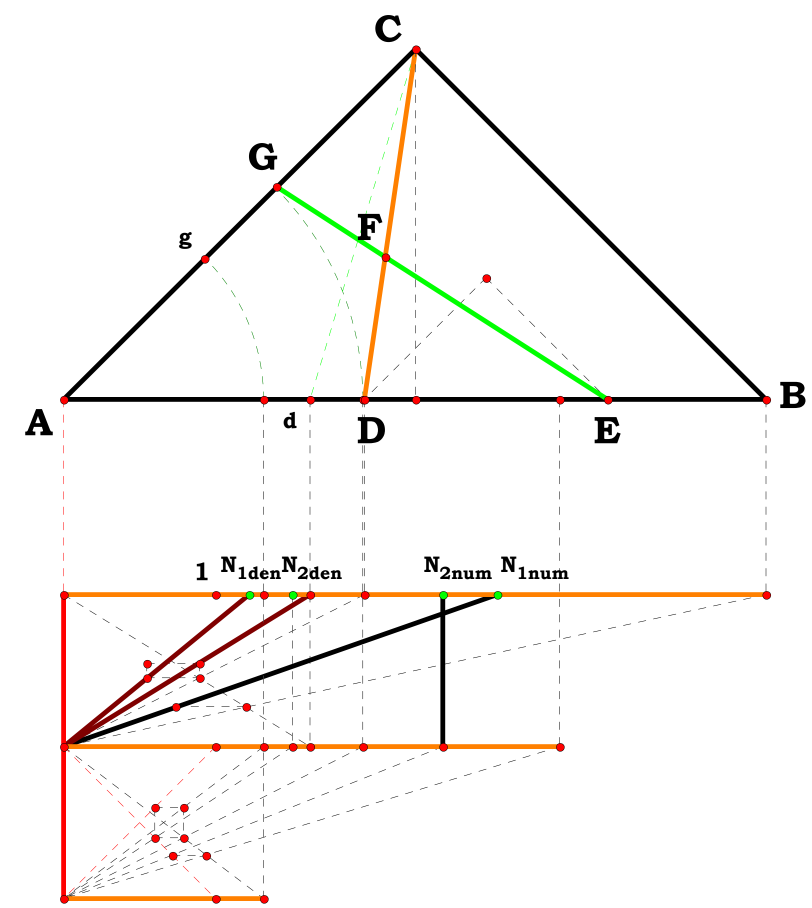
$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0 \quad \frac{CD}{DF} - \frac{N_2{}^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1}$

$N_1 = 2.32900$

$N_2 = 1.65606$

$$\frac{N_{1\ num}}{N_{1\ den}} = 2.32900$$

$$\frac{N_{2\ num}}{N_{2\ dem}} = 1.65606$$



$$\frac{EG}{FG} = 3.02573 \qquad \frac{(N_2+N_1)-2}{N_2-1} = 3.02573 \qquad \frac{EG}{FG}-\frac{(N_2+N_1)-2}{N_2-1} = 0.00000$$

$$\frac{CD}{DF} = 2.47357 \qquad \frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 2.47357 \qquad \frac{CD}{DF}-\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 0.00000$$

072593 Pyramid of Ratios III

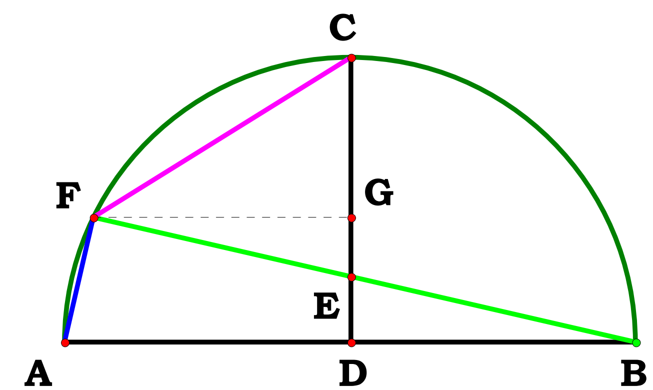
Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?

$AB := 1 \qquad N := 5 \qquad CD := \dfrac{AB}{2} \qquad DE := \dfrac{CD}{N}$

$BE := \sqrt{CD^2 + DE^2} \qquad BF := \dfrac{CD \cdot AB}{BE} \qquad AF := \dfrac{DE \cdot BF}{CD}$

$DG := \dfrac{DE \cdot BF}{BE} \qquad CG := CD - DG \qquad FG := \dfrac{CD \cdot (DG - DE)}{DE}$

$CF := \sqrt{FG^2 + CG^2}$



$\dfrac{BE}{BF} = 0.52 \qquad \dfrac{N^2+1}{2 \cdot N^2} = 0.52 \qquad \dfrac{AF}{CF} = 0.354 \qquad \dfrac{\sqrt{2}}{N-1} = 0.354$

$\dfrac{BE}{BF} - \dfrac{N^2+1}{2 \cdot N^2} = 0 \qquad \dfrac{AF}{CF} - \dfrac{\sqrt{2}}{N-1} = 0$

$\dfrac{N_{um}}{N_{den}} = 2.99984$

$N = 2.99984$

BE = 1.75445 in.

BF = 3.15797 in.

$\dfrac{BE}{BF} = 0.55556$

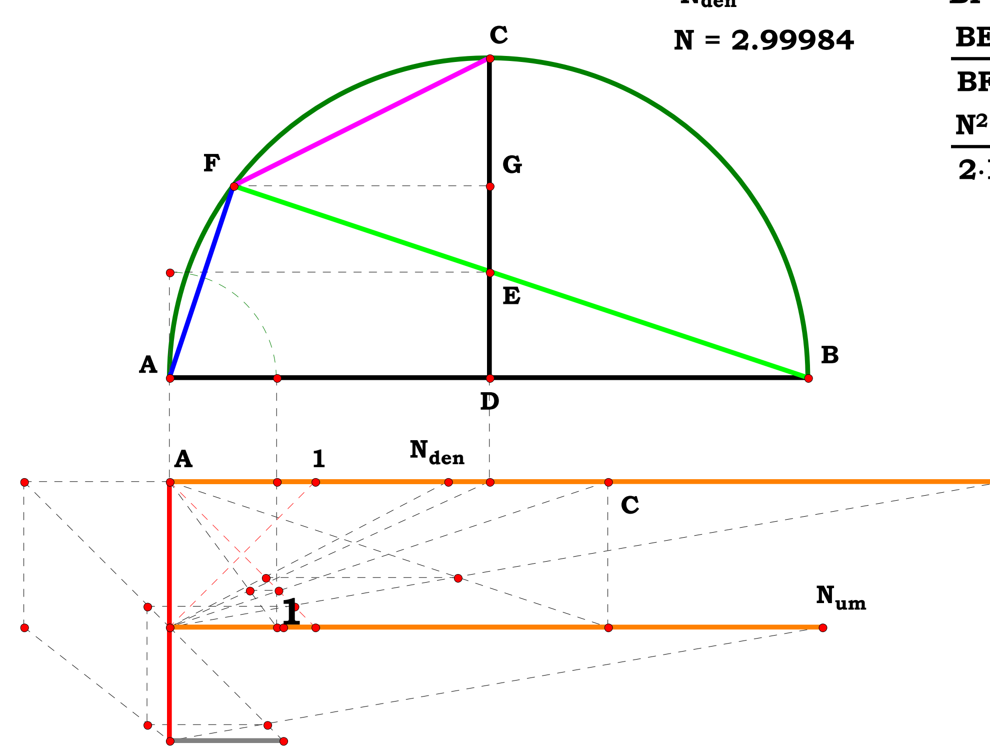
$\dfrac{N^2+1}{2 \cdot N^2} - \dfrac{BE}{BF} = 0.00000$

AF = 1.05271 in.

CF = 1.48864 in.

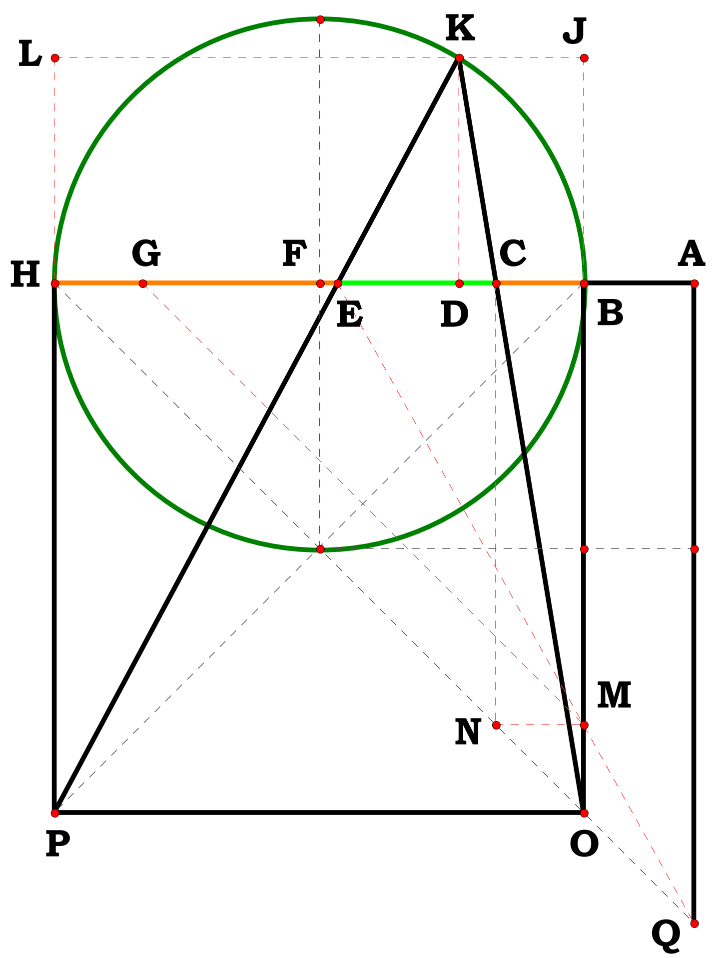
$\dfrac{AF}{CF} = 0.70716$

$\dfrac{\sqrt{2}}{N-1} - \dfrac{AF}{CF} = 0.00000$



110693 Gruntwork I on the Delian Solution

Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/3} = AE$?



$$N := 5 \quad BH := 3 \quad BF := \frac{BH}{2} \quad BD := \frac{BF}{N} \quad DH := BH - BD$$

$$DK := \sqrt{BD \cdot DH} \quad JO := BH + DK \quad BC := \frac{BD \cdot BH}{JO}$$

$$BG := BH - BC \quad EH := \frac{DH \cdot BH}{JO} \quad BE := BH - EH \quad EG := EH - BC$$

$$GM := \sqrt{2 \cdot BG^2} \quad HO := \sqrt{2 \cdot BH^2} \quad HQ := \frac{GM \cdot EH}{EG} \quad OQ := HQ - HO$$

$$AB := \frac{OQ}{\sqrt{2}} \quad AC := AB + BC \quad AE := AB + BE \quad AH := AB + BH$$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$$

$$BF - \frac{1}{2} = 1 \quad BD - \frac{BH}{2 \cdot N} = 0 \quad DH - \frac{BH \cdot (2 \cdot N - 1)}{2 \cdot N} = 0 \quad DK - \frac{BH \cdot \sqrt{2 \cdot N - 1}}{(2 \cdot N)} = 0$$

$$JO - \frac{BH \cdot \left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)}{2 \cdot N} = 0 \quad BC - \frac{BH}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \quad BG - \frac{BH \cdot \left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$$

$$EH - \frac{BH \cdot (2 \cdot N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \quad BE - \frac{BH \cdot \left(\sqrt{2 \cdot N - 1} + 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \quad EG - \frac{2 \cdot BH \cdot (N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$$

$$GM - \sqrt{2} \cdot \frac{BH \cdot \left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)} = 0 \quad HO - \sqrt{2} \cdot BH = 0$$

$$HQ - \frac{\sqrt{2}\cdot BH\cdot(2\cdot N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

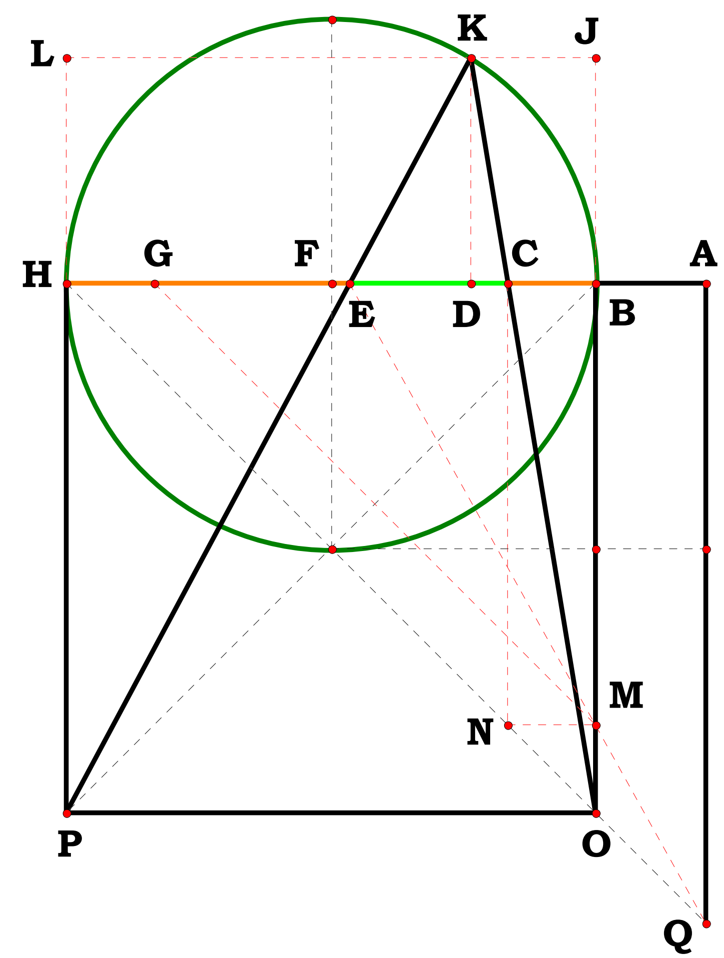
$$OQ - \frac{\sqrt{2}\cdot BH\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{\left(2\cdot N+\sqrt{2\cdot N-1}\right)\cdot(2\cdot N-2)} = 0$$

$$AB - \frac{BH\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AC - \frac{BH\cdot\left[2\cdot N+2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AE - \frac{BH\cdot\left[2\cdot N+(2\cdot N-1)^{\frac{3}{2}}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AH - \frac{BH\cdot\left[(2\cdot N-1)^{\frac{3}{2}}-4\cdot N+4\cdot N^2+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$



BF = 1.37917 in.

BD = 0.65310 in.

$\frac{BF}{BD}$ = 2.11173

N = 2.11173

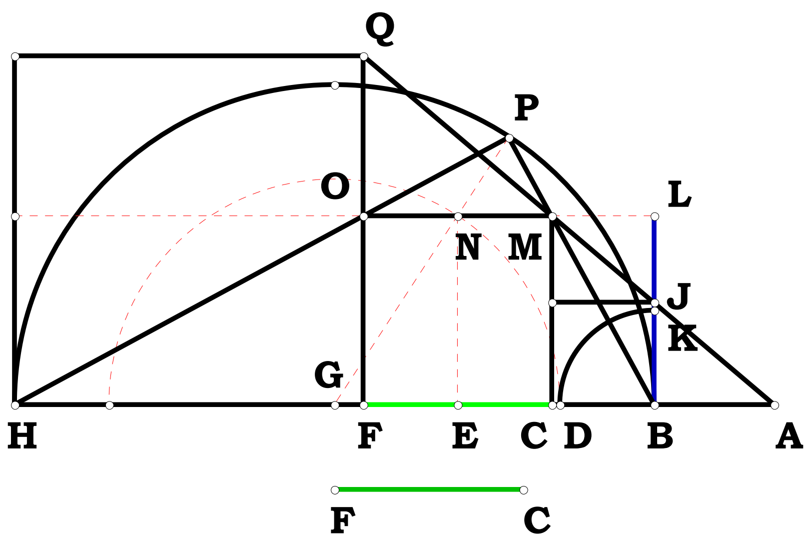
BH = 2.75833 in.

AE = 1.85725 in.

$$\frac{BH\cdot\left(\left(2\cdot N+(2\cdot N-1)^{\frac{3}{2}}\right)-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} - AE = 0.00000 \text{ in.}$$

110993 Solve For Cube Root Placement

With straight edge and compass only, solve the given problem.
BH is the difference between the segments AH and AB.
CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



$N_1 := 2 \quad N_2 := 4$

$BH := N_1 \quad BG := \frac{BH}{2} \quad CF := \frac{BH}{N_2} \quad BL := CGP := BG$

$BK := \frac{BL}{2} \quad BD := BK \quad NP := BD \quad GN := GP - NP \quad EN := BL$

$GE := \sqrt{GN^2 - EN^2} \quad CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD \quad FH := GH + GE - EF \quad FQ := FH$

$FO := BL \quad OQ := FQ - MO := CF \quad AF := \frac{MO \cdot FQ}{OQ} \quad AC := AF - CF \quad AH := AF + FH \quad AB := AH - BH$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \quad \frac{AH}{AB} = 17.944271909999$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

Algebraic Names

$$BH - N_1 = 0 \quad BG - \frac{N_1}{2} = 0 \quad CF - \frac{N_1}{N_2} = 0 \quad BK - \frac{N_1}{2 \cdot N_2} = 0 \quad GN - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0$$

$$GE - \frac{N_1 \cdot \sqrt{(N_2+1)\cdot(N_2-3)}}{2 \cdot N_2} = 0 \quad BC - \frac{N_1 \cdot \left[N_2 - \left[\sqrt{(N_2+1)\cdot(N_2-3)}\right] - 1\right]}{2 \cdot N_2} = 0$$

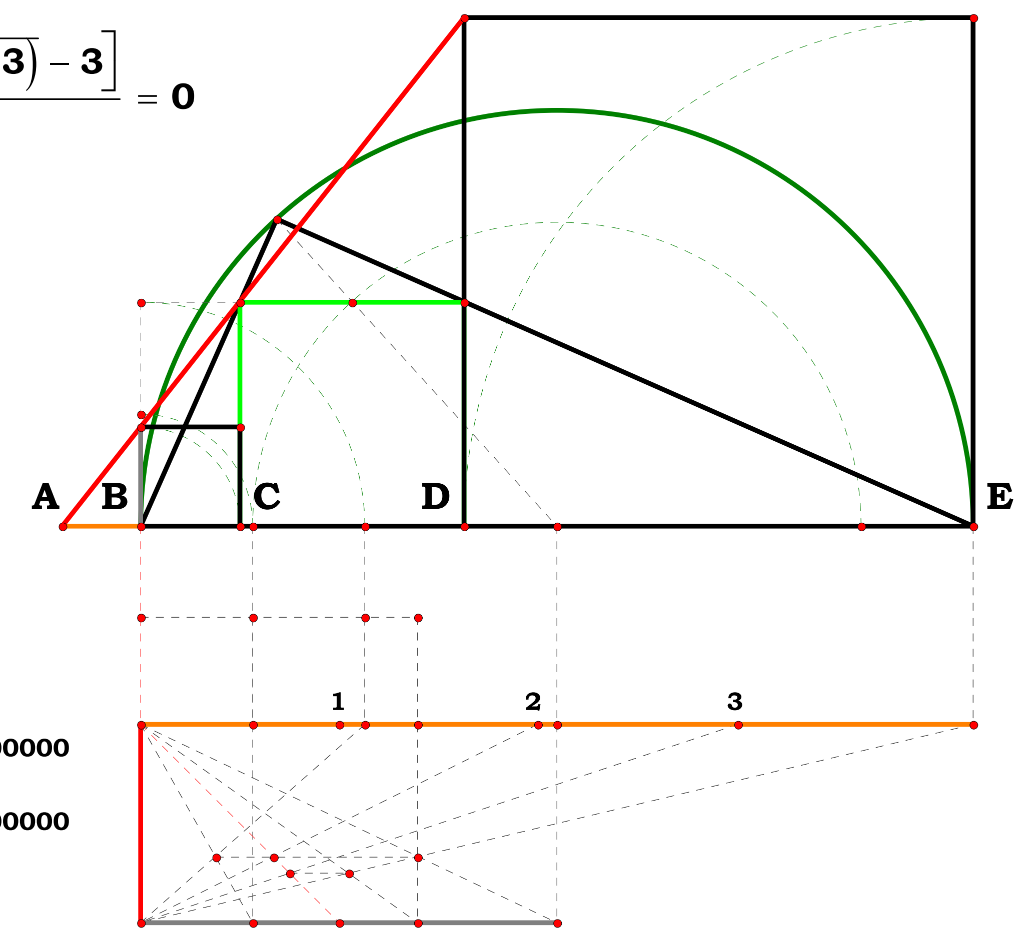
$$FH - \frac{N_1 \cdot \left[N_2 + \left[\sqrt{(N_2+1)\cdot(N_2-3)}\right] - 1\right]}{2 \cdot N_2} = 0 \quad OQ - \frac{N_1 \cdot \left[N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 3\right]}{2 \cdot N_2} = 0$$

$$AF - \frac{N_1}{N_2} \cdot \frac{N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 1}{N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 3} = 0$$

$$AC - \frac{2N_1}{N_2 \cdot \left[N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 3\right]} = 0$$

$$AH - \frac{N_1 \cdot \left[N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 1\right]^2}{2 \cdot N_2 \cdot \left[N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 3\right]} = 0$$

$$AB - \frac{N_1 \cdot \left[N_2 - \sqrt{(N_2+1)\cdot(N_2-3)} - 1\right]}{N_2 \cdot \left[N_2 + \sqrt{(N_2+1)\cdot(N_2-3)} - 3\right]} = 0$$



AB = 0.40540 in.
AE = 4.74189 in.
AC = 0.92025 in.
AD = 2.08896 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

$\frac{AE}{AB} = 11.69681$

111093 Gruntwork II on the Delian Solution

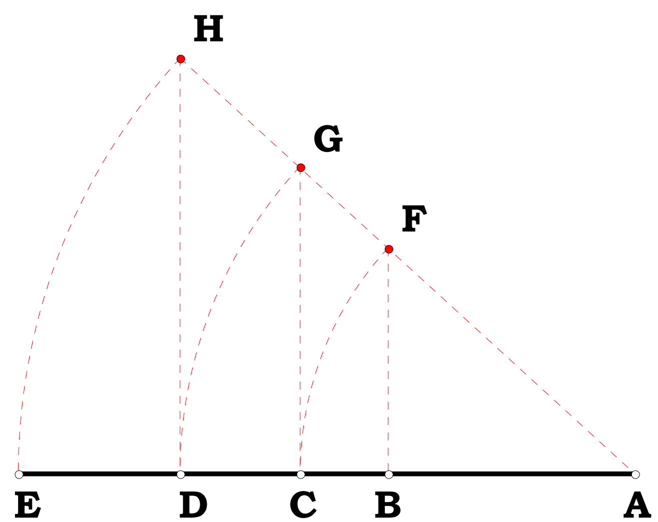
$N := 4 \qquad AE := 1$

$DE := \frac{AE}{N} \qquad AD := AE - DE$

$AH := AE \qquad AG := AD$

$AC := \frac{AD \cdot AD}{AE} \qquad AF := AC$

$AB := \frac{AC \cdot AC}{AD}$



$$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$$

$$\frac{AE}{AB} = 2.37 \qquad \frac{AD}{AB} = 1.778 \qquad \frac{AC}{AB} = 1.333$$

Albebraic Names:

$$\frac{1}{N} - DE = 0 \qquad 1 - \frac{1}{N} - AD = 0 \qquad \frac{(N-1)^2}{N^2} - AC = 0$$

$$\frac{(N-1)^3}{N^3} - AB = 0$$

$\frac{N_2}{N_1} = 0.73339$

$N = 0.73339$

$AE = 5.10000$ in.
$AD = 1.85398$ in.
$AC = 0.67397$ in.
$AB = 0.24500$ in.
$DE = 6.95398$ in.

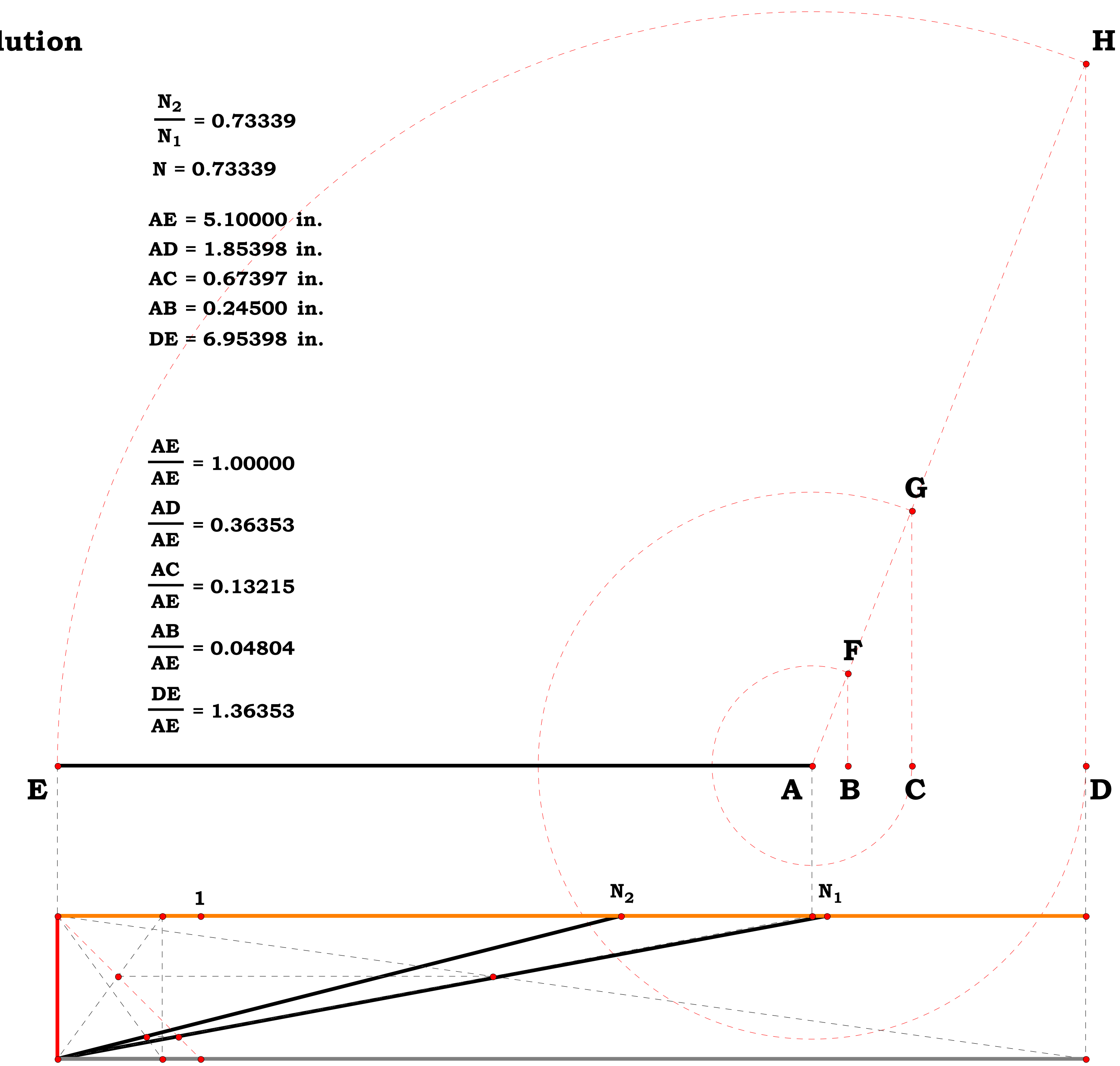
$\frac{AE}{AE} = 1.00000$

$\frac{AD}{AE} = 0.36353$

$\frac{AC}{AE} = 0.13215$

$\frac{AB}{AE} = 0.04804$

$\frac{DE}{AE} = 1.36353$

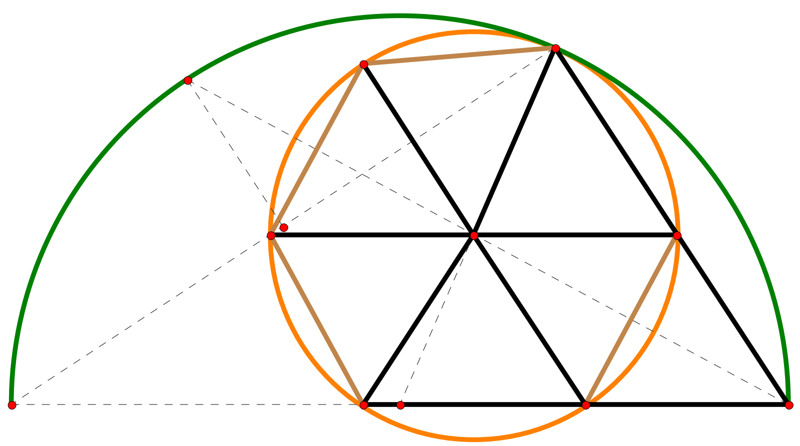


$$\sqrt{\frac{1}{N}^2} = 1.36353 \qquad \sqrt{\left(1-\frac{1}{N}\right)^2} = 0.36353 \qquad \frac{(N-1)^2}{N^2} = 0.13215 \qquad \sqrt{\frac{(N-1)^{3\,2}}{N^3}} = 0.04804$$

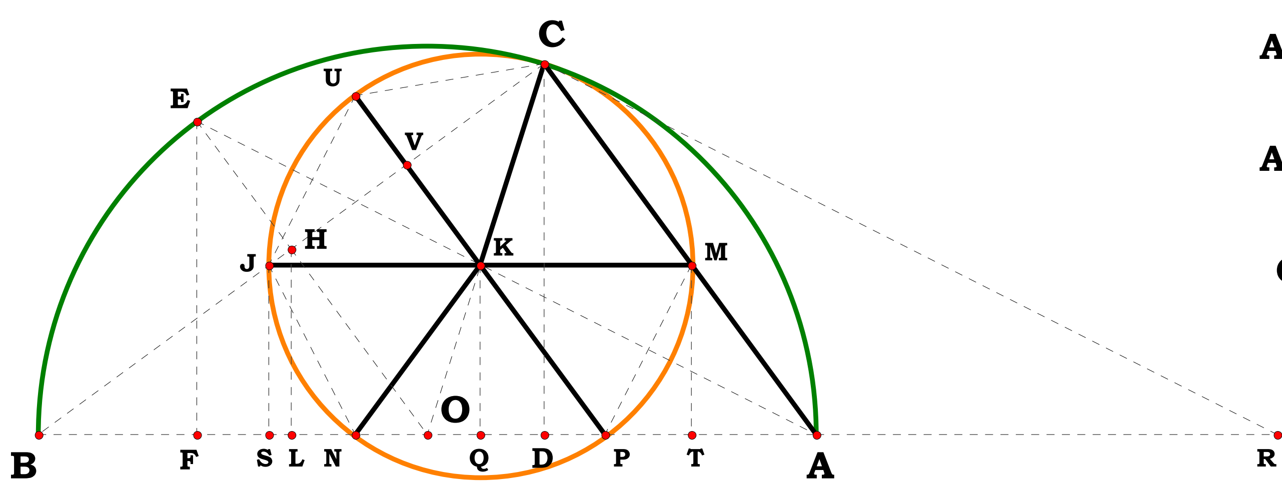
$$\sqrt{\frac{1}{N}^2} - \frac{DE}{AE} = 0.00000 \qquad \sqrt{\left(1-\frac{1}{N}\right)^2} - \frac{AD}{AE} = 0.00000 \qquad \frac{(N-1)^2}{N^2} - \frac{AC}{AE} = 0.00000 \qquad \sqrt{\frac{(N-1)^{3\,2}}{N^3}} - \frac{AB}{AE} = 0.00000$$

111193 The Archimedean Paper Trisector

When I looked up the Archimedean Paper Trisector, which is all I found. I did not find where anyone had bothered to complete the figure, for it was obvious to me that the figure was simply not complete. The first task then in writing up the figure is to simply complete the figure.



Once one understands that the angle on the center is twice the angle from the circumference one can then start to work filling in the figure to include the APT. One can see, not only here, but in other figures that trisection is involved with the right triangle and square roots.



First Principles: $AB := 4.05 \quad N := 3$

Descriptions:

$$AD := \frac{AB}{N} \quad BD := AB - AD \quad CD := \sqrt{AD \cdot BD} \quad BC := \sqrt{CD^2 + BD^2}$$

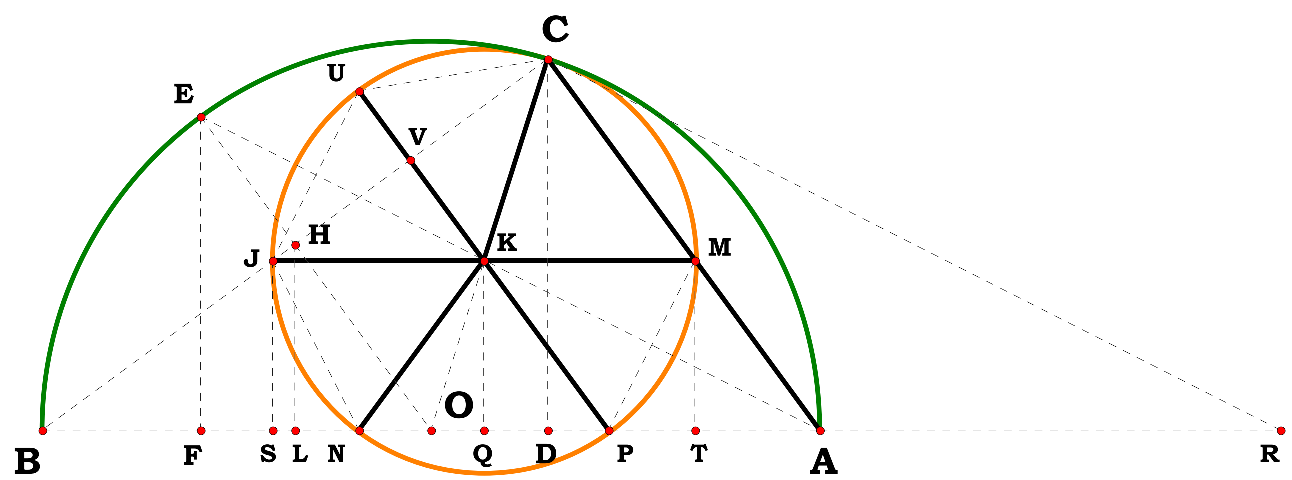
$$AC := \sqrt{AD^2 + CD^2} \quad BH := \frac{BC}{2} \quad AO := \frac{AB}{2} \quad HO := \frac{AC}{2} \quad HL := \frac{CD}{2} \quad LO := \frac{AD}{2}$$

$$OF := \frac{LO \cdot AO}{HO} \quad AF := AO + OF \quad BF := AB - AF \quad EF := \sqrt{BF \cdot AF} \quad DR := \frac{AF \cdot CD}{EF}$$

$$DO := AO - AD \quad OR := DR + DO \quad KQ := \frac{CD \cdot AO}{OR} \quad OK := \frac{AO \cdot KQ}{CD}$$

$$CK := AO - OK \quad QP := \sqrt{CK^2 - KQ^2} \quad OQ := \frac{DO \cdot KQ}{CD} \quad EH := AO - HO$$

$$AP := AO - (OQ + QP) \quad AP - CK = 0$$



$$CJ := \frac{BC \cdot CK}{AO} \qquad AQ := AO - OQ \qquad SO := CK - OQ \qquad BS := AO - SO$$

$$BJ := BC - CJ \qquad JS := \sqrt{BJ^2 - BS^2} \qquad JS - KQ = 0 \qquad SN := SO + OQ - QP \qquad JN := \sqrt{JS^2 + SN^2}$$

$$UV := \frac{EH \cdot CJ}{BC} \qquad JV := \frac{CJ}{2} \qquad JU := \sqrt{JV^2 + UV^2} \qquad CU := JU$$

$$AT := \frac{AD \cdot CK}{AC} \qquad PT := AP - AT \qquad MT := \frac{CD \cdot AP}{AC} \qquad MP := \sqrt{PT^2 + MT^2}$$

$$MP - JN = 0 \qquad MP - JU = 0 \qquad MP - CU = 0$$

Algebraic Names or Definitions:

$$AD - \frac{AB}{N} = 0 \qquad BD - \frac{AB \cdot (N-1)}{N} = 0 \qquad CD - \frac{AB \cdot \sqrt{N-1}}{N} = 0 \qquad BC - \frac{AB \cdot \sqrt{N-1}}{\sqrt{N}} = 0 \qquad AC - \frac{AB}{\sqrt{N}} = 0 \qquad BH - \frac{AB \cdot \sqrt{N-1}}{2 \cdot \sqrt{N}} = 0$$

$$AO - \frac{AB}{2} = 0 \qquad HO - \frac{AB}{2 \cdot \sqrt{N}} = 0 \qquad HL - \frac{AB \cdot \sqrt{N-1}}{2 \cdot N} = 0 \qquad LO - \frac{AB}{2 \cdot N} = 0 \qquad OF - \frac{AB}{2 \cdot \sqrt{N}} = 0 \qquad AF - \frac{AB \cdot \left(\sqrt{N}+1\right)}{2 \cdot \sqrt{N}} = 0 \qquad BF - \frac{AB \cdot \left(\sqrt{N}-1\right)}{2 \cdot \sqrt{N}} = 0$$

$$EF - \frac{AB \cdot \sqrt{(N-1)}}{2 \cdot \sqrt{N}} = 0 \qquad DR - \frac{AB \cdot \sqrt{N-1} \cdot \left(\sqrt{N}+1\right)}{N \cdot \sqrt{\left(\sqrt{N}-1\right) \cdot \left(\sqrt{N}+1\right)}} = 0 \qquad DO - \frac{AB \cdot \sqrt{(N-2)^2}}{2 \cdot N} = 0 \qquad OR - \frac{AB \cdot \left(\sqrt{N}+2\right)}{2 \cdot \sqrt{N}} = 0 \qquad KQ - \frac{AB \cdot \sqrt{N-1}}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0$$

$$OK - \frac{AB \cdot \sqrt{N}}{2 \cdot \left(\sqrt{N}+2\right)} = 0 \qquad QP - \frac{AB}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad OQ - \frac{AB \cdot \left[\sqrt{(N-2)^2}\right]}{2 \cdot \sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad EH - \frac{AB \cdot \left(\sqrt{N}-1\right)}{2 \cdot \sqrt{N}} = 0$$

$$AP - \frac{AB}{\sqrt{N}+2} = 0 \qquad CK - \frac{AB}{\sqrt{N}+2} = 0$$

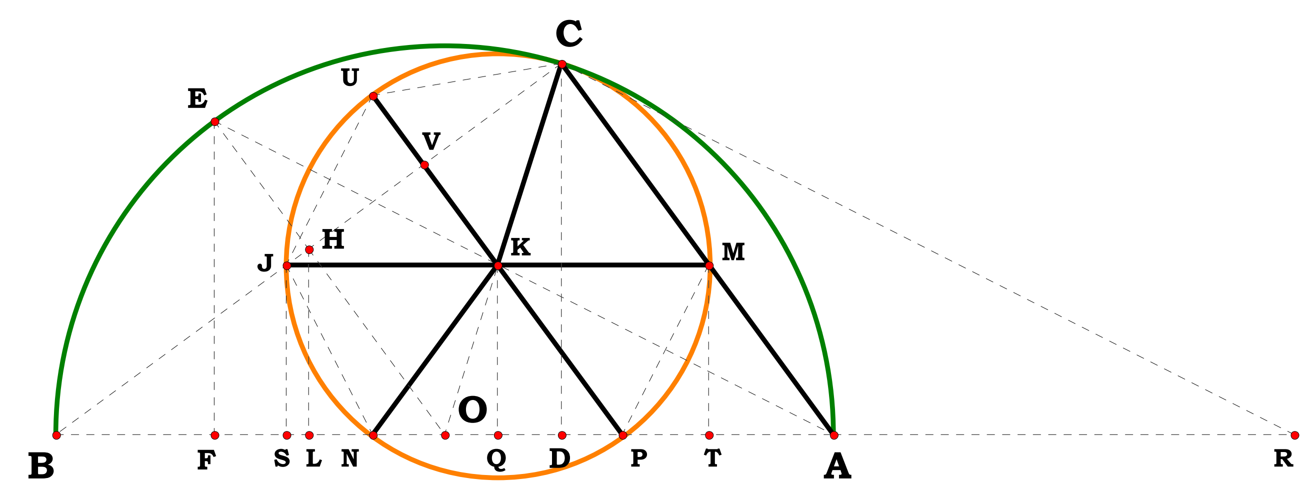
In logic, things which have the same name are equal. CK equals AP.

$$CJ - \frac{2\cdot AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AQ - \frac{AB\cdot\left(\sqrt{N}+1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SO - \frac{AB\cdot\left(2\cdot\sqrt{N}-N+2\right)}{2\cdot N+4\cdot\sqrt{N}} = 0 \qquad BS - \frac{AB\cdot(N-1)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad BJ - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}+2} = 0$$

$$JS - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad JS - KQ = 0 \qquad SN - \frac{AB\cdot\left(\sqrt{N}-1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad JN - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad UV - \frac{AB\cdot\left(\sqrt{N}-1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad JV - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JU - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad CU - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AT - \frac{AB}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad PT - \frac{AB\cdot\left(\sqrt{N}-1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MT - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

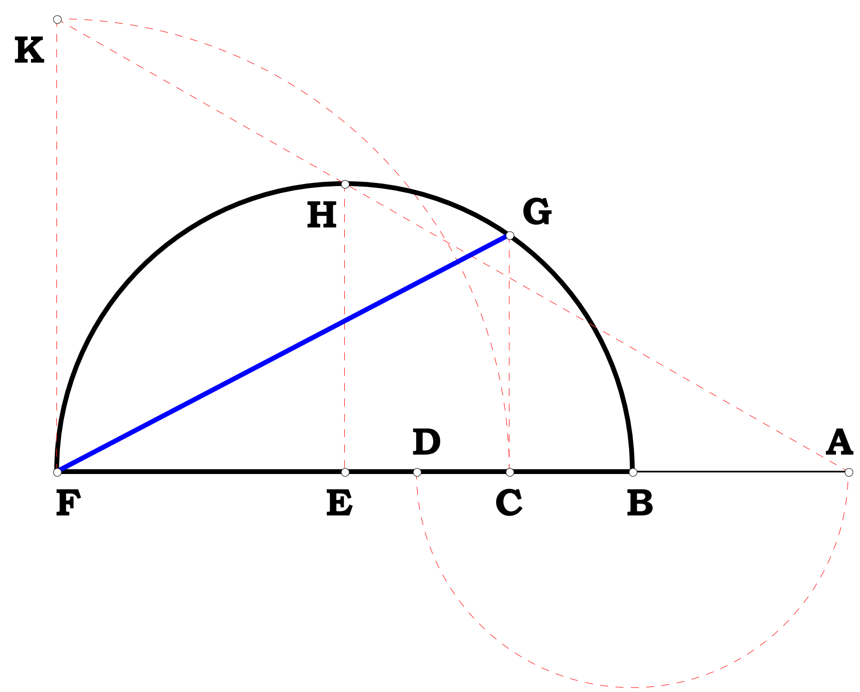
$$MP - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - JN = 0 \qquad MP - JU = 0 \qquad MP - CU = 0$$



111293 To Square A Circle Off The Base Of A Right Triangle.

Sometime in 1992, I remembered reading that some man spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost the figure, so I set out to find it - or something that could pass for it. It took a couple hours so I wonder what he did with the rest of his time?

Using the approximation, π = 22/7, square the circle off the base of a right triangle.



$$BF := 1 \quad BE := \frac{BF}{2} \quad EH := BE \quad BD := \frac{3}{4}\cdot BE \quad AB := BD$$

$$AE := AB + BE \quad AF := AB + BF \quad FK := \frac{EH\cdot AF}{AE} \quad CF := FK \quad BC := BF - CF$$

$$CG := \sqrt{BC\cdot CF} \quad FG := \sqrt{CF^2 + CG^2} \quad \pi\_A := \frac{FG^2}{BE^2}$$

$$\pi = 3.14159265359$$

$$\pi\_A = 3.142857142857$$

$$\frac{\pi}{\pi\_A} = 0.999597662505843 \qquad \pi\_A - \frac{22}{7} = 0$$

DE = 0.39839 in.
BF = 3.18715 in.
AB = 1.19518 in.
AB·BF = 3.80922 in^2
FG = 2.82511 in.
BE = 1.59357 in.
BF = 3.18715 in.

$$\frac{FG^2}{BF} = 0.78571$$

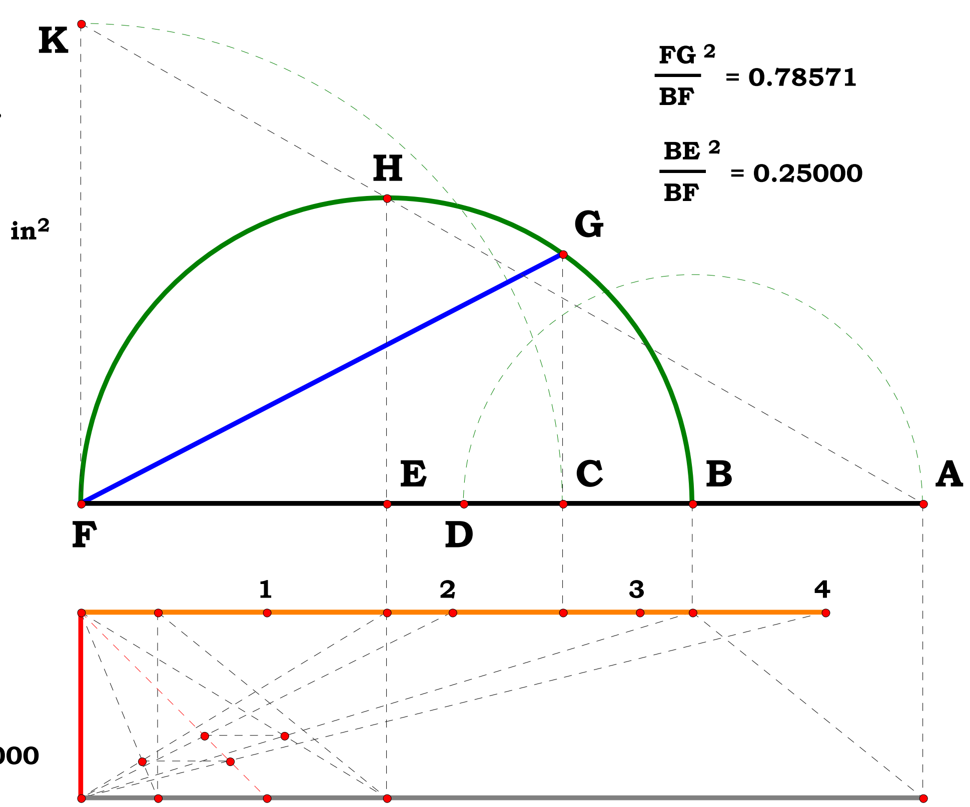
$$\frac{BE^2}{BF} = 0.25000$$

$$\frac{22}{\frac{FG^2}{BF}} = 28.00000$$

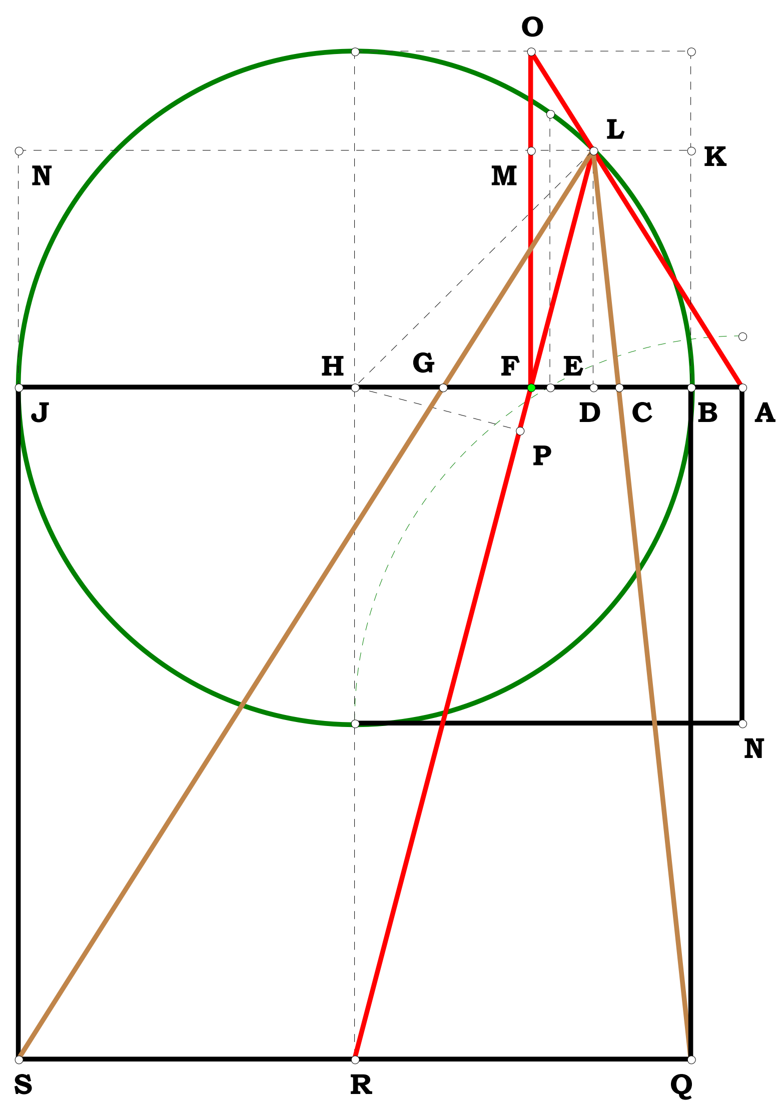
$$\frac{7}{\frac{BE^2}{BF}} = 28.00000$$

FK = 2.50419 in.

$$\left(\frac{FK}{BF}\right)\cdot 28 = 22.00000$$



111893A



Given:

$N_1 := 2 \quad N_2 := 1.5$

describe AB.

$$BJ := N_1 \quad BH := \frac{BJ}{2} \quad BD := \frac{BH}{N_2} \quad HJ := BH$$

$$DH := BH - BD \quad HR := BJ \quad DJ := DH + HJ$$

$$DL := \sqrt{BD \cdot DJ} \quad DF := \frac{DH \cdot DL}{DL + HR} \quad FO := BH \quad BF := BD + DF$$

$$MO := FO - DL \quad LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$$

Define each step in the description of AB.

$$BJ - N_1 = 0 \quad BH - \frac{N_1}{2} = 0 \quad BD - \frac{N_1}{2 \cdot N_2} = 0 \quad HJ - \frac{N_1}{2} = 0 \quad DH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0$$

$$HR - N_1 = 0 \quad DJ - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2} = 0 \quad DL - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2} = 0$$

$$DF - \frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \quad FO - \frac{N_1}{2} = 0 \quad BF - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} + 2\right)}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0$$

$$MO - \frac{N_1 \cdot \left(N_2 - \sqrt{2 \cdot N_2 - 1}\right)}{2 \cdot N_2} = 0 \quad LM - \frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0$$

$$AF - \frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right) \cdot \left(2 \cdot N_2 - 2 \cdot \sqrt{2 \cdot N_2 - 1}\right)} = 0 \quad AB - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} - 1\right)}{2 \cdot \left(2 \cdot {N_2}^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1} + 1\right)} = 0$$

$$BH-\frac{N_1}{2} = 0.00000 \text{ in.}$$

$$BD-\frac{N_1}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$HJ-\frac{N_1}{2} = 0.00000 \text{ in.}$$

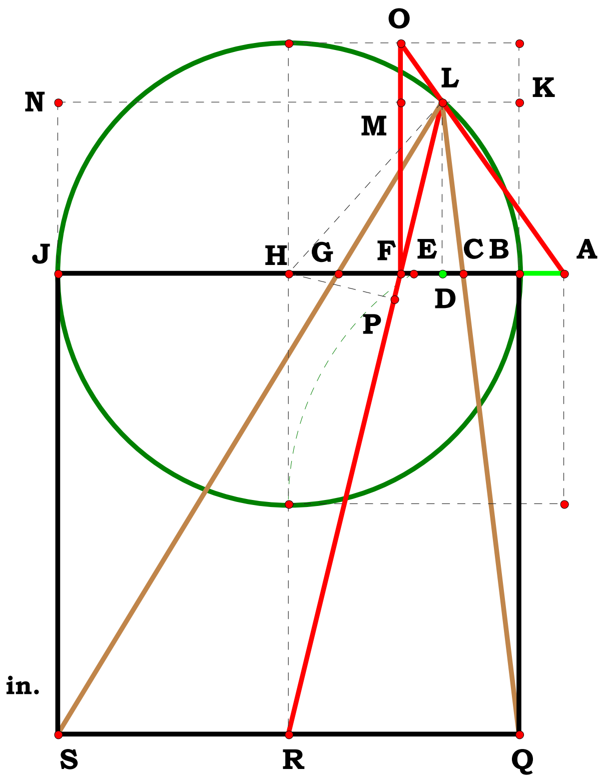
$$DH-\frac{N_1\cdot(N_2-1)}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$HR-N_1 = 0.00000 \text{ in.}$$

$$DJ-\frac{N_1\cdot(2\cdot N_2-1)}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$DL-\frac{N_1\cdot\sqrt{2\cdot N_2-1}}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$DF-\frac{N_1\cdot(N_2-1)\cdot\sqrt{2\cdot N_2-1}}{2\cdot N_2\cdot\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$



$$FO-\frac{N_1}{2} = 0.00000 \text{ in.}$$

$$BF-\frac{N_1\cdot\left(\sqrt{2\cdot N_2-1}+2\right)}{2\cdot\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$

$$MO-\frac{N_1\cdot\left(N_2-\sqrt{2\cdot N_2-1}\right)}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$LM-\frac{N_1\cdot(N_2-1)\cdot\sqrt{2\cdot N_2-1}}{2\cdot N_2\cdot\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$

$$AF-\frac{N_1\cdot(N_2-1)\cdot\sqrt{2\cdot N_2-1}}{\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)\cdot\left(2\cdot N_2-2\cdot\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$

$$AB-\frac{N_1\cdot\left(\sqrt{2\cdot N_2-1}-1\right)}{2\cdot\left(\left(2\cdot N_2^2-2\cdot N_2-N_2\cdot\sqrt{2\cdot N_2-1}\right)+1\right)} = 0.00000 \text{ in.}$$

$N_1 = 2.40000$ in.

$N_2 = 3.01550$

$BJ = 2.40000$ in.

$BH = 1.20000$ in.

$BD = 0.39794$ in.

$HJ = 1.20000$ in.

$$\frac{BH}{BD} = 3.01550$$

$DH = 0.80206$ in.

$HR = 2.40000$ in.

$DJ = 2.00206$ in.

$DL = 0.89258$ in.

$DF = 0.21743$ in.

$FO = 1.20000$ in.

$BF = 0.61537$ in.

$MO = 0.30742$ in.

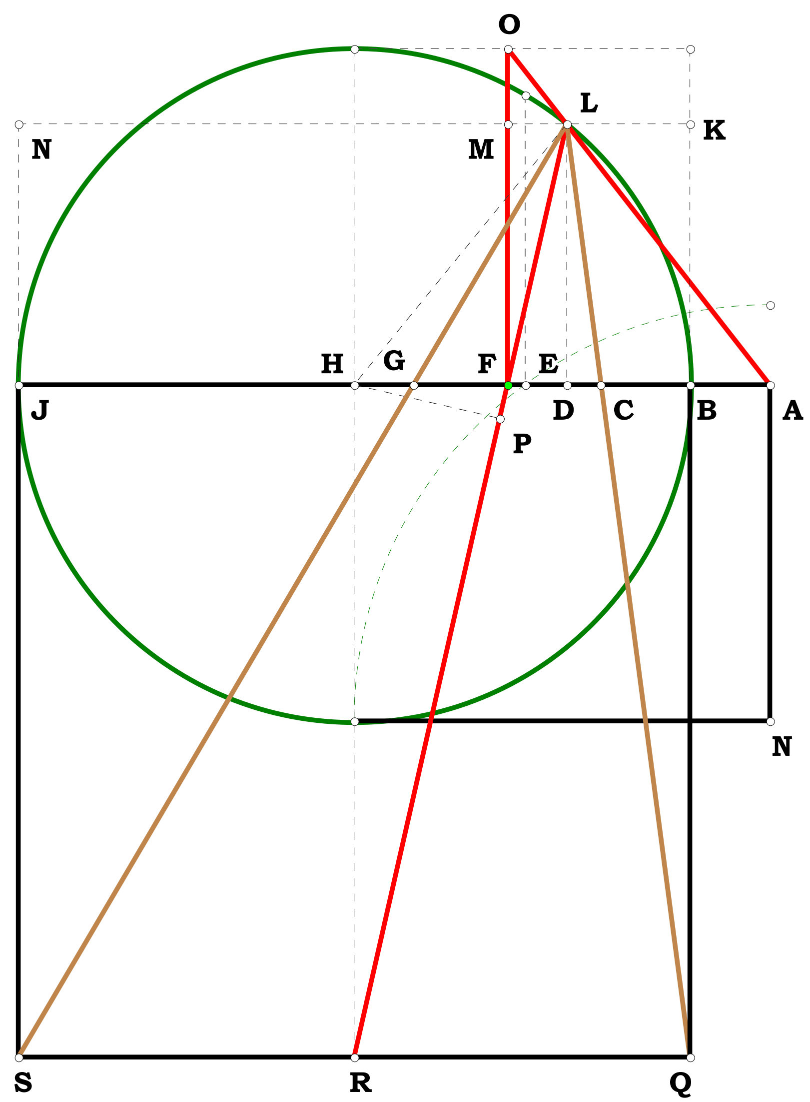
$LM = 0.21743$ in.

$AF = 0.84873$ in.

$AB = 0.23336$ in.

111893 Exploring Cube Roots Plate B

Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root. Notice in this write-up I chose the wrong point to proportion. I get the right answers, but the equations are more complicated. Compare features to plate A.



$$N_1 := 2.4 \quad N_2 := 3.01550$$

$$BJ := N_1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N_2} \quad FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$$

$$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2} \quad FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$$

$$FO := BH \quad FM := DL \quad MO := FO - FM \quad LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF \quad BQ := BJ$$

$$BK := DL \quad BD := BF - DF \quad KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad DJ := BJ - BD \quad LN := DJ \quad JS := BJ$$

$$JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC \quad AG := AB + BG \quad AJ := AB + BJ$$

$$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$$

$$BJ - N_1 = 0 \quad BH - \frac{N_1}{2} = 0 \quad HL - \frac{N_1}{2} = 0 \quad BF - \frac{N_1}{2 \cdot N_2} = 0 \quad FH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0 \quad HR - N_1 = 0$$

$$FR - \frac{N_1 \cdot \sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}}{2 \cdot N_2} = 0 \quad FP - \frac{N_1 \cdot (N_2 - 1)^2}{2 \cdot N_2 \cdot \sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}} = 0 \quad PH - \frac{N_1 \cdot (N_2 - 1)}{\sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}} = 0$$

$$LP - \frac{N_1\cdot\sqrt{N_2^{2}+6\cdot N_2-3}}{2\cdot\sqrt{5\cdot N_2^{2}-2\cdot N_2+1}} = 0 \qquad FL - \frac{N_1\cdot\left(2\cdot N_2-N_2^{2}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-1\right)}{2\cdot N_2\cdot\sqrt{5\cdot N_2^{2}-2\cdot N_2+1}} = 0$$

$$DF - \frac{N_1\cdot\left(N_2-1\right)\cdot\left(2\cdot N_2-N_2^{2}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-1\right)}{2\cdot N_2\cdot\left(5\cdot N_2^{2}-2\cdot N_2+1\right)} = 0 \qquad DL - \frac{N_1\cdot\left(2\cdot N_2-N_2^{2}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-1\right)}{5\cdot N_2^{2}-2\cdot N_2+1} = 0$$

$$FO - \frac{N_1}{2} = 0 \qquad FM - \frac{N_1\cdot\left(2\cdot N_2-N_2^{2}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-1\right)}{5\cdot N_2^{2}-2\cdot N_2+1} = 0 \qquad MO - \frac{N_1\cdot\left(7\cdot N_2^{2}-6\cdot N_2-2\cdot N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}+3\right)}{2\cdot\left(5\cdot N_2^{2}-2\cdot N_2+1\right)} = 0$$

$$LM - \frac{N_1\cdot\left(N_2-1\right)\cdot\left(2\cdot N_2-N_2^{2}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-1\right)}{2\cdot N_2\cdot\left(5\cdot N_2^{2}-2\cdot N_2+1\right)} = 0 \qquad AF - \frac{N_1\cdot\left(N_2-1\right)\cdot\left(N_2^{2}-2\cdot N_2-N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}+1\right)}{2\cdot N_2\cdot\left(6\cdot N_2-7\cdot N_2^{2}+2\cdot N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-3\right)} = 0$$

$$AB - \frac{N_1\cdot\left(3\cdot N_2+N_2^{2}\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-4\cdot N_2^{2}-N_2^{3}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-2\right)}{2\cdot\left(3\cdot N_2-2\cdot N_2^{2}\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-6\cdot N_2^{2}+7\cdot N_2^{3}\right)} = 0 \qquad BQ - N_1 = 0$$

$$BK - \frac{N_1\cdot\left(2\cdot N_2-N_2^{2}+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-1\right)}{5\cdot N_2^{2}-2\cdot N_2+1} = 0 \qquad BD - \frac{N_1\cdot\left[2\cdot N_2+N_2^{2}+\left(1-N_2\right)\cdot\sqrt{N_2^{2}+6\cdot N_2-3}+1\right]}{2\cdot\left(5\cdot N_2^{2}-2\cdot N_2+1\right)} = 0$$

$$KQ - \frac{N_1\cdot N_2\cdot\left(4\cdot N_2+\sqrt{N_2^{2}+6\cdot N_2-3}\right)}{5\cdot N_2^{2}-2\cdot N_2+1} = 0 \qquad KL - \frac{N_1\cdot\left[2\cdot N_2+N_2^{2}+\left(1-N_2\right)\cdot\sqrt{N_2^{2}+6\cdot N_2-3}+1\right]}{2\cdot\left(5\cdot N_2^{2}-2\cdot N_2+1\right)} = 0$$

$$BC - \frac{N_1\cdot\left[2\cdot N_2+N_2^{2}-\sqrt{N_2^{2}+6\cdot N_2-3}\cdot\left(N_2-1\right)+1\right]}{2\cdot N_2\cdot\left(4\cdot N_2+\sqrt{N_2^{2}+6\cdot N_2-3}\right)} = 0 \qquad DJ - \frac{N_1\cdot\left(9\cdot N_2^{2}-6\cdot N_2+N_2\cdot\sqrt{N_2^{2}+6\cdot N_2-3}-\sqrt{N_2^{2}+6\cdot N_2-3}+1\right)}{2\cdot\left(5\cdot N_2^{2}-2\cdot N_2+1\right)} = 0$$

$$LN - \frac{N_1 \cdot \left(9 \cdot N_2^{\,2} - 6 \cdot N_2 + N_2 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} - \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} + 1\right)}{2 \cdot \left(5 \cdot N_2^{\,2} - 2 \cdot N_2 + 1\right)} = 0 \qquad JS - N_1 = 0$$

$$JN - \frac{N_1 \cdot \left(2 \cdot N_2 - N_2^{\,2} + N_2 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} - 1\right)}{5 \cdot N_2^{\,2} - 2 \cdot N_2 + 1} = 0 \qquad NS - \frac{N_1 \cdot N_2 \cdot \left(4 \cdot N_2 + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3}\right)}{5 \cdot N_2^{\,2} - 2 \cdot N_2 + 1} = 0$$

$$GJ - \frac{N_1 \cdot \left(9 \cdot N_2^{\,2} - 6 \cdot N_2 + N_2 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} - \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} + 1\right)}{2 \cdot N_2 \cdot \left(4 \cdot N_2 + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3}\right)} = 0 \qquad BG - \frac{3 \cdot N_1 - N_1 \cdot N_2 + N_1 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3}}{6 \cdot N_2} = 0$$

$$AC - \frac{3 \cdot N_1 - 5 \cdot N_1 \cdot N_2^{\,2} + 9 \cdot N_1 \cdot N_2^{\,3} + 6 \cdot N_1 \cdot N_2^{\,4} + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(9 \cdot N_1 \cdot N_2^{\,2} - 6 \cdot N_1 \cdot N_2^{\,3} - 8 \cdot N_1 \cdot N_2 + N_1\right) - 5 \cdot N_1 \cdot N_2}{36 \cdot N_2^{\,2} - 72 \cdot N_2^{\,3} + 52 \cdot N_2^{\,4} - \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(2 \cdot N_2^{\,3} + 12 \cdot N_2^{\,2} - 6 \cdot N_2\right)} = 0$$

$$AG - \frac{N_1 + N_1 \cdot N_2^{\,2} - 4 \cdot N_1 \cdot N_2^{\,3} + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(4 \cdot N_1 \cdot N_2^{\,2} - 3 \cdot N_1 \cdot N_2 + N_1\right) - 2 \cdot N_1 \cdot N_2}{6 \cdot N_2 - 12 \cdot N_2^{\,2} + 14 \cdot N_2^{\,3} - 4 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot N_2^{\,2}} = 0$$

$$AJ - \frac{13 \cdot N_1 \cdot N_2^{\,3} - 16 \cdot N_1 \cdot N_2^{\,2} - 2 \cdot N_1 + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(N_1 \cdot N_2 - 3 \cdot N_1 \cdot N_2^{\,2}\right) + 9 \cdot N_1 \cdot N_2}{6 \cdot N_2 - 12 \cdot N_2^{\,2} + 14 \cdot N_2^{\,3} - 4 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot N_2^{\,2}} = 0$$

Etc.

$$BF-\frac{N_1}{2\cdot N_2} = 0.00000\ \text{in.}$$

$$AF-\frac{N_1\cdot(N_2-1)\cdot\left(\left(N_2^2-2\cdot N_2-N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)+1\right)}{2\cdot N_2\cdot\left(\left(\left(6\cdot N_2-7\cdot N_2^2\right)+2\cdot N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-3\right)} = 0.00000\ \text{in.}$$

$$AB-\frac{N_1\cdot\left(\left(\left(\left(3\cdot N_2+N_2^2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-4\cdot N_2^2-N_2^3\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-2\right)}{2\cdot\left(\left(3\cdot N_2-2\cdot N_2^2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}-6\cdot N_2^2\right)+7\cdot N_2^3\right)} = 0.00000\ \text{in}$$

Animate Point

$N_1 = 2.40000$ in.
$N_2 = 1.86107$

$$FH-\frac{N_1\cdot(N_2-1)}{2\cdot N_2} = 0.00000\ \text{in.}$$

$$FR-\frac{N_1\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}}{2\cdot N_2} = 0.00000\ \text{in.}$$

$$FP-\frac{N_1\cdot(N_2-1)^2}{2\cdot N_2\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\ \text{in.}$$

$$PH-\frac{N_1\cdot(N_2-1)}{\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\ \text{in.}$$

$$LP-\frac{N_1\cdot\sqrt{(N_2^2+6\cdot N_2)-3}}{2\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\ \text{in.}$$

$$FL-\frac{N_1\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{2\cdot N_2\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\ \text{in.}$$

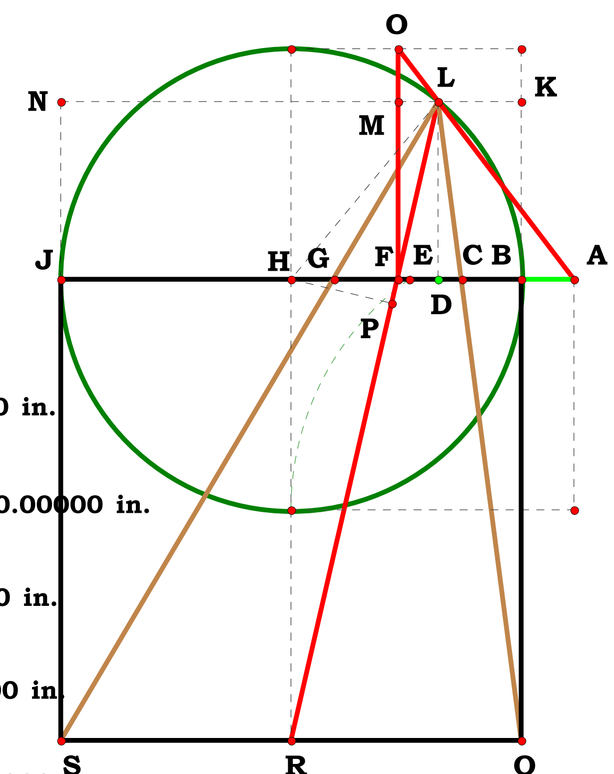
$$DF-\frac{N_1\cdot(N_2-1)\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{2\cdot N_2\cdot\left(\left(5\cdot N_2^2-2\cdot N_2\right)+1\right)} = 0.00000\ \text{in.}$$

$$DL-\frac{N_1\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{(5\cdot N_2^2-2\cdot N_2)+1} = 0.00000\ \text{in.}$$

$$FM-\frac{N_1\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{(5\cdot N_2^2-2\cdot N_2)+1} = 0.00000\ \text{in.}$$

$$MO-\frac{N_1\cdot\left(\left(7\cdot N_2^2-6\cdot N_2-2\cdot N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)+3\right)}{2\cdot\left(\left(5\cdot N_2^2-2\cdot N_2\right)+1\right)} = 0.00000\ \text{in.}$$

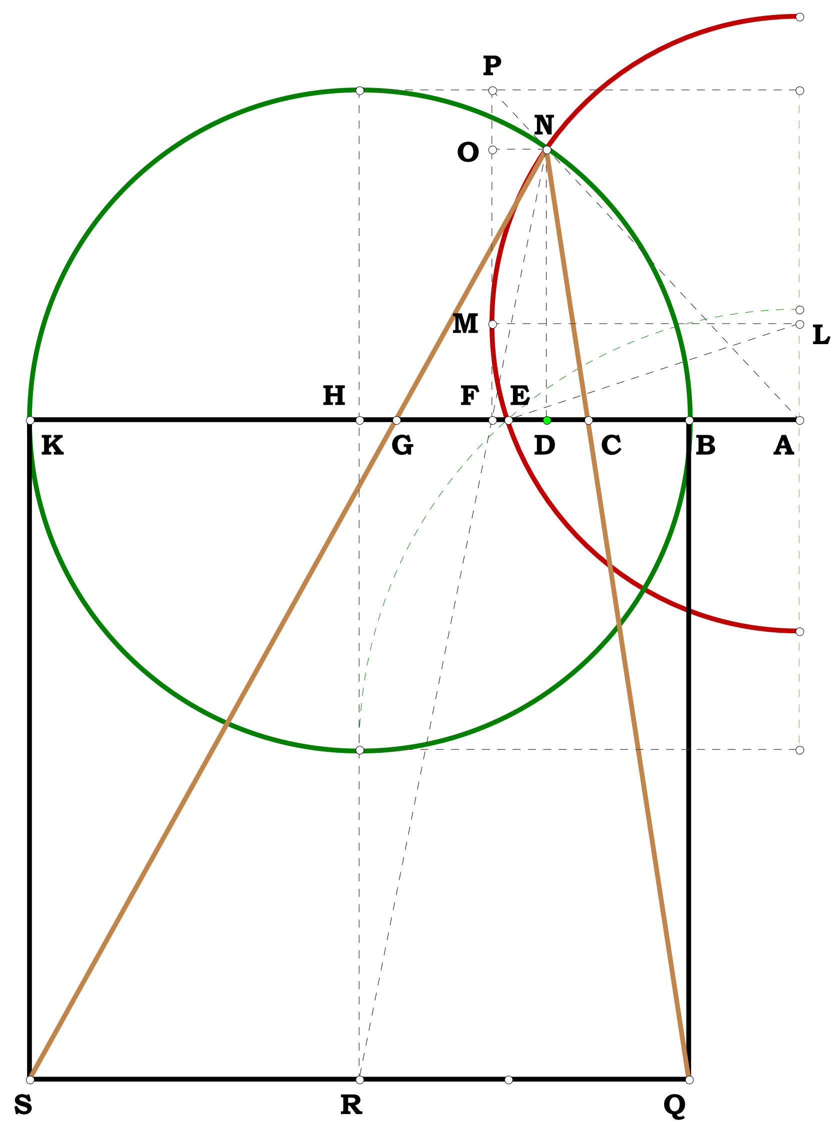
$$LM-\frac{N_1\cdot(N_2-1)\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{2\cdot N_2\cdot\left(\left(5\cdot N_2^2-2\cdot N_2\right)+1\right)} = 0.00000\ \text{in.}$$



BJ = 2.40000 in.
BH = 1.20000 in.
BF = 0.64479 in.
$\frac{BH}{BF} = 1.86107$
FH = 0.55521 in.
FR = 2.46338 in.
FP = 0.12514 in.
PH = 0.54092 in.
LP = 1.07117 in.
FL = 0.94603 in.
DF = 0.21322 in.
DL = 0.92169 in.
FM = 0.92169 in.
MO = 0.27831 in.
LM = 0.21322 in.
AF = 0.91936 in.
AB = 0.27457 in.

111893 Exploring Cube Roots Plate C

If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.



$$N_1 := 4 \qquad N_2 := 1.2 \qquad BK := N_1$$

$$BH := \frac{BK}{2} \qquad BD := \frac{BH}{N_2} \qquad DK := BK - BD$$

$$DN := \sqrt{BD \cdot DK} \qquad BQ := BK \qquad KS := BK \qquad HR := BK$$

$$BC := \frac{BD \cdot BQ}{BQ + DN} \qquad GK := \frac{DK \cdot KS}{KS + DN} \qquad BG := BK - GK$$

$$DH := BH - BD \qquad FH := \frac{DH \cdot HR}{HR + DN} \qquad BF := BH - FH$$

$$CF := BF - BC \qquad AL := CF \qquad DF := BF - BD$$

$$NO := DF \qquad FP := BH \qquad PO := FP - DN$$

$$AD := \frac{NO \cdot DN}{PO} \qquad AB := AD - BD$$

$$AF := AB + BF \qquad LM := AF \qquad EL := AF \qquad AK := AD + DK$$

$$AE_1 := \sqrt{EL^2 - AL^2} \qquad AE_2 := \sqrt{AB \cdot AK} \qquad AE_1 - AE_2 = 0$$

$N_1 := 4 \qquad N_2 := 1.2 \qquad BK := N_1$

$$BH - \frac{N_1}{2} = 0 \qquad BD - \frac{N_1}{2 \cdot N_2} = 0 \qquad DK - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2} = 0 \qquad DN - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2} = 0 \qquad BQ - N_1 = 0 \qquad KS - N_1 = 0$$

$$HR - N_1 = 0 \qquad BC - \frac{N_1}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \qquad GK - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \qquad BG - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} + 1\right)}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \qquad DH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0$$

$$FH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \qquad BF - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} + 2\right)}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \qquad CF - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \qquad AL - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0$$

$$DF - \frac{\sqrt{2 \cdot N_2 - 1} \cdot N_1 \cdot (N_2 - 1)}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \qquad NO - \frac{\sqrt{2 \cdot N_2 - 1} \cdot N_1 \cdot (N_2 - 1)}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \qquad FP - \frac{N_1}{2} = 0 \qquad PO - \frac{N_1 \cdot \left(N_2 - \sqrt{2 \cdot N_2 - 1}\right)}{2 \cdot N_2} = 0$$

$$AD - \frac{N_1 \cdot (N_2 - 1) \cdot \left(\sqrt{2 \cdot N_2 - 1}\right)^2}{2 \cdot N_2 \cdot \left(N_2 - \sqrt{2 \cdot N_2 - 1}\right) \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \qquad AB - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} - 1\right)}{2 \cdot \left(2 \cdot N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1} + 1\right)} = 0$$

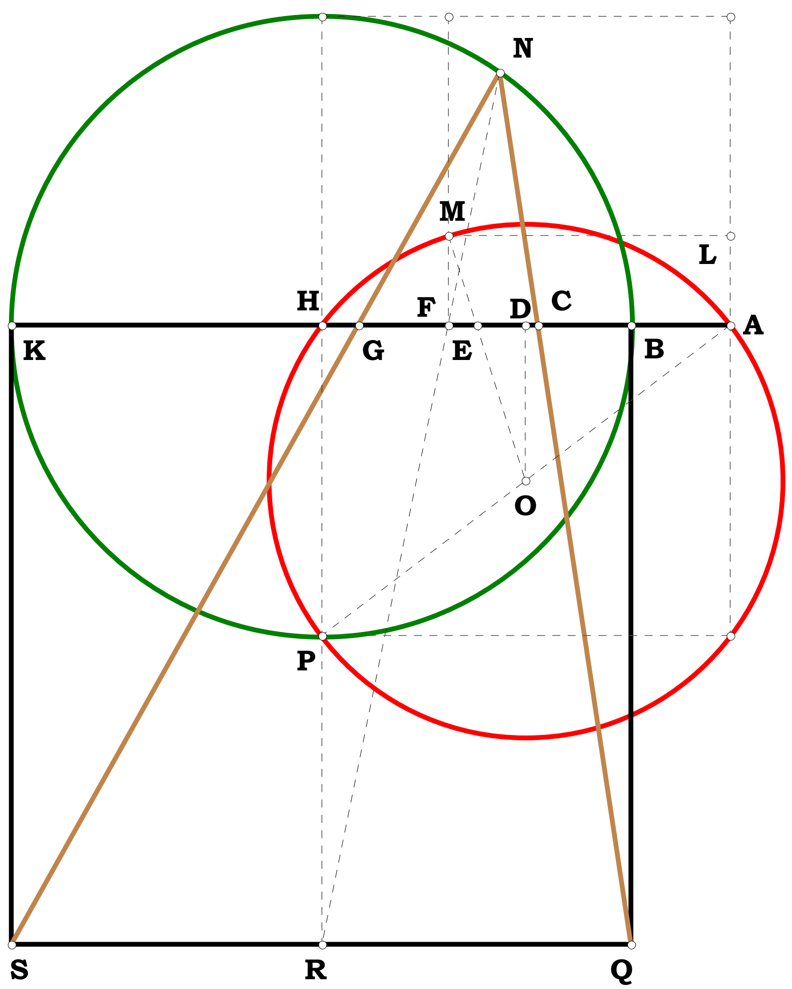
$$AF - \frac{N_1 \cdot (N_2 - 1) \cdot \left(2 \cdot N_2 + 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1} - 1\right)}{2 \cdot \left(3 \cdot N_2 + \sqrt{2 \cdot N_2 - 1} - 6 \cdot N_2^2 + 4 \cdot N_2^3 - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1}\right)} = 0 \qquad EL - AF = 0 \qquad LM - AF = 0 \qquad AK - \frac{N_1 \cdot \left[4 \cdot N_2^2 - (2 \cdot N_2 - 1)^{\frac{3}{2}} - 4 \cdot N_2 + 1\right]}{2 \cdot \left(2 \cdot N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1} + 1\right)} = 0$$

$$AE_1 - \sqrt{\left[\frac{N_1^2 \cdot (N_2 - 1)^2 \cdot \left(2 \cdot N_2 + 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1} - 1\right)^2}{4 \cdot \left(3 \cdot N_2 + \sqrt{2 \cdot N_2 - 1} - 6 \cdot N_2^2 + 4 \cdot N_2^3 - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1}\right)^2} - \frac{N_1^2 \cdot (2 \cdot N_2 - 1)}{4 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)^2}\right]} = 0$$

$$AE_2 - \sqrt{\left[\frac{N_1^2 \cdot \left(\sqrt{2 \cdot N_2 - 1} - 1\right) \cdot \left[4 \cdot N_2^2 - (2 \cdot N_2 - 1)^{\frac{3}{2}} - 4 \cdot N_2 + 1\right]}{4 \cdot \left(2 \cdot N_2 - 2 \cdot N_2^2 + N_2 \cdot \sqrt{2 \cdot N_2 - 1} - 1\right)^2}\right]} = 0 \qquad AE_1 - AE_2 = 0$$

111893D Exploring Cube Roots

The circle AO passes through point M. FM equals half of CG.



$N_1 := 3 \quad N_2 := 2$

$AB := N_1 \quad BK := N_2 \quad AK := BK + AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

$CG := AG - AC \quad CF := \frac{CG}{2} \quad BH := \frac{BK}{2}$

$AH := AB + BH \quad HP := BH \quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \quad DO := \frac{HP}{2} \quad AF := AC + CF \quad AD := \frac{AH}{2}$

$DF := AF - AD \quad FM := CF \quad MO := AO$

$$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$$

$$AK-(N_1+N_2)=0$$

$$AC-\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}=0 \quad AG-\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}=0 \quad CG-\left[\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}-\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}\right]=0$$

$$CF-\frac{\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}-\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}}{2}=0 \quad BH-\frac{N_2}{2}=0 \quad AH-\frac{2\cdot N_1+N_2}{2}=0 \quad HP-\frac{N_2}{2}=0 \quad AP-\frac{\sqrt{2\cdot N_1{}^2+2\cdot N_1\cdot N_2+N_2{}^2}}{\sqrt{2}}=0$$

$$AO-\frac{\sqrt{2}}{4}\cdot\sqrt{2\cdot N_1{}^2+2\cdot N_1\cdot N_2+N_2{}^2}=0 \quad DO-\frac{N_2}{4}=0 \quad AF-\frac{\left[\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}+\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}\right]}{2}=0 \quad AD-\frac{2\cdot N_1+N_2}{4}=0$$

$$DF-\frac{\left[2\cdot\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}-N_2-2\cdot N_1+2\cdot\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}\right]}{4}=0 \quad FM-\frac{\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}-\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}}{2}=0$$

$$MO-\frac{\sqrt{2}}{4}\cdot\sqrt{2\cdot N_1{}^2+2\cdot N_1\cdot N_2+N_2{}^2}=0$$

$$MO^2-\frac{\begin{array}{l}4\cdot\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{2}{3}}+2\cdot N_1{}^2+N_2{}^2-4\cdot N_1\cdot\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}-4\cdot N_2\cdot\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}\ \ldots\\ +\ 2\cdot N_1\cdot N_2+4\cdot\left(N_1{}^3+N_2\cdot N_1{}^2\right)^{\frac{2}{3}}-4\cdot N_1\cdot\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}\end{array}}{8}=0$$

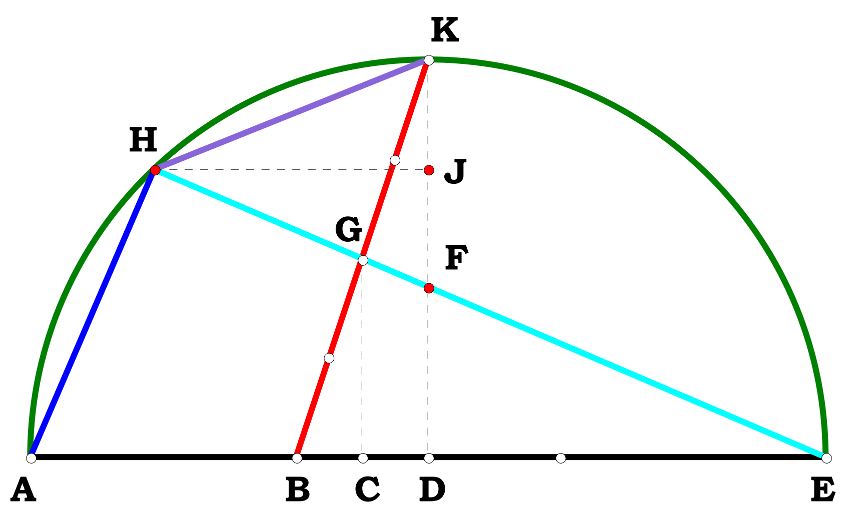
$$\frac{\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{2}{3}}}{2}+\frac{\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{2}{3}}}{2}-\frac{N_1\cdot\left[N_1\cdot(N_1+N_2)^2\right]^{\frac{1}{3}}}{2}-\frac{N_1\cdot\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}}{2}-\frac{N_2\cdot\left[N_1{}^2\cdot(N_1+N_2)\right]^{\frac{1}{3}}}{2}=0$$

Generalize the work of 07/25/93 for dividing the base AE with K constant.

$N_1 := 3 \quad N_2 := 5 \quad AE := 1$

$\alpha := 1 .. N_1 - 1 \quad \beta := 1 .. N_2 - 1$

112493 POR Series IV



$AB := \dfrac{AE}{N_1} \quad AD := \dfrac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \dfrac{BK}{N_2} \quad BC := \dfrac{BD \cdot BG}{BK}$

$CG := \dfrac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \dfrac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \dfrac{DF \cdot AE}{EF} \quad EH := \dfrac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

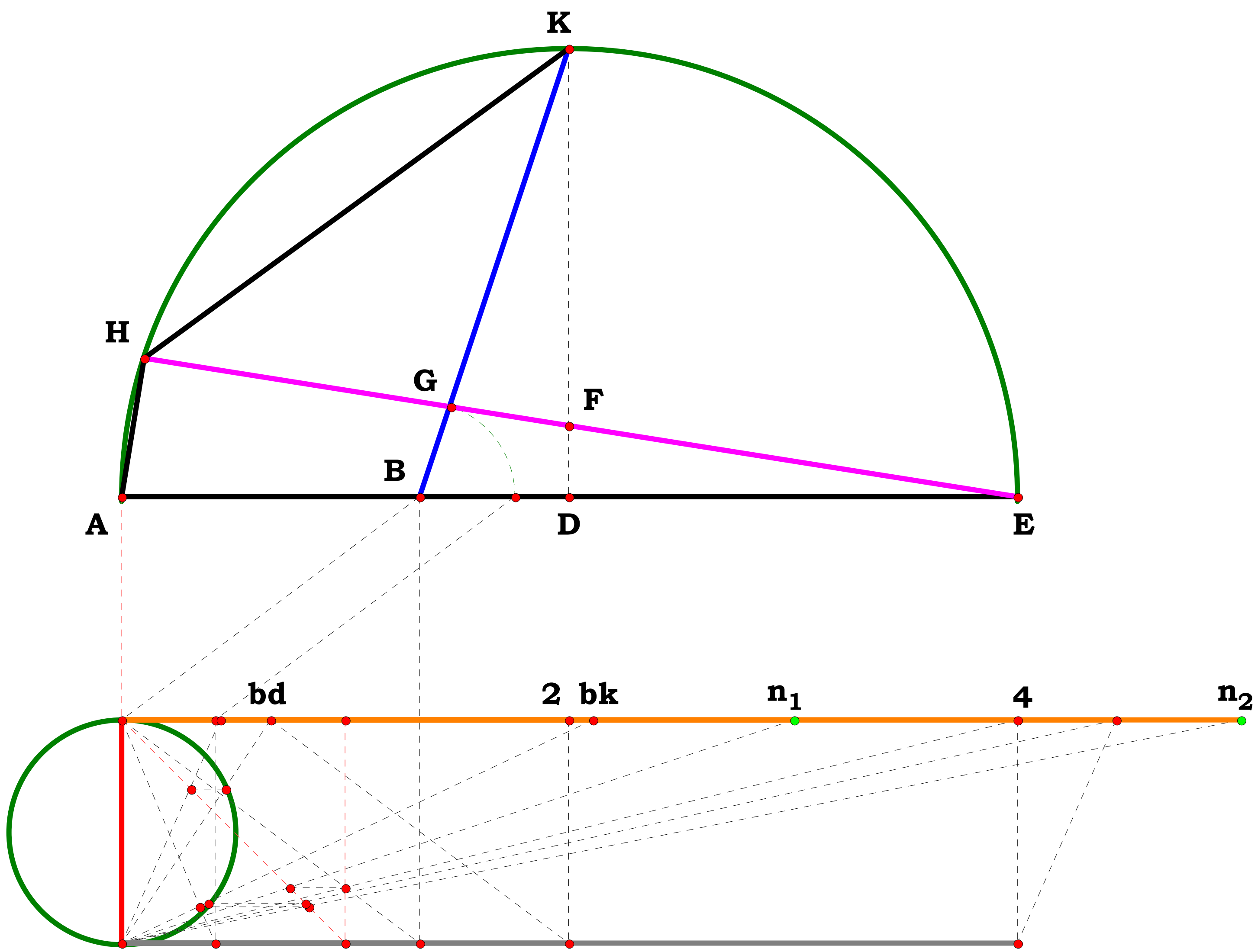
$FJ := \dfrac{DF \cdot FH}{EF} \quad HJ := \dfrac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

$\dfrac{AH}{HK} = 0.265 \qquad \dfrac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \qquad SeriesAH_{\alpha, \beta} := \dfrac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)} \qquad SeriesAH = \begin{pmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{pmatrix}$

$\dfrac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \dfrac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot \left(2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^{\,2} - 2 \cdot N_1 + 2\right)} = 2.85$

$SeriesEH_{\alpha, \beta} := N_1 \cdot N_2 \cdot \dfrac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot \left(2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^{\,2} \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta\right)}$

$SeriesEH = \begin{pmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{pmatrix}$



$HK = 2.35269$

$$\frac{AH}{HK} = 0.26517$$

$$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} = 0.26517$$

$$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} - \frac{AH}{HK} = 0.00000$$

$EH = 3.95105$

$GH = 1.38633$

$$\frac{EH}{GH} = 2.85000$$

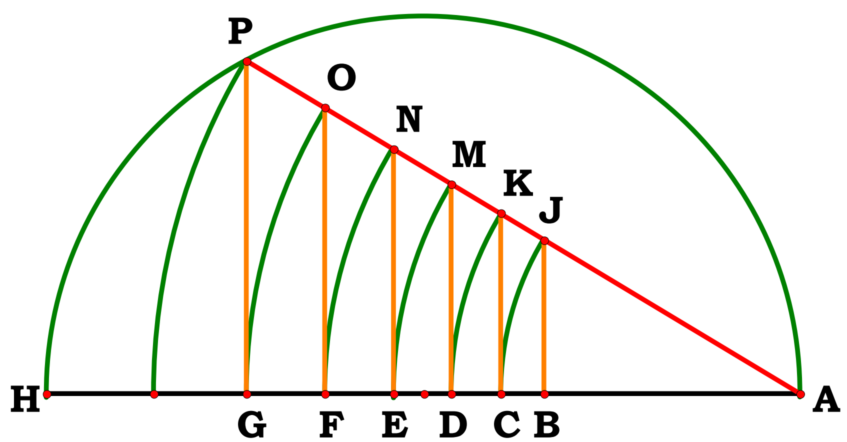
$$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} = 2.85000$$

$$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} - \frac{EH}{GH} = 0.00000$$

120293.MCD POR Roots and Powers
(Pyramid of Ratio Series V)

Is the progression noticed in 11_29_93 a continuous phenomenon?

$AH := 5 \qquad \delta := 1 .. 7 \qquad AP_\delta := \dfrac{AH}{\delta}$



$AG_\delta := \dfrac{(AP_\delta)^2}{AH} \qquad AO_\delta := AG_\delta \qquad AF_\delta := \dfrac{(AG_\delta)^2}{AP_\delta}$

$AE_\delta := \dfrac{(AF_\delta)^2}{AO_\delta} \qquad AN_\delta := AF_\delta \qquad AD_\delta := \dfrac{(AE_\delta)^2}{AN_\delta} \qquad AM_\delta := AE_\delta$

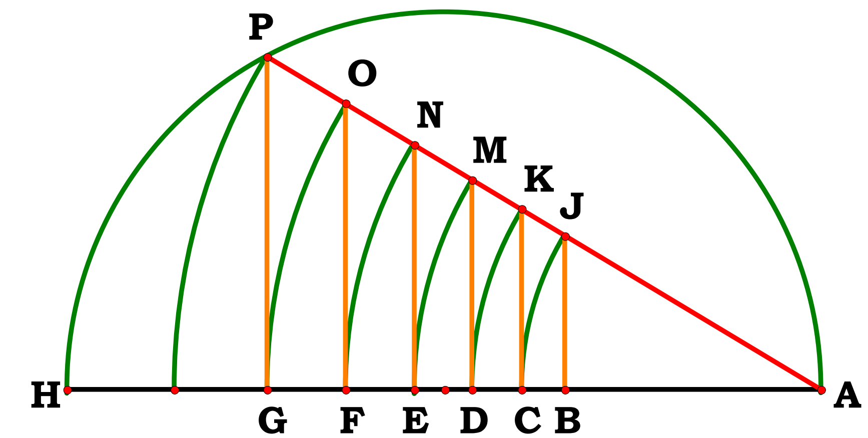
$AC_\delta := \dfrac{(AD_\delta)^2}{AM_\delta} \qquad AK_\delta := AD_\delta \qquad AB_\delta := \dfrac{(AC_\delta)^2}{AK_\delta} \qquad \Delta := 1 .. 6$

$$\frac{AH}{AP_\Delta} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} =$$

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6

$$\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}} = \frac{AC_\Delta}{AB_\Delta} =$$

1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6



$$\begin{pmatrix} \frac{AH}{1}, & \frac{AH}{1^2}, & \frac{AH}{1^3}, & \frac{AH}{1^4}, & \text{etc} \\ \frac{AH}{2}, & \frac{AH}{2^2}, & \frac{AH}{2^3}, & \frac{AH}{2^4}, & \text{etc} \\ \frac{AH}{3}, & \frac{AH}{3^2}, & \frac{AH}{3^3}, & \frac{AH}{3^4}, & \text{etc} \\ \frac{AH}{4}, & \frac{AH}{4^2}, & \frac{AH}{4^3}, & \frac{AH}{4^4}, & \text{etc} \\ \frac{AH}{5}, & \frac{AH}{5^2}, & \frac{AH}{5^3}, & \frac{AH}{5^4}, & \text{etc} \end{pmatrix}$$

$AP_\Delta =$	$AG_\Delta =$	$AF_\Delta =$	$AE_\Delta =$	$AD_\Delta =$	$AC_\Delta =$	$AB_\Delta =$
5	5	5	5	5	5	5
2.5	1.25	0.625	0.313	0.156	0.078	0.039
1.667	0.556	0.185	0.062	0.021	$6.859\cdot10^{-3}$	$2.286\cdot10^{-3}$
1.25	0.313	0.078	0.02	$4.883\cdot10^{-3}$	$1.221\cdot10^{-3}$	$3.052\cdot10^{-4}$
1	0.2	0.04	$8\cdot10^{-3}$	$1.6\cdot10^{-3}$	$3.2\cdot10^{-4}$	$6.4\cdot10^{-5}$
0.833	0.139	0.023	$3.858\cdot10^{-3}$	$6.43\cdot10^{-4}$	$1.072\cdot10^{-4}$	$1.786\cdot10^{-5}$

$$AP_\Delta = \quad AG_\Delta\cdot\frac{AH}{AP_\Delta} = AF_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^2 = AE_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^3 = AD_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^4 = AC_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^5 = AB_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^6 =$$

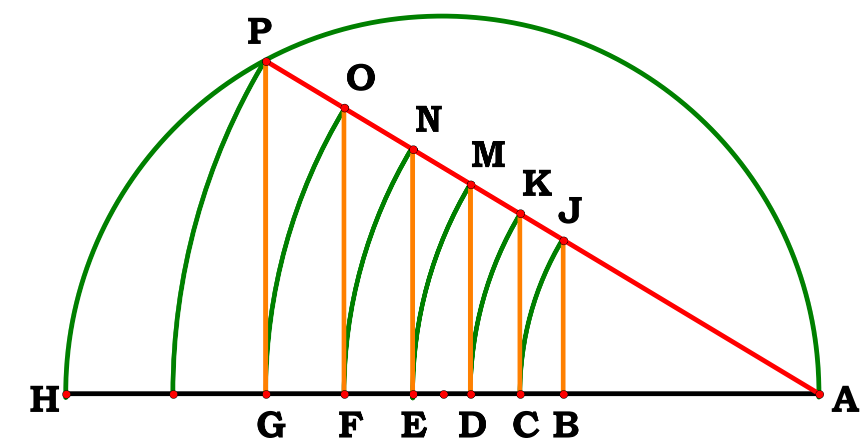
5	5	5	5	5	5	5
2.5	2.5	2.5	2.5	2.5	2.5	2.5
1.667	1.667	1.667	1.667	1.667	1.667	1.667
1.25	1.25	1.25	1.25	1.25	1.25	1.25
1	1	1	1	1	1	1
0.833	0.833	0.833	0.833	0.833	0.833	0.833

$$\frac{AH^2}{AP_\triangle} = \left(\frac{AH^3}{AG_\triangle}\right)^{\frac{1}{2}} = \left(\frac{AH^4}{AF_\triangle}\right)^{\frac{1}{3}} = \left(\frac{AH^5}{AE_\triangle}\right)^{\frac{1}{4}} = \left(\frac{AH^6}{AD_\triangle}\right)^{\frac{1}{5}} = \left(\frac{AH^7}{AC_\triangle}\right)^{\frac{1}{6}} = \left(\frac{AH^8}{AB_\triangle}\right)^{\frac{1}{7}} = \left(\frac{AH}{AB_\triangle}\right)^{\frac{1}{7}} \cdot AH =$$

5	5	5	5	5	5	5	5
10	10	10	10	10	10	10	10
15	15	15	15	15	15	15	15
20	20	20	20	20	20	20	20
25	25	25	25	25	25	25	25
30	30	30	30	30	30	30	30

$$\left(\frac{AH}{AB_\triangle}\right)^{\frac{3}{7}} = \left(\frac{AH}{AC_\triangle}\right)^{\frac{3}{6}} = \left(\frac{AH}{AD_\triangle}\right)^{\frac{3}{5}} = \left(\frac{AH}{AE_\triangle}\right)^{\frac{3}{4}} = \left(\frac{AH}{AF_\triangle}\right)^{\frac{3}{3}} = \left(\frac{AH}{AG_\triangle}\right)^{\frac{3}{2}} = \left(\frac{AH}{AP_\triangle}\right)^{3} =$$

1	1	1	1	1	1	1
8	8	8	8	8	8	8
27	27	27	27	27	27	27
64	64	64	64	64	64	64
125	125	125	125	125	125	125
216	216	216	216	216	216	216



$$\left(\frac{AH}{AB_\triangle}\right)^{\frac{7}{7}} = \left(\frac{AH}{AC_\triangle}\right)^{\frac{7}{6}} = \left(\frac{AH}{AD_\triangle}\right)^{\frac{7}{5}} = \left(\frac{AH}{AE_\triangle}\right)^{\frac{7}{4}} = \left(\frac{AH}{AF_\triangle}\right)^{\frac{7}{3}} = \left(\frac{AH}{AG_\triangle}\right)^{\frac{7}{2}} = \left(\frac{AH}{AP_\triangle}\right)^{\frac{7}{1}} =$$

1	1	1	1	1	1	1
128	128	128	128	128	128	128
$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$
$1.638\cdot10^4$	$1.638\cdot10^4$	$1.638\cdot10^4$	$1.638\cdot10^4$	$1.638\cdot10^4$	$1.638\cdot10^4$	$1.638\cdot10^4$
$7.812\cdot10^4$	$7.812\cdot10^4$	$7.812\cdot10^4$	$7.812\cdot10^4$	$7.813\cdot10^4$	$7.813\cdot10^4$	$7.813\cdot10^4$
$2.799\cdot10^5$	$2.799\cdot10^5$	$2.799\cdot10^5$	$2.799\cdot10^5$	$2.799\cdot10^5$	$2.799\cdot10^5$	$2.799\cdot10^5$

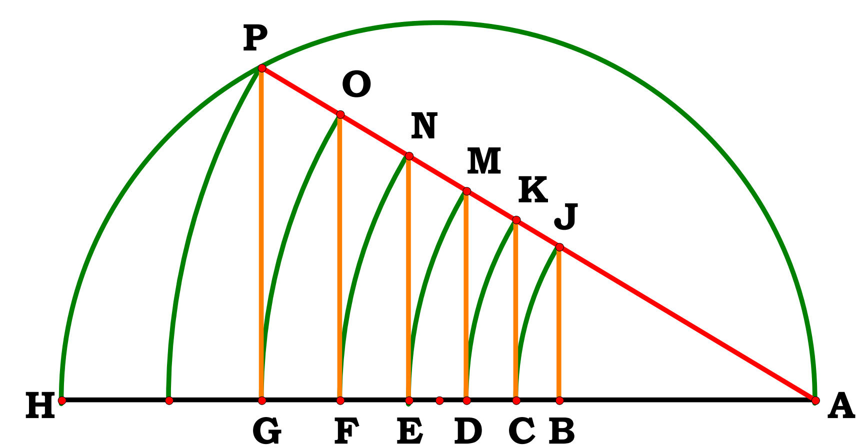
$$GH_\delta := AH - AG_\delta \quad GP_\delta := \sqrt{AG_\delta \cdot GH_\delta} \quad FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta} \quad EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta} \quad DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta}$$

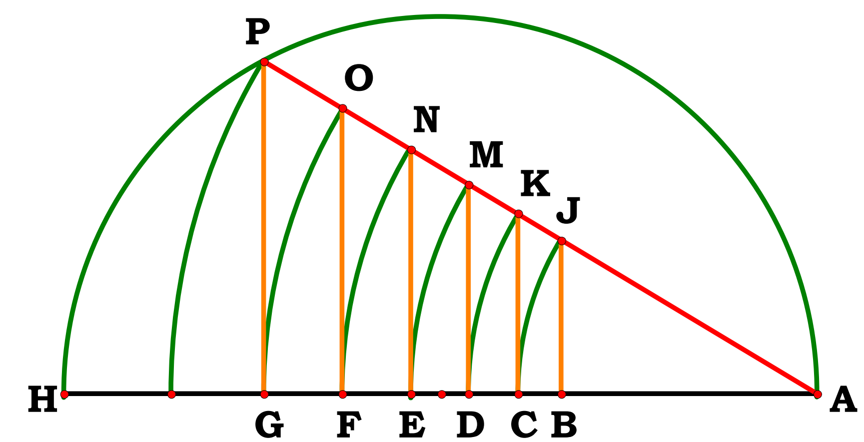
$$CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta} \quad BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$$

$GP_\Delta =$	$FO_\Delta =$	$EN_\Delta =$	$DM_\Delta =$	$CK_\Delta =$	$BJ_\Delta =$
0	0	0	0	0	0
2.165	1.083	0.541	0.271	0.135	0.068
1.571	0.524	0.175	0.058	0.019	$6.466\cdot10^{-3}$
1.21	0.303	0.076	0.019	$4.728\cdot10^{-3}$	$1.182\cdot10^{-3}$
0.98	0.196	0.039	$7.838\cdot10^{-3}$	$1.568\cdot10^{-3}$	$3.135\cdot10^{-4}$
0.822	0.137	0.023	$3.804\cdot10^{-3}$	$6.34\cdot10^{-4}$	$1.057\cdot10^{-4}$

$\frac{GP_\Delta}{\Delta^5} =$	$\frac{FO_\Delta}{\Delta^4} =$	$\frac{EN_\Delta}{\Delta^3} =$	$\frac{DM_\Delta}{\Delta^2} =$	$\frac{CK_\Delta}{\Delta} =$	$BJ_\Delta =$
0	0	0	0	0	0
0.068	0.068	0.068	0.068	0.068	0.068
$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$
$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$
$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$
$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$

$\frac{GP_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^5} =$	$\frac{FO_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^4} =$	$\frac{EN_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^3} =$	$\frac{DM_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^2} =$	$\frac{CK_\Delta}{\frac{AH}{AP_\Delta}} =$	$BJ_\Delta =$
0	0	0	0	0	0
0.068	0.068	0.068	0.068	0.068	0.068
$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$	$6.466\cdot10^{-3}$
$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$	$1.182\cdot10^{-3}$
$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$	$3.135\cdot10^{-4}$
$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$	$1.057\cdot10^{-4}$

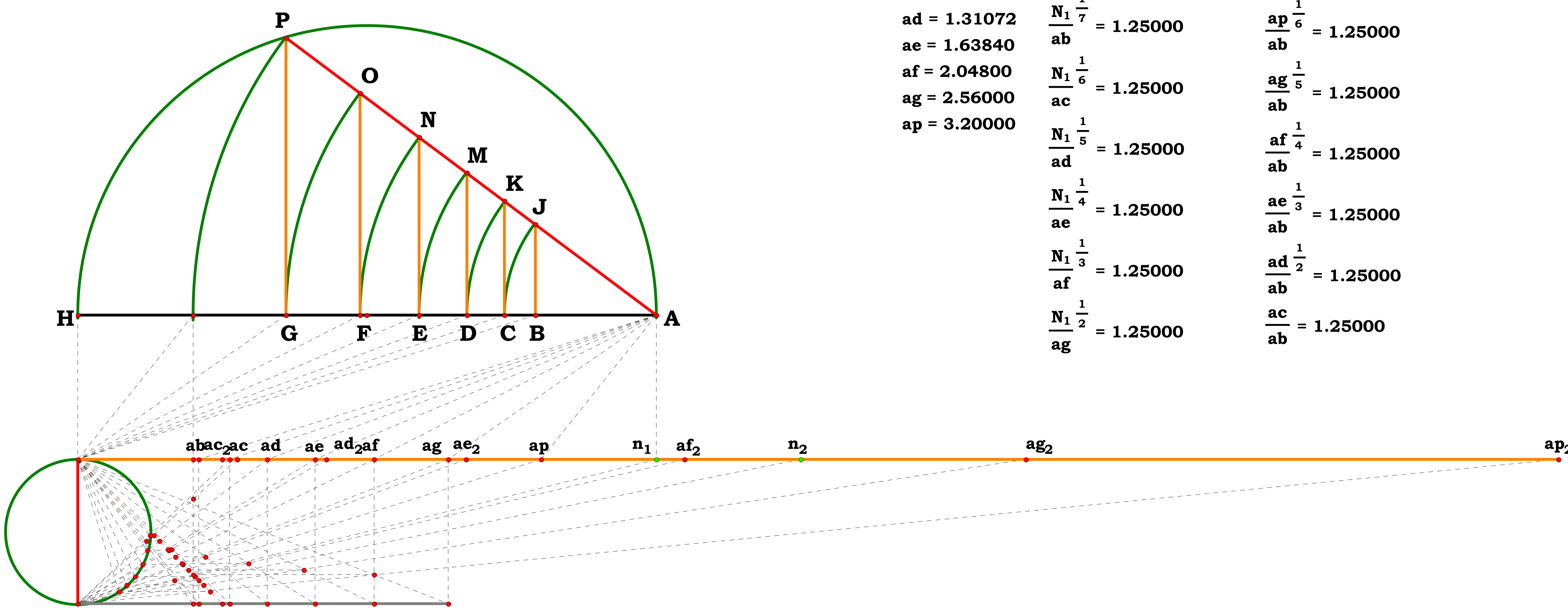




GP_{Δ} =	FO_{Δ} =	EN_{Δ} =	DM_{Δ} =	CK_{Δ} =	BJ_{Δ} =
0	0	0	0	0	0
2.165	1.083	0.541	0.271	0.135	0.068
1.571	0.524	0.175	0.058	0.019	$6.466\cdot10^{-3}$
1.21	0.303	0.076	0.019	$4.728\cdot10^{-3}$	$1.182\cdot10^{-3}$
0.98	0.196	0.039	$7.838\cdot10^{-3}$	$1.568\cdot10^{-3}$	$3.135\cdot10^{-4}$
0.822	0.137	0.023	$3.804\cdot10^{-3}$	$6.34\cdot10^{-4}$	$1.057\cdot10^{-4}$

$$\frac{\sqrt{AG_2\cdot GH_2}}{2} = 1.083 \qquad \frac{\sqrt{AG_2\cdot GH_2}}{2^2} = 0.541 \qquad \frac{\sqrt{AG_2\cdot GH_2}}{2^3} = 0.271$$

$$\frac{\sqrt{AG_3\cdot GH_3}}{3} = 0.524 \qquad \frac{\sqrt{AG_3\cdot GH_3}}{3^2} = 0.175 \qquad \frac{\sqrt{AG_3\cdot GH_3}}{3^3} = 0.058$$



ab = 0.83886
ac = 1.04858
ad = 1.31072
ae = 1.63840
af = 2.04800
ag = 2.56000
ap = 3.20000

$\frac{N_1}{ap} = 1.25000$

$\left(\frac{N_1}{ab}\right)^{\frac{1}{7}} = 1.25000$

$\left(\frac{N_1}{ac}\right)^{\frac{1}{6}} = 1.25000$

$\left(\frac{N_1}{ad}\right)^{\frac{1}{5}} = 1.25000$

$\left(\frac{N_1}{ae}\right)^{\frac{1}{4}} = 1.25000$

$\left(\frac{N_1}{af}\right)^{\frac{1}{3}} = 1.25000$

$\left(\frac{N_1}{ag}\right)^{\frac{1}{2}} = 1.25000$

$\left(\frac{ap}{ab}\right)^{\frac{1}{6}} = 1.25000$

$\left(\frac{ag}{ab}\right)^{\frac{1}{5}} = 1.25000$

$\left(\frac{af}{ab}\right)^{\frac{1}{4}} = 1.25000$

$\left(\frac{ae}{ab}\right)^{\frac{1}{3}} = 1.25000$

$\left(\frac{ad}{ab}\right)^{\frac{1}{2}} = 1.25000$

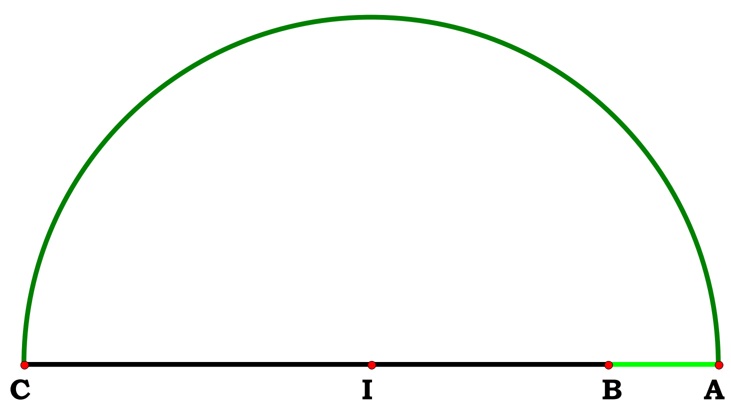
$\frac{ac}{ab} = 1.25000$

120493 Exponential Series M^(1/2^N)

A number is no more than a digital name used with the stipulation that the indexing system is further qualified by using as standard difference. In other words that the concept of ratio will employ a name called a number. Ratio, however, is independent of the naming convention. Given any ration, say, M, describe a two prime exponential series, M^1/2^N, where N is any whole ratio.

To use the digital indexing system to apply names, let AC be the thing with which we seek to name an exponential series on. AB is our unit. As a number is a ratio, numbers are two dimensional.

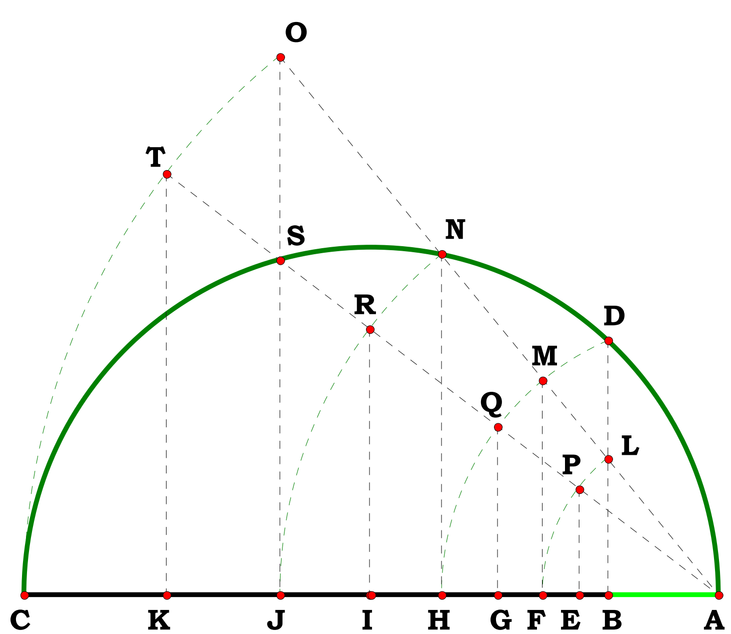
$N_1 := 1 \qquad N_2 := 5 \qquad AB := N_1 \qquad AC := N_2$



The circle is a two dimensional object which is capable of producing every ratio between two differences.

$BC := AC - AB \quad BD := \sqrt{BC \cdot AB} \quad AH := \sqrt{AB^2 + BD^2} \quad CH := AC - AH \quad HN := \sqrt{CH \cdot AH}$

$AJ := \sqrt{AH^2 + HN^2} \quad CJ := AC - AJ \quad JS := \sqrt{CJ \cdot AJ} \quad AK := \sqrt{JS^2 + AJ^2}$



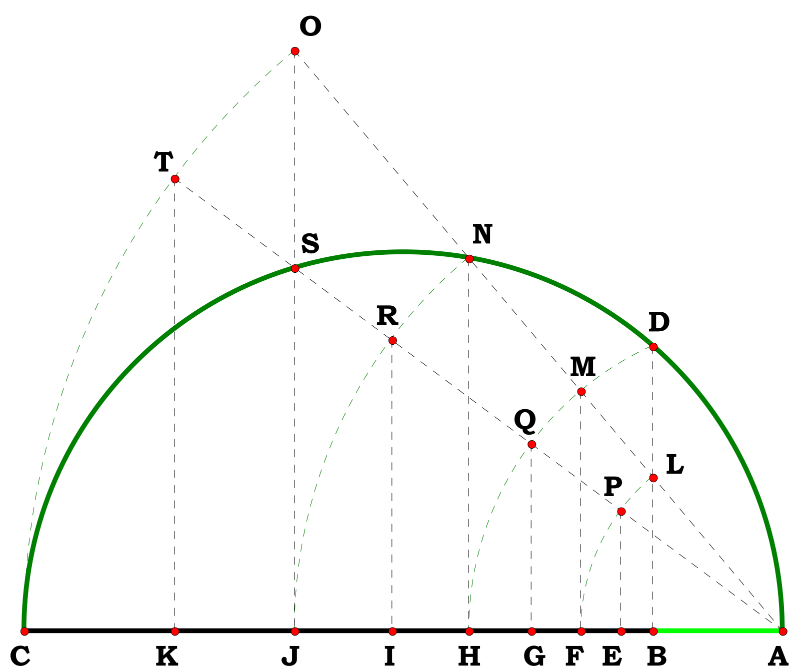
$$AI := \frac{AJ^2}{AK} \quad AG := \frac{AH^2}{AI} \quad AF := \frac{AH^2}{AJ} \quad AE := \frac{AF^2}{AG} \quad \left(\frac{AC}{AB}\right)^0 - \frac{AB}{AB} = 0$$

$$AC^{\frac{1}{8}} - AE = 0 \quad AC^{\frac{2}{8}} - AF = 0 \quad AC^{\frac{3}{8}} - AG = 0 \quad AC^{\frac{4}{8}} - AH = 0$$

$$AC^{\frac{5}{8}} - AI = 0 \quad AC^{\frac{6}{8}} - AJ = 0 \quad AC^{\frac{7}{8}} - AK = 0 \quad AC^{\frac{8}{8}} - AC = 0$$

$$\left(\frac{AC}{AB}\right)^{\frac{1}{8}} - \frac{AE}{AB} = 0 \quad \left(\frac{AC}{AB}\right)^{\frac{2}{8}} - \frac{AF}{AB} = 0 \quad \left(\frac{AC}{AB}\right)^{\frac{3}{8}} - \frac{AG}{AB} = 0 \quad \left(\frac{AC}{AB}\right)^{\frac{4}{8}} - \frac{AH}{AB} = 0$$

$$\left(\frac{AC}{AB}\right)^{\frac{5}{8}} - \frac{AI}{AB} = 0 \quad \left(\frac{AC}{AB}\right)^{\frac{6}{8}} - \frac{AJ}{AB} = 0 \quad \left(\frac{AC}{AB}\right)^{\frac{7}{8}} - \frac{AK}{AB} = 0 \quad \left(\frac{AC}{AB}\right)^{\frac{8}{8}} - \frac{AC}{AB} = 0$$



$AC^0 = 1.00000$	$N_1 = 1.00000$	$\frac{AB}{AB} = 1.00000$	AC = 3.95000 in.
$AC^{\frac{1}{8}} = 1.24752$	AE = 1.24752	$\frac{AE}{AB} = 1.24752$	AB = 0.67333 in.
$AC^{\frac{2}{8}} = 1.55630$	AF = 1.55630	$\frac{AF}{AB} = 1.55630$	AE = 0.83998 in.
$AC^{\frac{3}{8}} = 1.94151$	AG = 1.94151	$\frac{AG}{AB} = 1.94151$	AF = 1.04790 in.
$AC^{\frac{4}{8}} = 2.42207$	AH = 2.42207	$\frac{AH}{AB} = 2.42207$	AG = 1.30727 in.
$AC^{\frac{5}{8}} = 3.02157$	AI = 3.02157	$\frac{AI}{AB} = 3.02157$	AH = 1.63084 in.
$AC^{\frac{6}{8}} = 3.76946$	AJ = 3.76946	$\frac{AJ}{AB} = 3.76946$	AI = 2.03450 in.
$AC^{\frac{7}{8}} = 4.70247$	AK = 4.70247	$\frac{AK}{AB} = 4.70247$	AJ = 2.53807 in.
$AC^{\frac{8}{8}} = 5.86641$	AC = 5.86641	$\frac{AC}{AB} = 5.86641$	AK = 3.16629 in.

$AC^0\text{-}N_1 = 0.00000$

$AC^{\frac{1}{8}}\text{-}AE = 0.00000$

$AC^{\frac{2}{8}}\text{-}AF = 0.00000$

$AC^{\frac{3}{8}}\text{-}AG = 0.00000$

$AC^{\frac{4}{8}}\text{-}AH = 0.00000$

$AC^{\frac{5}{8}}\text{-}AI = 0.00000$

$AC^{\frac{6}{8}}\text{-}AJ = 0.00000$

$AC^{\frac{7}{8}}\text{-}AK = 0.00000$

$AC^{\frac{8}{8}}\text{-}AC = 0.00000$

Alternate method of creating an exponential series.

$N_1 = 2.97175$

$N_1^{\frac{8}{8}} = 2.97175$

$N_1^{\frac{7}{8}} = 2.59350$

$N_1^{\frac{6}{8}} = 2.26339$

$N_1^{\frac{5}{8}} = 1.97530$

$N_1^{\frac{4}{8}} = 1.72388$

$N_1^{\frac{3}{8}} = 1.50446$

$N_1^{\frac{2}{8}} = 1.31296$

$N_1^{\frac{1}{8}} = 1.14585$

$N_1^{\frac{9}{8}} = 3.40517$

$N_1^{\frac{10}{8}} = 3.90181$

$N_1^{\frac{11}{8}} = 4.47087$

$N_1^{\frac{-1}{8}} = 0.87272$

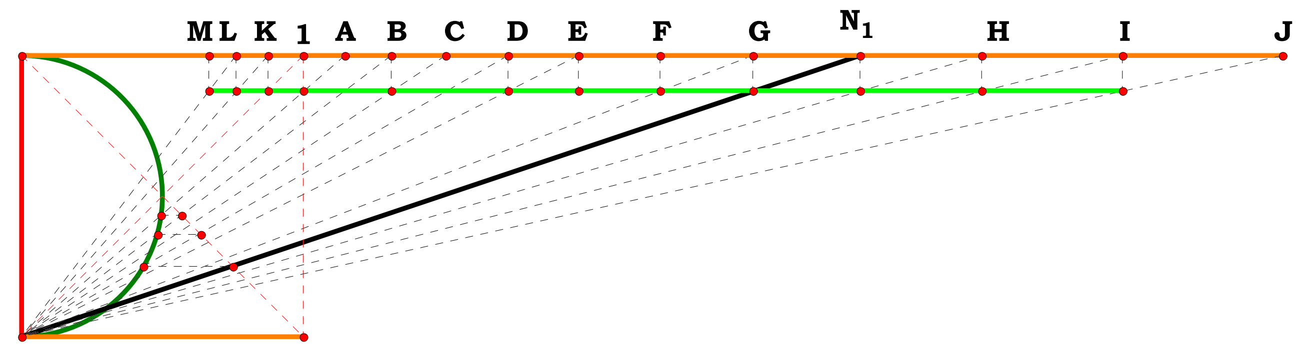
$N_1^{\frac{-2}{8}} = 0.76163$

$N_1^{\frac{-3}{8}} = 0.66469$

A = 1.14585
B = 1.31296
C = 1.50446
D = 1.72388
E = 1.97530
F = 2.26339
G = 2.59350
H = 3.40517
I = 3.90181
J = 4.47087
K = 0.87272
L = 0.76163
M = 0.66469

$N_1^0 = 1.00000$

$N_1^{\frac{8}{8}} - N_1 = 0.00000$



$N_1^{\frac{-1}{8}} - K = 0.00000$

$N_1^{\frac{-2}{8}} - L = 0.00000$

$N_1^{\frac{-3}{8}} - M = 0.00000$

$N_1^{\frac{1}{8}} - A = 0.00000$

$N_1^{\frac{2}{8}} - B = 0.00000$

$N_1^{\frac{3}{8}} - C = 0.00000$

$N_1^{\frac{4}{8}} - D = 0.00000$

$N_1^{\frac{5}{8}} - E = 0.00000$

$N_1^{\frac{6}{8}} - F = 0.00000$

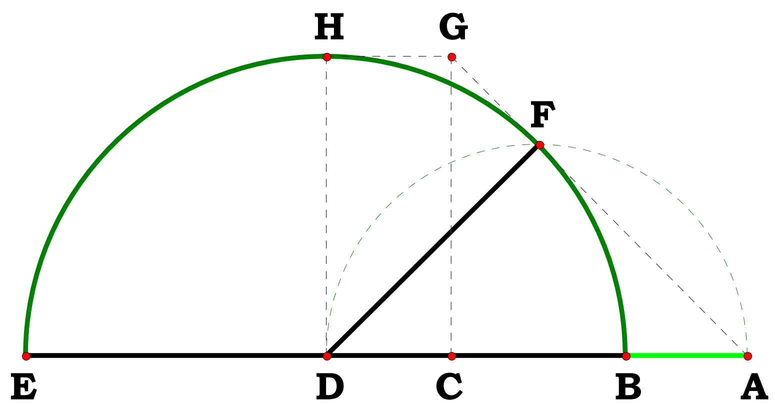
$N_1^{\frac{7}{8}} - G = 0.00000$

$N_1^{\frac{9}{8}} - H = 0.00000$

$N_1^{\frac{10}{8}} - I = 0.00000$

$N_1^{\frac{11}{8}} - J = 0.00000$

120693 Alternate Method: Square Root
Common Segment Common Endpoint



$\mathbf{N_1 := 6} \quad \mathbf{N_2 := 4}$

$\mathbf{AB := N_1} \quad \mathbf{BE := N_2}$

$\mathbf{AE := AB + BE} \quad \mathbf{BD := \dfrac{BE}{2}}$

$\mathbf{AE - \left(N_1 + N_2\right) = 0} \quad \mathbf{BD - \dfrac{N_2}{2} = 0}$

$\mathbf{DF := BD} \quad \mathbf{AD := BD + AB}$

$\mathbf{DF - \dfrac{N_2}{2} = 0} \quad \mathbf{AD - \dfrac{2 \cdot N_1 + N_2}{2} = 0}$

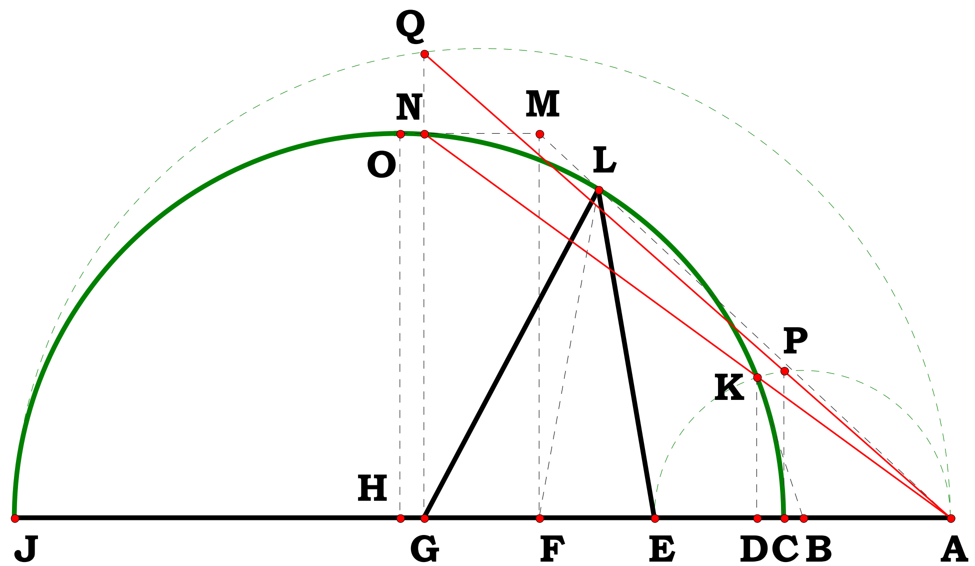
$\mathbf{AF := \sqrt{AD^2 - DF^2}} \quad \mathbf{AC := AF}$

$\mathbf{\sqrt{AB \cdot AE} - AC = 0}$

$\mathbf{AF - \sqrt{\left[N_1 \cdot \left(N_1 + N_2\right)\right]} = 0} \quad \mathbf{AC - \sqrt{\left[N_1 \cdot \left(N_1 + N_2\right)\right]} = 0}$

120693B Gruntwork IV on the Delian Solution.

Are A, P and Q collinear? Are A, K and N collinear?



$N_1 := 5 \qquad N_2 := 6$

$AC := N_1 \qquad CJ := N_2 \qquad AJ := AC + CJ$

$AE := \left(AC^2 \cdot AJ\right)^{\frac{1}{3}} \qquad AG := \left(AC \cdot AJ^2\right)^{\frac{1}{3}}$

$CG := AG - AC \quad GJ := CJ - CG \quad GN := \sqrt{CG \cdot GJ}$

$AB := \dfrac{AE}{2} \quad CE := AE - AC \quad CH := \dfrac{CJ}{2}$

$BK := AB \quad HK := CH \quad HJ := CH \quad AH := AJ - HJ \quad BH := AH - AB \quad BD := \dfrac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

$AD := AB + BD \quad DE := AE - AD \quad DK := \sqrt{AD \cdot DE} \quad GQ := \sqrt{AG \cdot GJ} \quad CP := \sqrt{AC \cdot CE}$

$$\frac{AG}{GN} - \frac{AD}{DK} = 0 \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$$

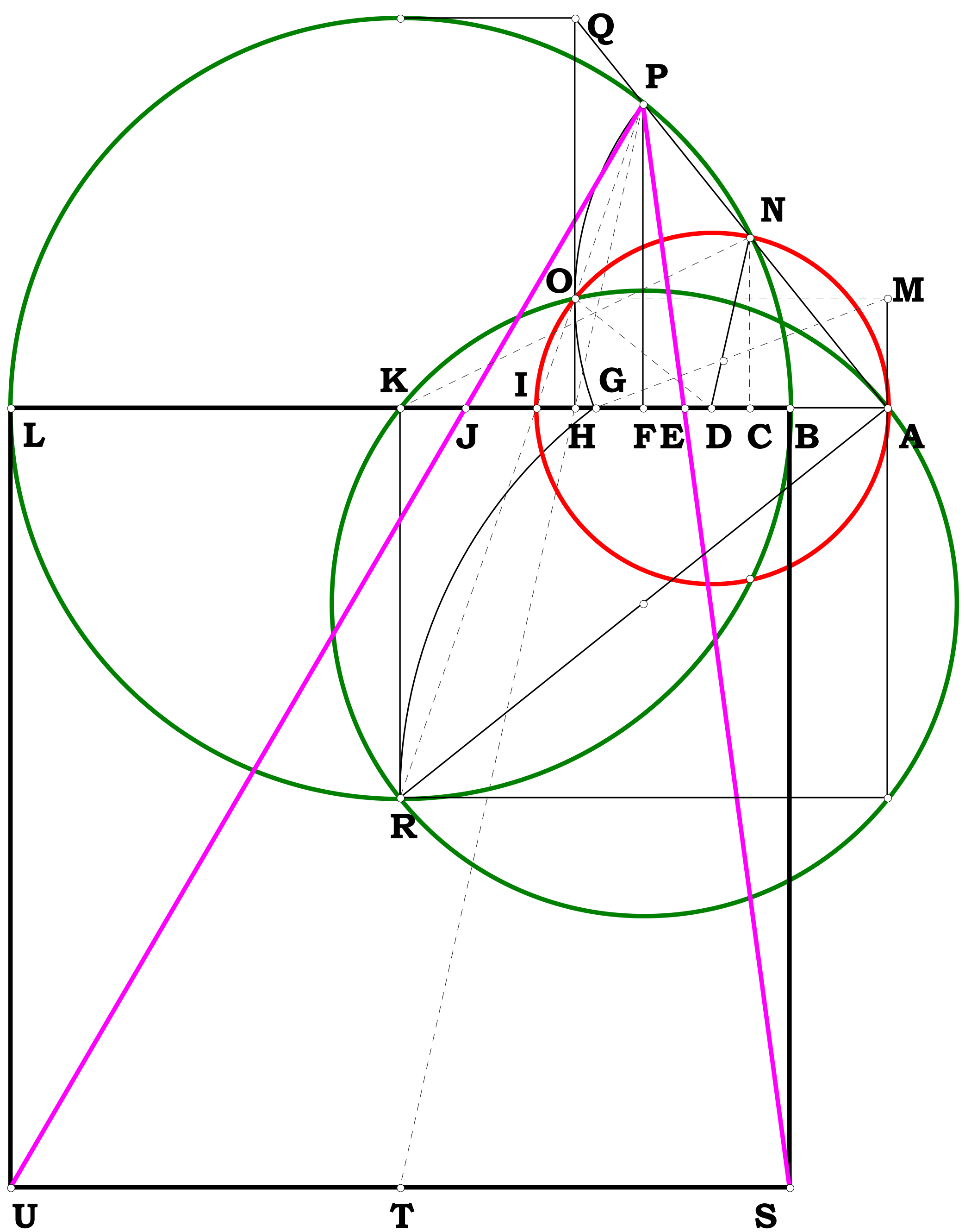
$$AJ - \left(N_1 + N_2\right) = 0 \qquad AE - \left(N_1{}^3 + N_1{}^2 \cdot N_2\right)^{\frac{1}{3}} = 0 \qquad AG - \left(N_1{}^3 + 2 \cdot N_1{}^2 \cdot N_2 + N_1 \cdot N_2{}^2\right)^{\frac{1}{3}} = 0$$

$$CG - \left[\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - N_1\right] = 0 \qquad GJ - \left[N_1 + N_2 - \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}}\right] = 0$$

$$GN - \sqrt{\left[\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - N_1\right] \cdot \left[N_1 + N_2 - \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}}\right]} = 0$$

121193

The structure in red appears to be a constant.



$N := 6 \quad AB := 1 \quad AL := AB \cdot N$

$BL := AL - AB \quad BK := \frac{BL}{2} \quad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB \quad BJ := AJ - AB \quad JL := BL - BJ \quad EJ := AJ - AE \quad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ \quad BF := BL - FL \quad FP := \sqrt{BF \cdot FL} \quad KR := BK \quad KL := BK$

$FK := FL - KL \quad IK := \frac{FK \cdot KR}{KR + FP} \quad AK := BK + AB \quad AI := AK - IK \quad AD := \frac{AI}{2}$

$KT := BL \quad FH := \frac{FK \cdot FP}{KT + FP} \quad AF := BF + AB \quad AH := AF + FH \quad HI := AI - AH$

$HO := \sqrt{AH \cdot HI} \quad DN := AD \quad KN := BK \quad DK := AK - AD \quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK \quad CI := AI - AC \quad CN := \sqrt{AC \cdot CI} \quad \frac{KR}{IK} - \frac{HO}{HI} = 0 \quad \frac{AF}{FP} - \frac{AC}{CN} = 0$

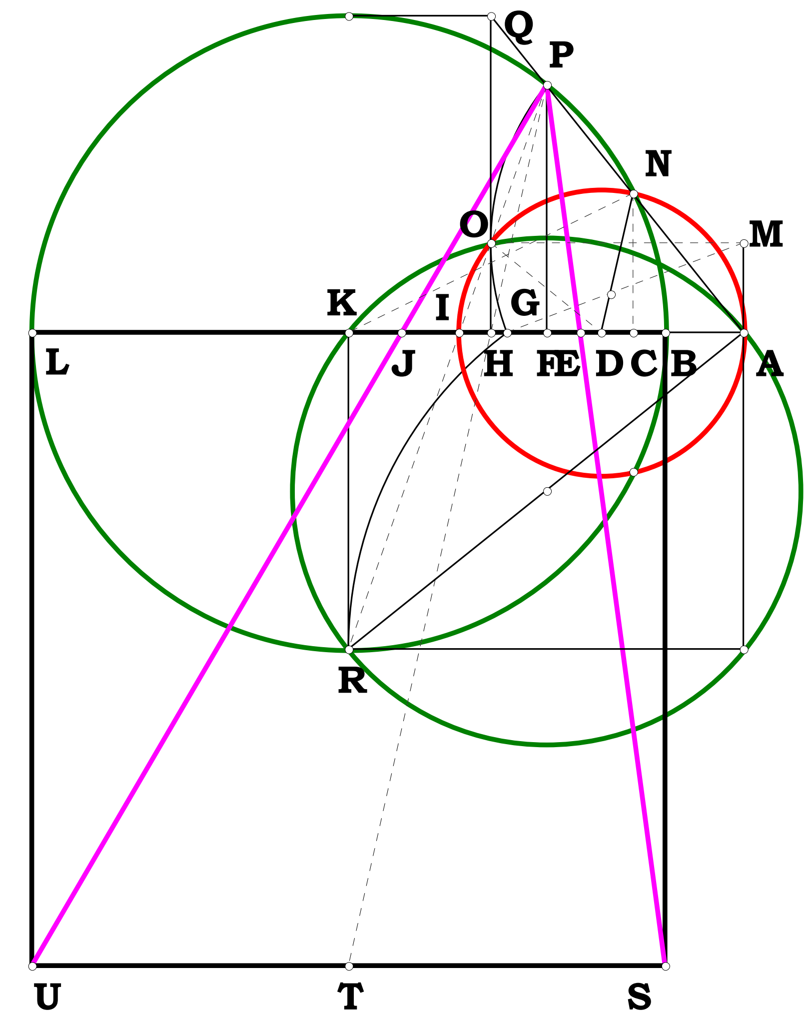
$AB := 1 \quad AL - N = 0 \quad BL - (N-1) = 0 \quad BK - \frac{N-1}{2} = 0 \quad AE - N^{\frac{1}{3}} = 0 \quad AJ - N^{\frac{2}{3}} = 0$

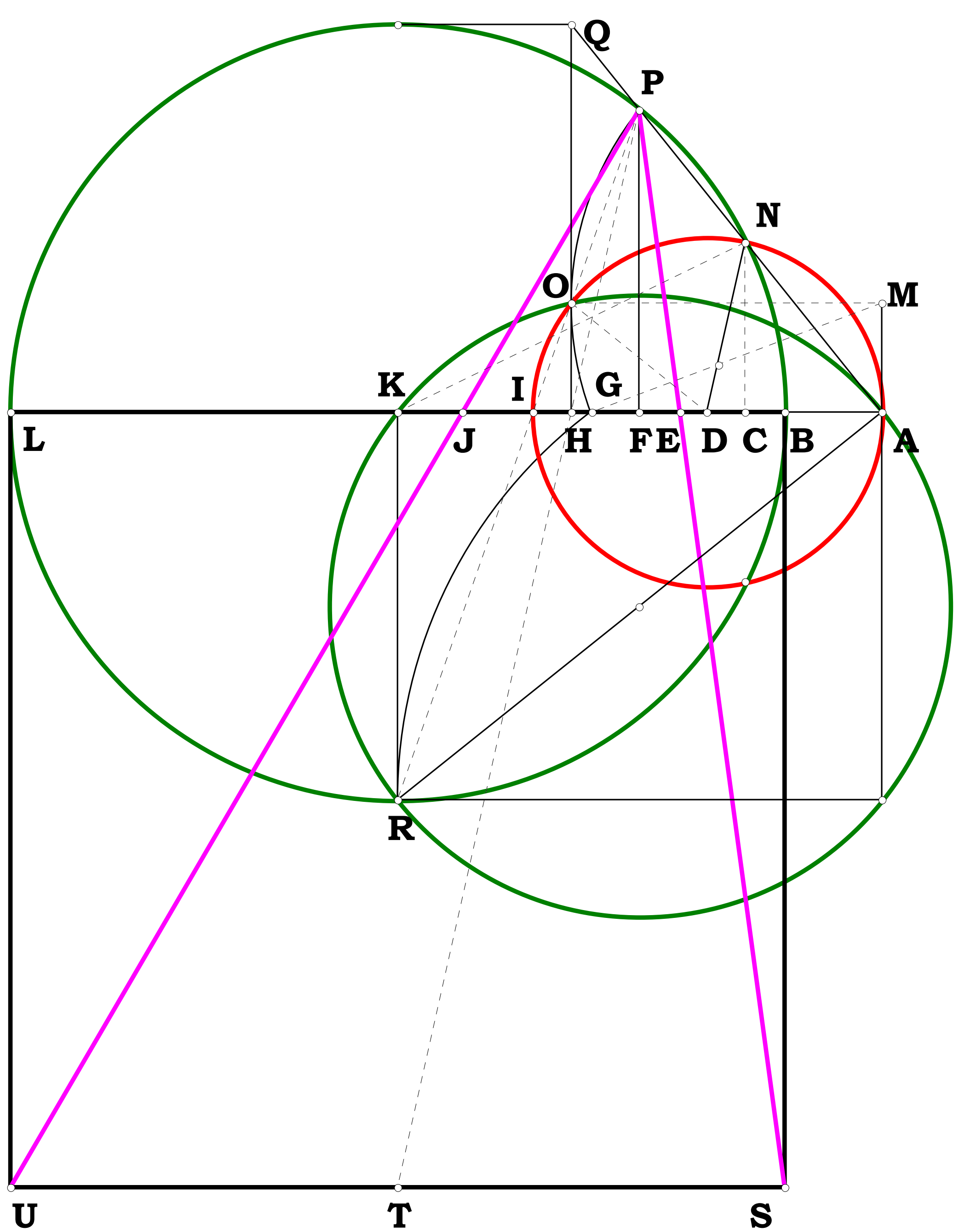
$BE - \left(N^{\frac{1}{3}} - 1\right) = 0 \quad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \quad JL - \left(N - N^{\frac{2}{3}}\right) = 0 \quad EJ - N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) = 0$

$$FJ - \frac{N \cdot \left(N^{\frac{1}{3}} - 1\right)}{N^{\frac{2}{3}} + 1} = 0 \quad FL - \frac{N^{\frac{2}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0 \quad BF - \frac{N-1}{N^{\frac{2}{3}} + 1} = 0 \quad FP - \frac{N^{\frac{1}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0$$

$$FK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \quad IK - \frac{\left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$AK - \frac{N+1}{2} = 0 \quad AI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{2}{3}} + 1\right)}{N^{\frac{1}{3}} + 1} = 0 \quad AD - \frac{N + N^{\frac{1}{3}}}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \quad FH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \quad AF - \frac{\left(N^{\frac{1}{3}}\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{2}{3}} + 1} = 0$$





$$AH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{2} = 0$$

$$HI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$HO - \frac{\sqrt{N^{\frac{2}{3}} - 2 \cdot N + N^{\frac{4}{3}}}}{2} = 0$$

$$DK - \frac{N^{\frac{4}{3}} + 1}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$CK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$AC - \frac{\left(N^{\frac{1}{3}}\right)^3 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{4}{3}} + 1} = 0$$

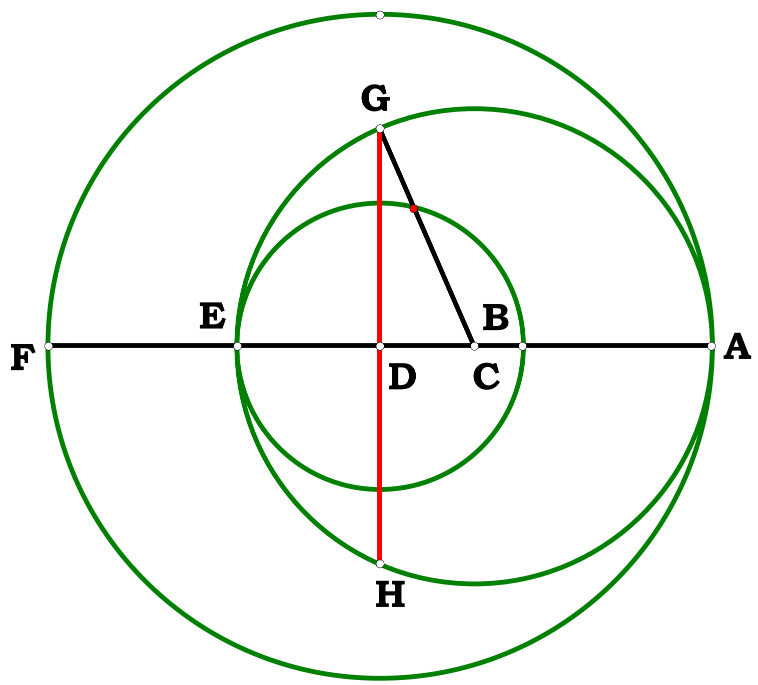
$$CI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)^2}{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$CN - \frac{N^{\frac{2}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)}{\left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$\frac{KR}{IK} - \frac{N^{\frac{1}{3}} + 1}{N^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{N - 1} = 0$$

121293 The Square Root

Square root by common segment common midpoint. Given AFand BE is GH their root?



$N_1 := 5 \quad N_2 := 3$

$AF := N_1 \quad BE := N_2$

$AD := \frac{AF}{2} \quad BD := \frac{BE}{2} \quad AB := AD - BD$

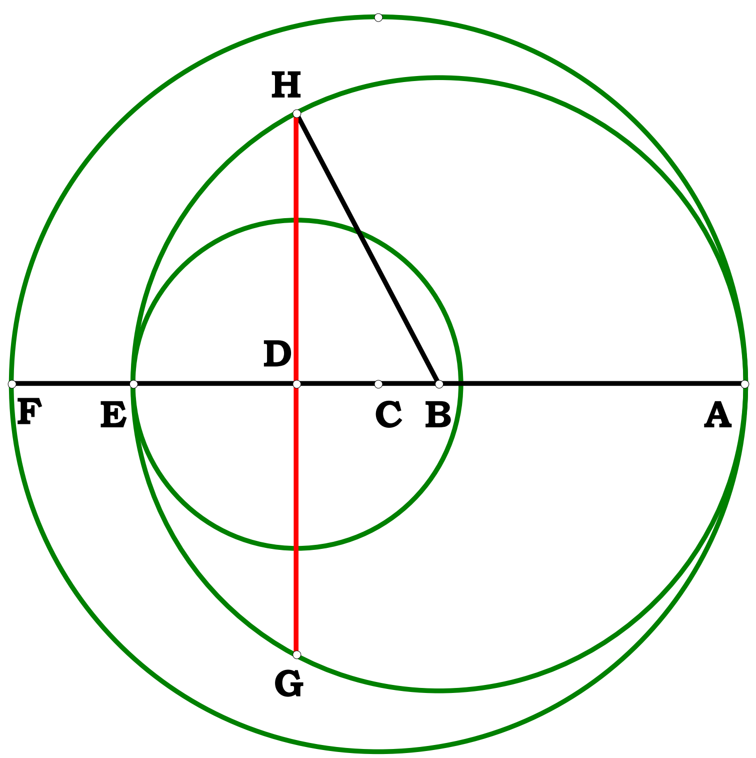
$AE := BE + AB \quad AC := \frac{AE}{2} \quad CG := AC$

$CD := AD - AC \quad GH := 2 \cdot \sqrt{CG^2 - CD^2}$

$GH - \sqrt{AF \cdot BE} = 0$

$GH - \sqrt{N_1 \cdot N_2} = 0$

121293B Generalize The Previous Square Root Figure



$$N_1 := 1 \quad N_2 := 3 \quad N_3 := 2$$

$$AF := N_1 \quad DF := \frac{AF}{N_2} \quad AD := AF - DF$$

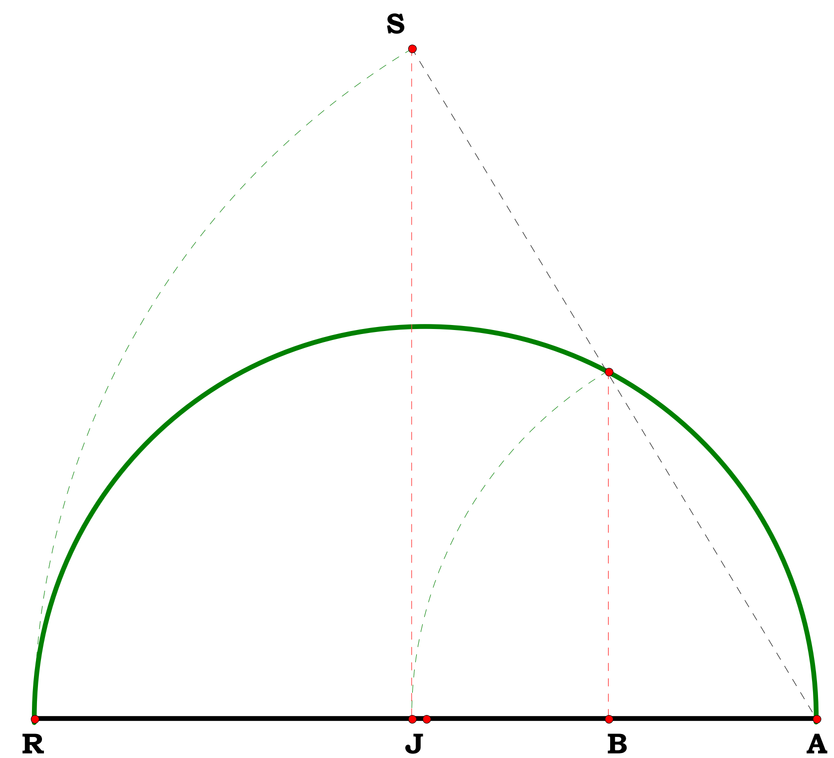
$$DE := \frac{DF}{N_3} \quad AE := AD + DE \quad AB := \frac{AE}{2}$$

$$BD := AD - AB \quad BH := AB$$

$$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$$

$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$

12_16_93, using 12_04_93.MCD



$\mathbf{AR} := \mathbf{10} \quad \Delta := \mathbf{5} \quad \delta := \mathbf{2} .. \Delta + \mathbf{1} \quad \mathbf{AB}_\delta := \dfrac{\mathbf{AR}}{\delta}$

$\mathbf{AJ}_\delta := \sqrt{\mathbf{AB}_\delta \cdot \mathbf{AR}} \quad \mathbf{JR}_\delta := \mathbf{AR} - \mathbf{AJ}_\delta$

$\mathbf{JW}_\delta := \sqrt{\mathbf{AJ}_\delta \cdot \mathbf{JR}_\delta} \quad \mathbf{AW}_\delta := \sqrt{\left(\mathbf{AJ}_\delta\right)^{\mathbf{2}} + \left(\mathbf{JW}_\delta\right)^{\mathbf{2}}}$

$\mathbf{AB}_\delta =$	$\mathbf{AJ}_\delta =$
5	7.071
3.333	5.774
2.5	5
2	4.472
1.667	4.082

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.

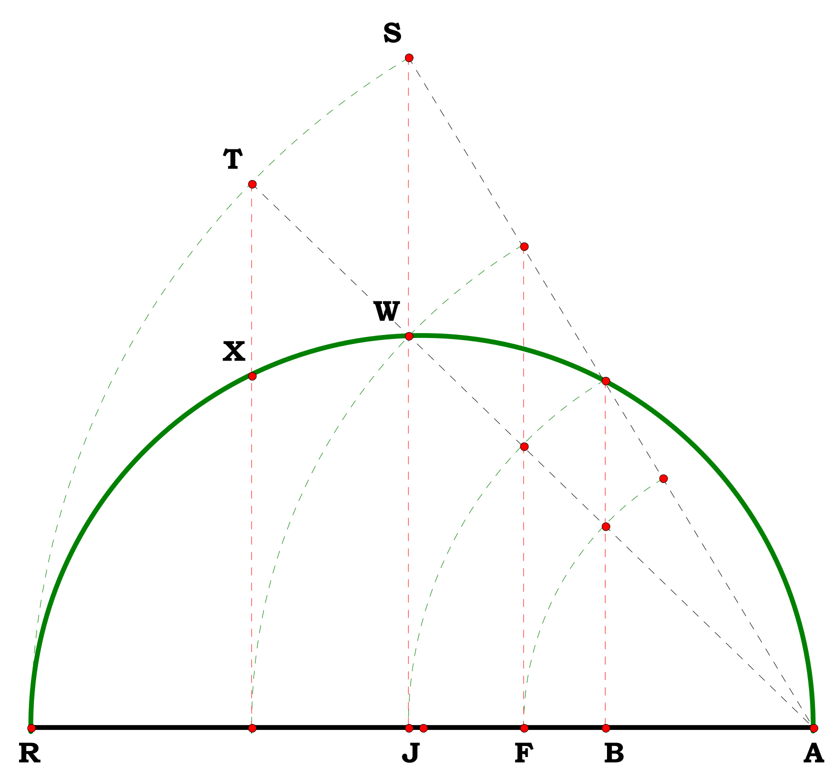
Euclidean Exponential Series

$\mathbf{AT} := \mathbf{AR} \quad \mathbf{AN}_\delta := \mathbf{AW}_\delta \quad \mathbf{AF}_\delta := \dfrac{\left(\mathbf{AJ}_\delta\right)^{\mathbf{2}}}{\mathbf{AW}_\delta}$

$\mathbf{NR}_\delta := \mathbf{AR} - \mathbf{AN}_\delta \quad \mathbf{NX}_\delta := \sqrt{\mathbf{AN}_\delta \cdot \mathbf{NR}_\delta}$

$\mathbf{AX}_\delta := \sqrt{\left(\mathbf{AN}_\delta\right)^{\mathbf{2}} + \left(\mathbf{NX}_\delta\right)^{\mathbf{2}}}$

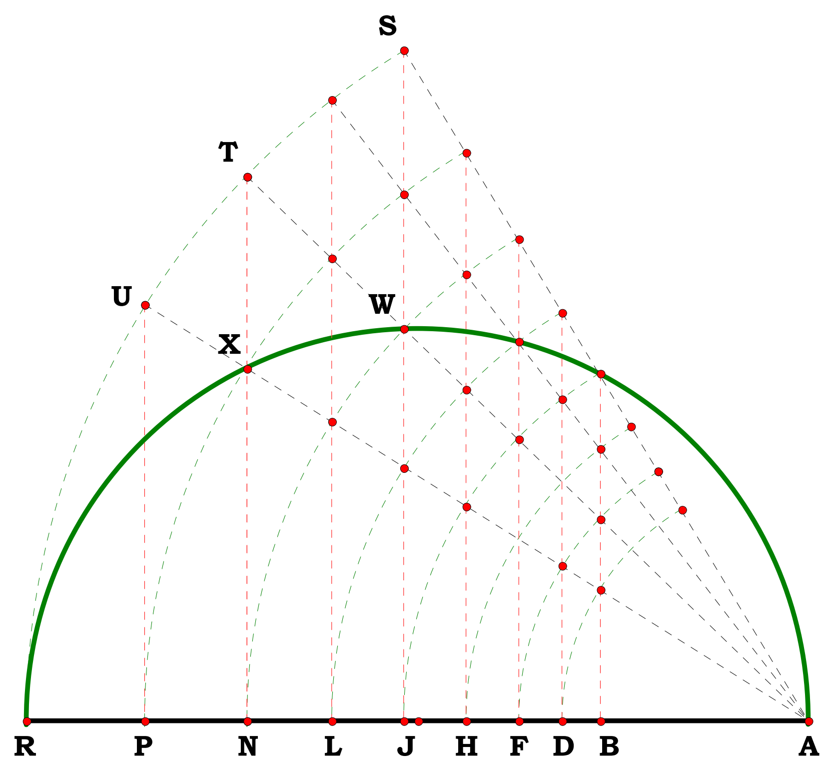
$\mathbf{AB}_\delta =$	$\mathbf{AF}_\delta =$	$\mathbf{AJ}_\delta =$	$\mathbf{AN}_\delta =$
5	5.946	7.071	8.409
3.333	4.387	5.774	7.598
2.5	3.536	5	7.071
2	2.991	4.472	6.687
1.667	2.608	4.082	6.389



What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.

$$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta} \quad AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$$

$$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta} \quad AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$$



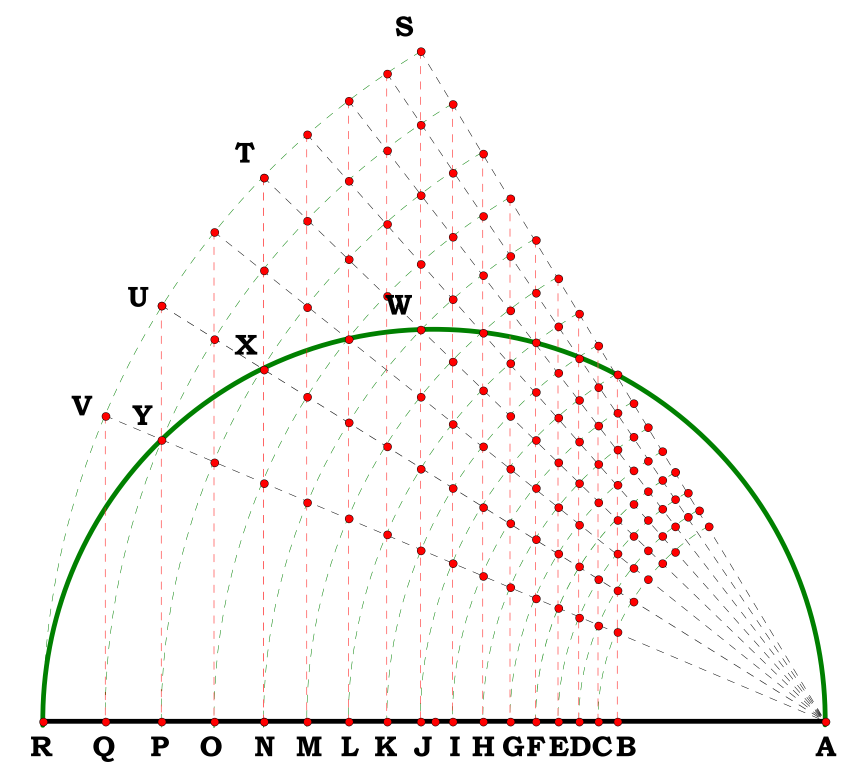
$AB_\delta =$	$AD_\delta =$	$AF_\delta =$	$AH_\delta =$	$AJ_\delta =$	$AL_\delta =$	$AN_\delta =$	$AP_\delta =$
5	5.453	5.946	6.484	7.071	7.711	8.409	9.17
3.333	3.824	4.387	5.033	5.774	6.623	7.598	8.717
2.5	2.973	3.536	4.204	5	5.946	7.071	8.409
2	2.446	2.991	3.657	4.472	5.469	6.687	8.178
1.667	2.085	2.608	3.263	4.082	5.107	6.389	7.993

$$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta} \quad AM_\delta := \frac{(AN_\delta)^2}{AO_\delta}$$

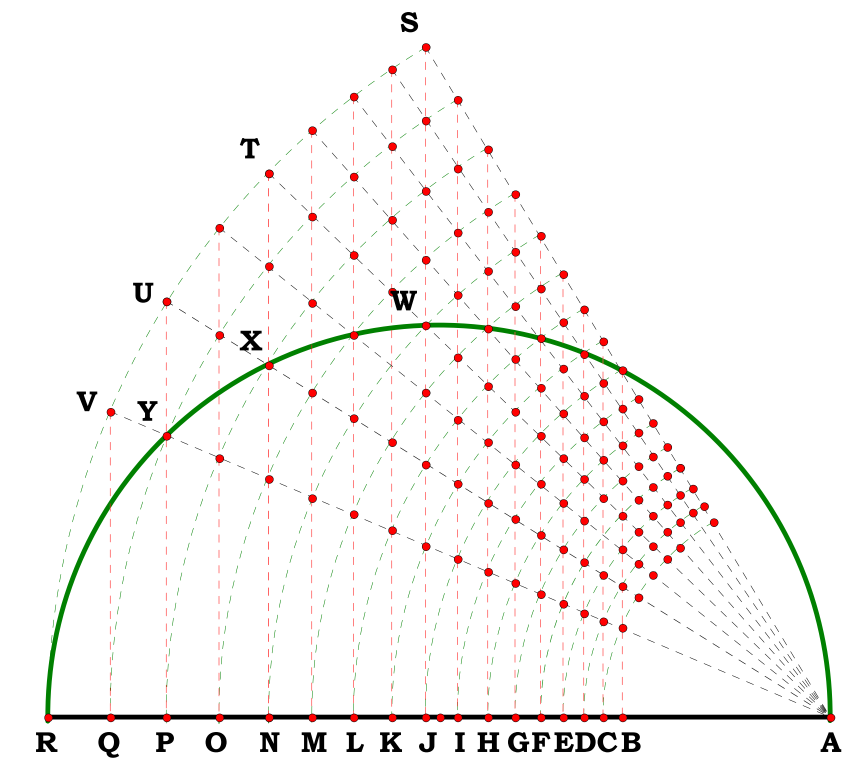
$$AK_\delta := \frac{(AL_\delta)^2}{AM_\delta} \quad AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$$

$$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$$

$AB_\delta =$	$AC_\delta =$	$AD_\delta =$	$AE_\delta =$	$AF_\delta =$
5	5.221	5.453	5.694	5.946
3.333	3.57	3.824	4.096	4.387
2.5	2.726	2.973	3.242	3.536
2	2.212	2.446	2.704	2.991
1.667	1.864	2.085	2.332	2.608



$AG_\delta =$	$AH_\delta =$	$AI_\delta =$	$AJ_\delta =$	$AK_\delta =$	$AL_\delta =$	$AM_\delta =$	$AN_\delta =$	$AO_\delta =$	$AP_\delta =$	$AQ_\delta =$
6.209	6.484	6.771	7.071	7.384	7.711	8.052	8.409	8.781	9.17	9.576
4.699	5.033	5.39	5.774	6.184	6.623	7.094	7.598	8.138	8.717	9.336
3.856	4.204	4.585	5	5.453	5.946	6.484	7.071	7.711	8.409	9.17
3.307	3.657	4.044	4.472	4.945	5.469	6.047	6.687	7.395	8.178	9.043
2.918	3.263	3.65	4.082	4.566	5.107	5.713	6.389	7.147	7.993	8.941



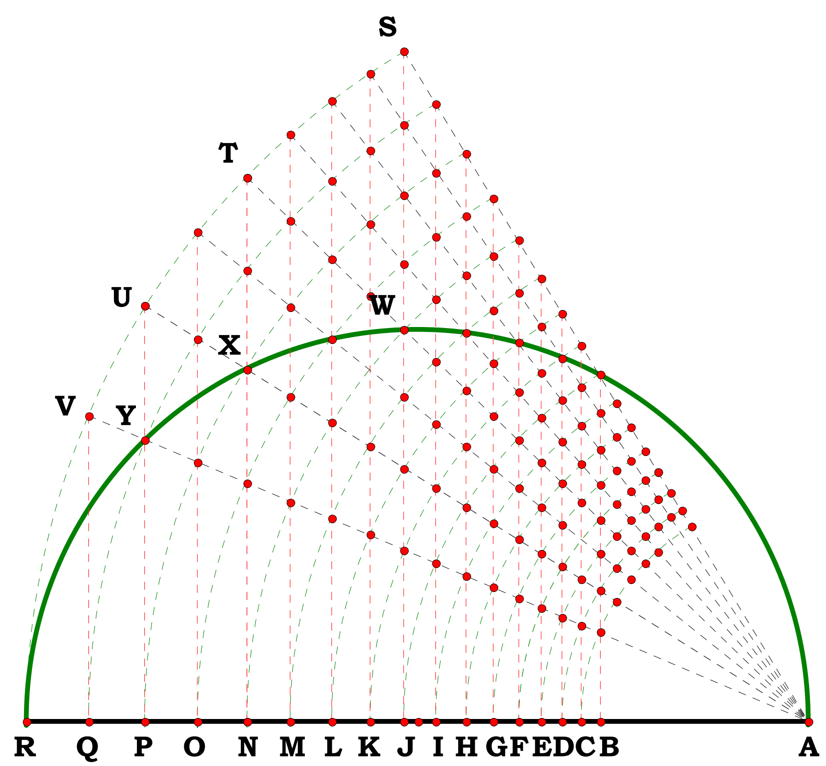
Values found by the investigator of 12_14_93

$AQ_\delta = \left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ $AP_\delta = \left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}} =$ $AN_\delta = \left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$ $AL_\delta = \left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$ $AI_\delta = \left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}} =$

AQ_δ	$\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$	AP_δ	$\left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}}$	AN_δ	$\left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$	AL_δ	$\left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$	AI_δ	$\left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}}$
9.576	9.576	9.17	9.17	8.409	8.409	7.711	7.711	6.771	6.771
9.336	9.336	8.717	8.717	7.598	7.598	6.623	6.623	5.39	5.39
9.17	9.17	8.409	8.409	7.071	7.071	5.946	5.946	4.585	4.585
9.043	9.043	8.178	8.178	6.687	6.687	5.469	5.469	4.044	4.044
8.941	8.941	7.993	7.993	6.389	6.389	5.107	5.107	3.65	3.65

$AH_\delta = \left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$ $AG_\delta = \left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ $AF_\delta = \left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}} =$

AH_δ	$\left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$	AG_δ	$\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$	AF_δ	$\left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}}$
6.484	6.484	6.209	6.209	5.946	5.946
5.033	5.033	4.699	4.699	4.387	4.387
4.204	4.204	3.856	3.856	3.536	3.536
3.657	3.657	3.307	3.307	2.991	2.991
3.263	3.263	2.918	2.918	2.608	2.608



$$AE_\delta = \left[(AB_\delta)^{13}\cdot AR^{3}\right]^{\frac{1}{16}}$$

5.694	5.694
4.096	4.096
3.242	3.242
2.704	2.704
2.332	2.332

$$AD_\delta = \left[(AB_\delta)^{7}\cdot AR\right]^{\frac{1}{8}}$$

5.453	5.453
3.824	3.824
2.973	2.973
2.446	2.446
2.085	2.085

$$AC_\delta = \left[(AB_\delta)^{15}\cdot AR\right]^{\frac{1}{16}} =$$

5.221	5.221
3.57	3.57
2.726	2.726
2.212	2.212
1.864	1.864

$$AQ_\delta = \left(AB_\delta\cdot AR^{15}\right)^{\frac{1}{16}}$$

9.576	9.576
9.336	9.336
9.17	9.17
9.043	9.043
8.941	8.941

$$AP_\delta = \left[(AB_\delta)^{2}\cdot AR^{14}\right]^{\frac{1}{16}}$$

9.17	9.17
8.717	8.717
8.409	8.409
8.178	8.178
7.993	7.993

$$AO_\delta = \left[(AB_\delta)^{3}\cdot AR^{13}\right]^{\frac{1}{16}} =$$

8.781	8.781
8.138	8.138
7.711	7.711
7.395	7.395
7.147	7.147

$$AN_\delta = \left[(AB_\delta)^{4}\cdot AR^{12}\right]^{\frac{1}{16}}$$

8.409	8.409
7.598	7.598
7.071	7.071
6.687	6.687
6.389	6.389

$$AM_\delta = \left[(AB_\delta)^{5}\cdot AR^{11}\right]^{\frac{1}{16}}$$

8.052	8.052
7.094	7.094
6.484	6.484
6.047	6.047
5.713	5.713

$$AL_\delta = \left[(AB_\delta)^{6}\cdot AR^{10}\right]^{\frac{1}{16}} =$$

7.711	7.711
6.623	6.623
5.946	5.946
5.469	5.469
5.107	5.107

$$AK_\delta = \left[(AB_\delta)^{7}\cdot AR^{9}\right]^{\frac{1}{16}}$$

7.384	7.384
6.184	6.184
5.453	5.453
4.945	4.945
4.566	4.566

$$AJ_\delta = \left[(AB_\delta)^{8}\cdot AR^{8}\right]^{\frac{1}{16}}$$

7.071	7.071
5.774	5.774
5	5
4.472	4.472
4.082	4.082

$$AI_\delta = \left[(AB_\delta)^{9}\cdot AR^{7}\right]^{\frac{1}{16}}$$

6.771	6.771
5.39	5.39
4.585	4.585
4.044	4.044
3.65	3.65

$$AH_\delta = \left[(AB_\delta)^{10}\cdot AR^{6}\right]^{\frac{1}{16}} =$$

6.484	6.484
5.033	5.033
4.204	4.204
3.657	3.657
3.263	3.263

$$AG_{\delta} = \left[(AB_{\delta})^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$$

AG_{δ}	$\left[(AB_{\delta})^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$
6.209	6.209
4.699	4.699
3.856	3.856
3.307	3.307
2.918	2.918

$$AF_{\delta} = \left[(AB_{\delta})^{12} \cdot AR^{4}\right]^{\frac{1}{16}} =$$

AF_{δ}	$\left[(AB_{\delta})^{12} \cdot AR^{4}\right]^{\frac{1}{16}}$
5.946	5.946
4.387	4.387
3.536	3.536
2.991	2.991
2.608	2.608

$$AE_{\delta} = \left[(AB_{\delta})^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$$

AE_{δ}	$\left[(AB_{\delta})^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$
5.694	5.694
4.096	4.096
3.242	3.242
2.704	2.704
2.332	2.332

$$AD_{\delta} = \left[(AB_{\delta})^{14} \cdot AR^{2}\right]^{\frac{1}{16}} =$$

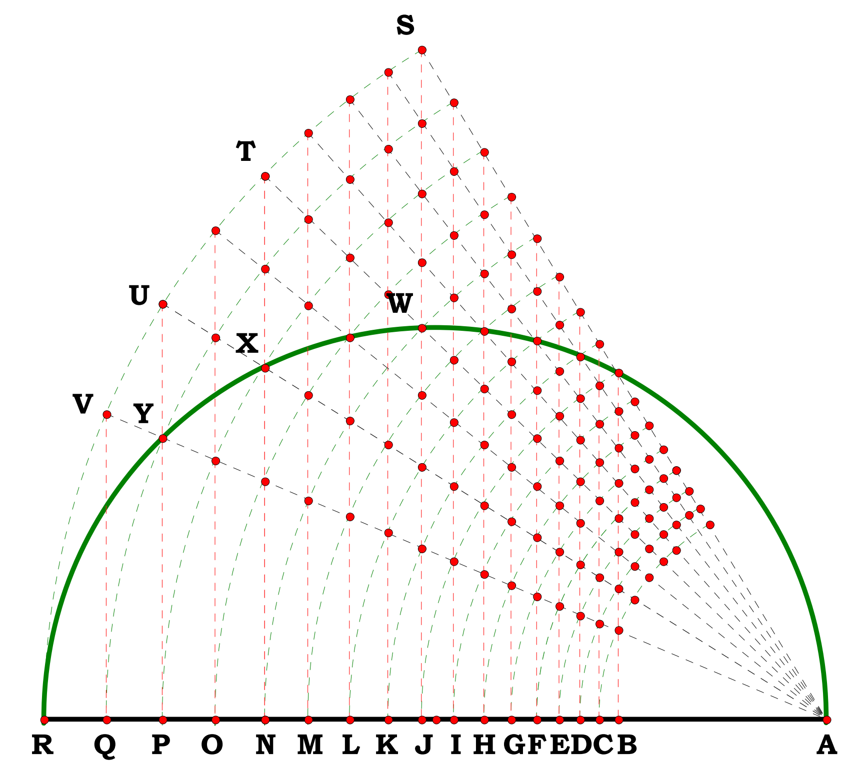
AD_{δ}	$\left[(AB_{\delta})^{14} \cdot AR^{2}\right]^{\frac{1}{16}}$
5.453	5.453
3.824	3.824
2.973	2.973
2.446	2.446
2.085	2.085

$$AC_{\delta} = \left[(AB_{\delta})^{15} \cdot AR\right]^{\frac{1}{16}}$$

AC_{δ}	$\left[(AB_{\delta})^{15} \cdot AR\right]^{\frac{1}{16}}$
5.221	5.221
3.57	3.57
2.726	2.726
2.212	2.212
1.864	1.864

$$AB_{\delta} = \left[(AB_{\delta})^{16} \cdot AR^{0}\right]^{\frac{1}{16}} =$$

AB_{δ}	$\left[(AB_{\delta})^{16} \cdot AR^{0}\right]^{\frac{1}{16}}$
5	5
3.333	3.333
2.5	2.5
2	2
1.667	1.667



Resultant Equation

$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

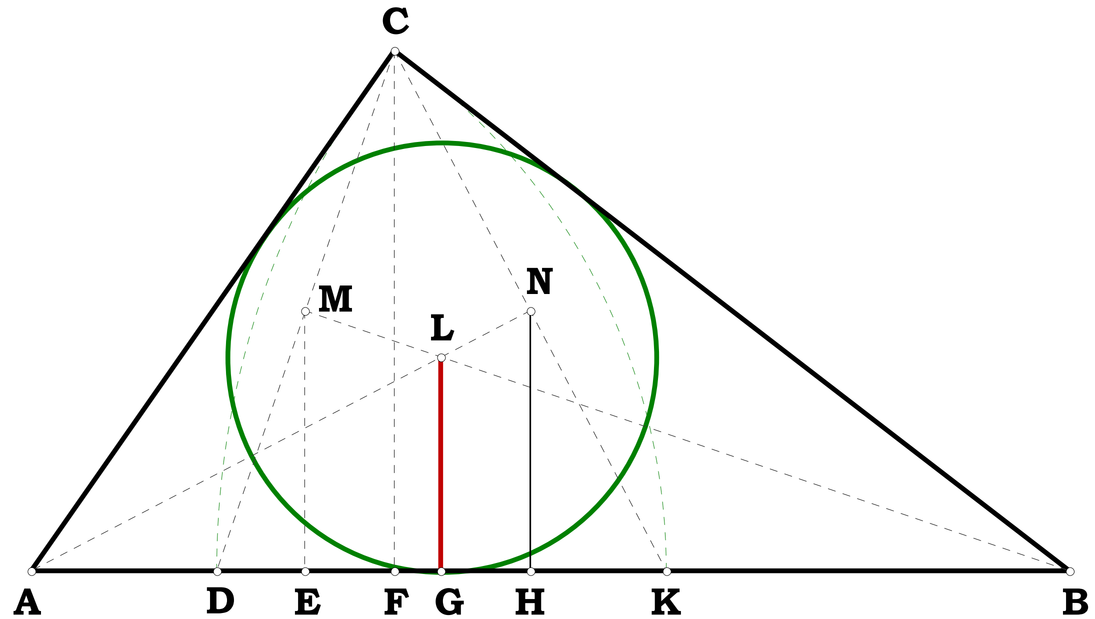
or

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

depending on direction of transcription.

040694 Inscribing A Circle In A Given Triangle

Given three sides of a triangle, what is the length of the inscribed radius?



$$\mathbf{AB} := 3 \qquad \mathbf{BC} := 4 \qquad \mathbf{AC} := 5$$

$$\mathbf{AK} := \mathbf{AC} \qquad \mathbf{BD} := \mathbf{BC} \qquad \mathbf{AF} := \frac{\mathbf{AC}^2 + \mathbf{AB}^2 - \mathbf{BC}^2}{2 \cdot \mathbf{AB}}$$

$$\mathbf{FK} := \mathbf{AK} - \mathbf{AF} \qquad \mathbf{CF} := \sqrt{\mathbf{AC}^2 - \mathbf{AF}^2}$$

$$\mathbf{CK} := \sqrt{\mathbf{FK}^2 + \mathbf{CF}^2} \qquad \mathbf{AN} := \frac{\mathbf{CF} \cdot \mathbf{AK}}{\mathbf{CK}}$$

$$\mathbf{AH} := \frac{\mathbf{CF} \cdot \mathbf{AN}}{\mathbf{CK}} \qquad \mathbf{HN} := \frac{\mathbf{FK} \cdot \mathbf{AN}}{\mathbf{CK}} \qquad \mathbf{BF} := \mathbf{AB} - \mathbf{AF}$$

$$\mathbf{DF} := \mathbf{BD} - \mathbf{BF} \qquad \mathbf{CD} := \sqrt{\mathbf{CF}^2 + \mathbf{DF}^2} \qquad \mathbf{BM} := \frac{\mathbf{CF} \cdot \mathbf{BD}}{\mathbf{CD}} \qquad \mathbf{BE} := \frac{\mathbf{CF} \cdot \mathbf{BM}}{\mathbf{CD}} \qquad \mathbf{GL} := \frac{\mathbf{HN} \cdot \mathbf{AB}}{\mathbf{AH} + \mathbf{BE}}$$

$$\mathbf{S1} := \begin{pmatrix} \mathbf{AB} \\ \mathbf{BC} \\ \mathbf{AC} \end{pmatrix} \quad \mathbf{S2} := \begin{pmatrix} \mathbf{BC} \\ \mathbf{AC} \\ \mathbf{AB} \end{pmatrix} \quad \mathbf{S3} := \begin{pmatrix} \mathbf{AC} \\ \mathbf{AB} \\ \mathbf{BC} \end{pmatrix} \quad \delta := 0 .. 2$$

$$\mathbf{Radius}_\delta := \frac{\sqrt{-\mathbf{S1}_\delta + \mathbf{S2}_\delta + \mathbf{S3}_\delta} \cdot \sqrt{\mathbf{S1}_\delta - \mathbf{S2}_\delta + \mathbf{S3}_\delta} \cdot \sqrt{\mathbf{S1}_\delta + \mathbf{S2}_\delta - \mathbf{S3}_\delta}}{2 \cdot \sqrt{\mathbf{S1}_\delta + \mathbf{S2}_\delta + \mathbf{S3}_\delta}} \qquad \mathbf{Radius} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$DE = 0.58830$

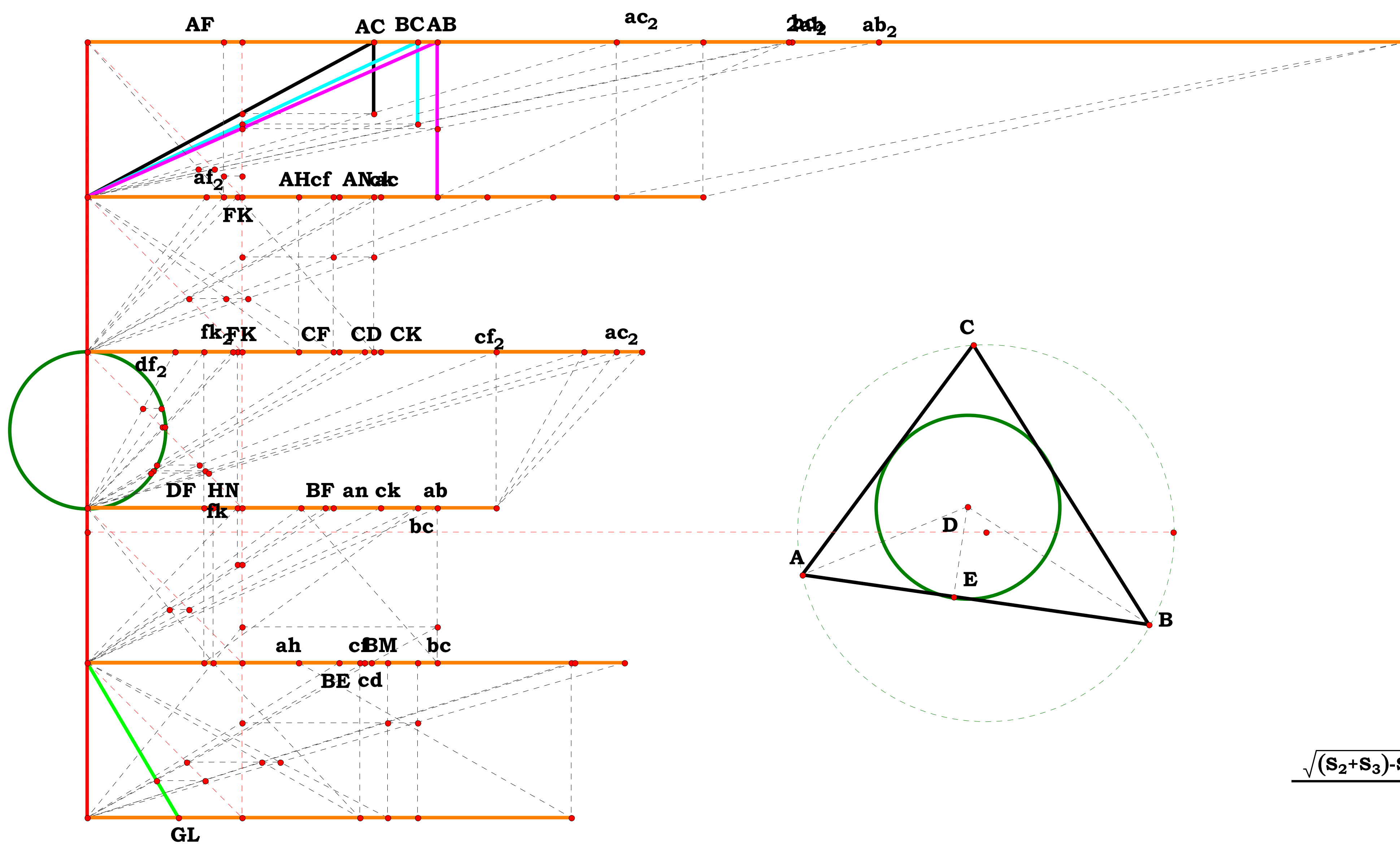
$GL = 0.58830$

$DE\text{-}GL = 0.00000$

$AB = 2.25959$

$BC = 2.13235$

$AC = 1.84623$

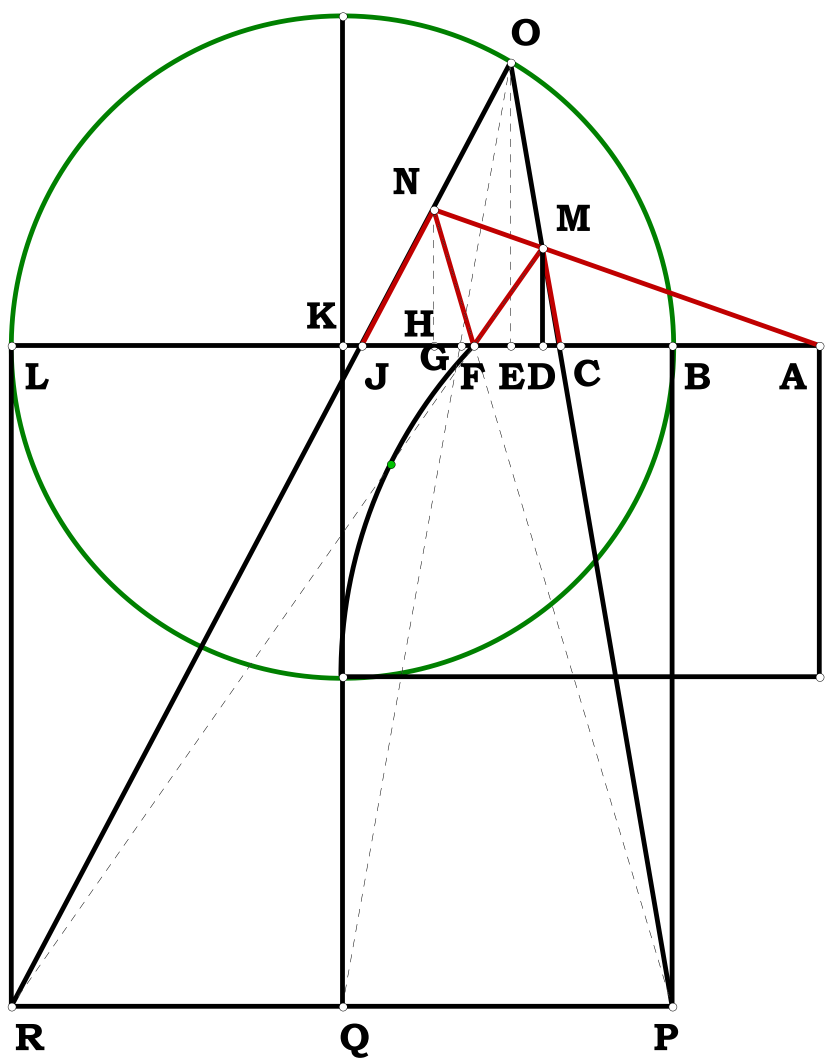


$S_1 = 2.25959$

$S_2 = 2.13235$

$S_3 = 1.84623$

$$\frac{\sqrt{(S_2+S_3)-S_1}\cdot\sqrt{(S_1+S_3)-S_2}\cdot\sqrt{(S_1+S_2)-S_3}}{2\cdot\sqrt{S_1+S_2+S_3}}-GL = 0.00000$$



042194 The Cradle

Are A, M, N colinear?

$N := 5 \quad AB := 1 \quad AL := AB \cdot N$

$AF := \sqrt{AB \cdot AL} \quad AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \quad BP := BL \quad LR := BL \quad FL := AL - AF$

$BC := AC - AB \quad BJ := AJ - AB \quad JL := BL - BJ$

$BF := AF - AB \quad FJ := AJ - AF \quad CF := AF - AC$

$FG := \dfrac{BF \cdot FJ}{BF + JL} \quad GN := \dfrac{BP \cdot FG}{BF} \quad CD := \dfrac{BC \cdot CF}{BC + FL}$

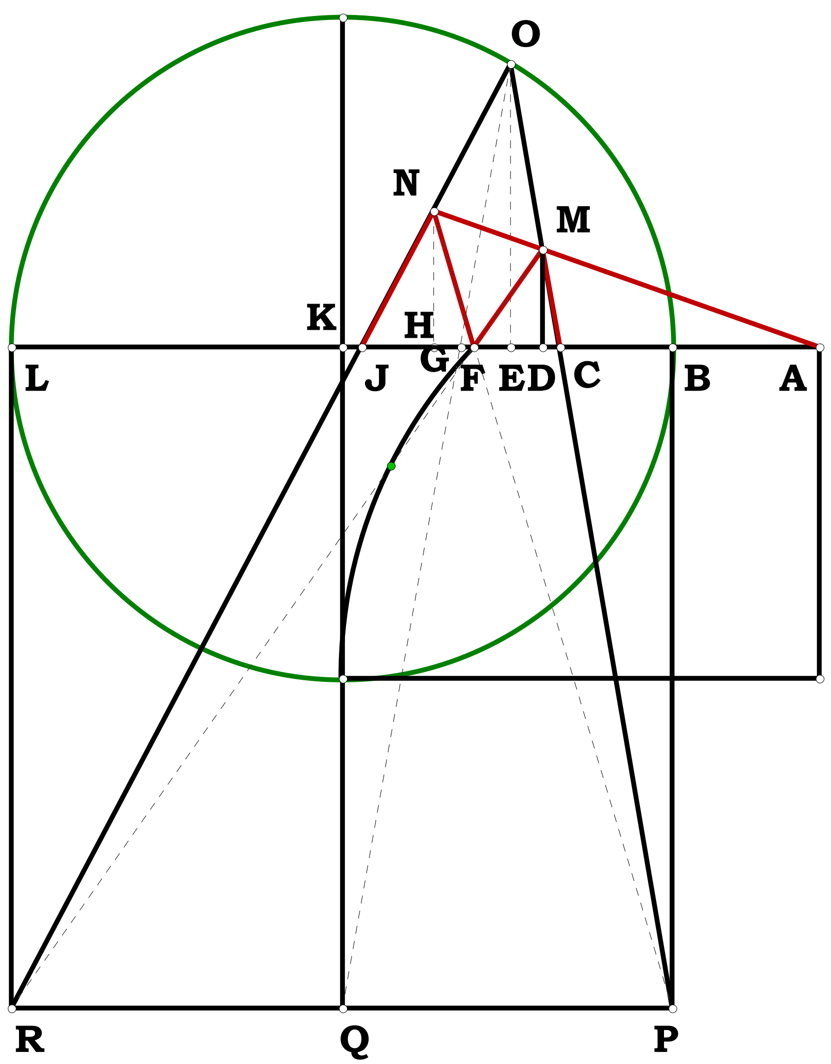
$DM := \dfrac{BP \cdot CD}{BC} \quad AD := AC + CD \quad AG := AF + FG$

$\dfrac{AG}{GN} - \dfrac{AD}{DM} = 0$

$$AL - N = 0 \qquad AF - \sqrt{N} = 0 \qquad AC - N^{\frac{1}{3}} = 0 \qquad AJ - N^{\frac{2}{3}} = 0 \qquad BL - (N - 1) = 0 \qquad FL - \left(N - \sqrt{N}\right) = 0$$

$$BC - \left(N^{\frac{1}{3}} - 1\right) = 0 \qquad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \qquad JL - \left[N - 1 - \left(N^{\frac{2}{3}} - 1\right)\right] = 0 \qquad BF - \left(\sqrt{N} - 1\right) = 0$$

$$FJ - \left(N^{\frac{2}{3}} - \sqrt{N}\right) = 0 \qquad CF - \left[\sqrt{N} - (N)^{\frac{1}{3}}\right] = 0 \qquad FG - \frac{\sqrt{N}\cdot\left(\sqrt{N} - 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$$



$$GN - \frac{\sqrt{N}\cdot(N - 1)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad CD - \frac{N^{\frac{1}{3}}\cdot\left(N^{\frac{1}{3}} - 1\right)}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$$

$$DM - \frac{N^{\frac{1}{3}}\cdot(N - 1)}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad AD - \frac{\left(N^{\frac{1}{6}}\right)^{3}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$$

$$AG - \frac{N^{\frac{2}{3}}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad \frac{AG}{GN} - \frac{N^{\frac{1}{6}}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{N - 1} = 0 \qquad \frac{AD}{DM} - \frac{N^{\frac{1}{6}}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{N - 1} = 0$$

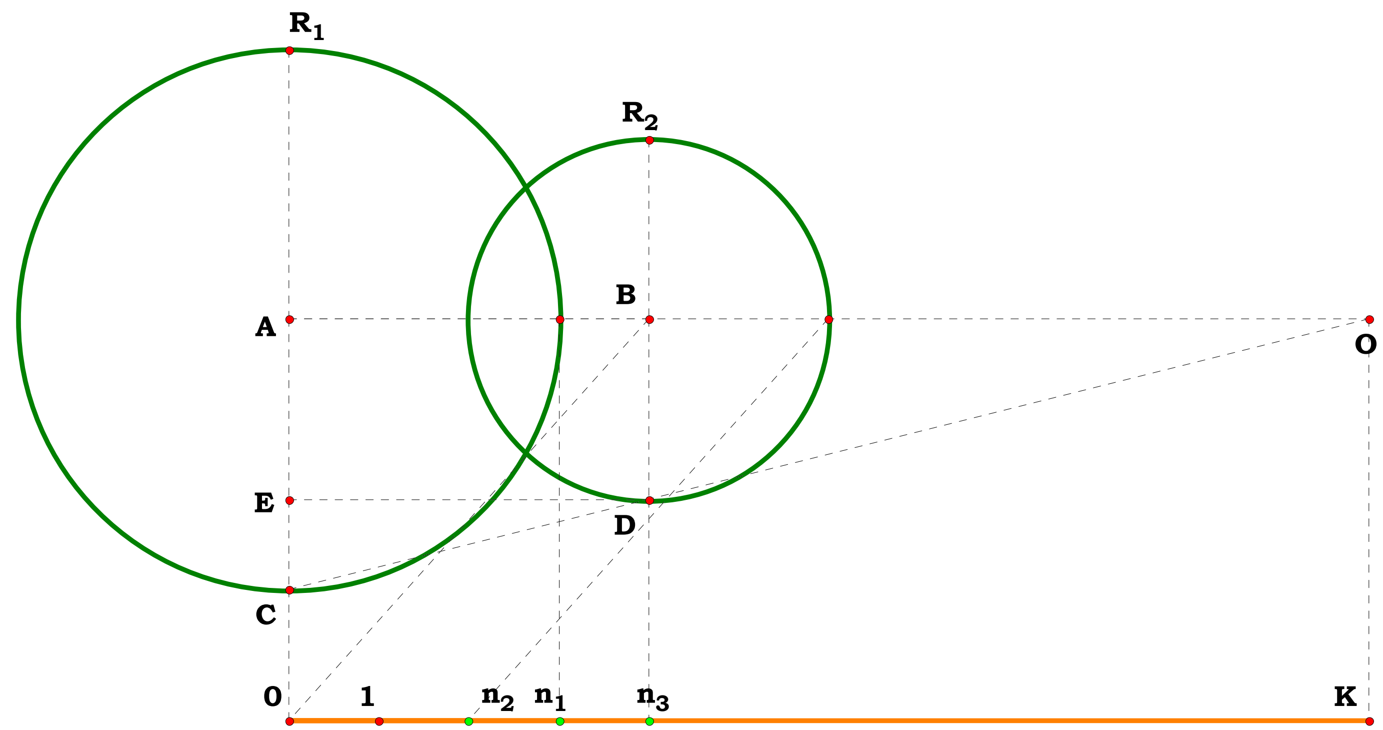
$N_1 := -3 \quad N_2 := 2 \qquad N_3 := -7$

$R_1 := \sqrt{N_1^{\,2}} \qquad R_2 := \sqrt{N_2^{\,2}}$

The above is one way using a numbered line for values that do not have negative values.

$AC := R_1 \quad BD := R_2 \quad AB := N_3$

042694 Tangents and Similarity Points.



What is the length of the AO, O being the similarity point?

$DE := AB \quad AE := BD \quad CE := AC - AE$

$AO := \frac{DE \cdot AC}{CE} \qquad AO = -21$

$$\frac{N_3 \cdot R_1}{R_1 - R_2} = -21$$

$$AO - \frac{N_3 \cdot R_1}{R_1 - R_2} = 0$$

What is the length of the tangent GO?

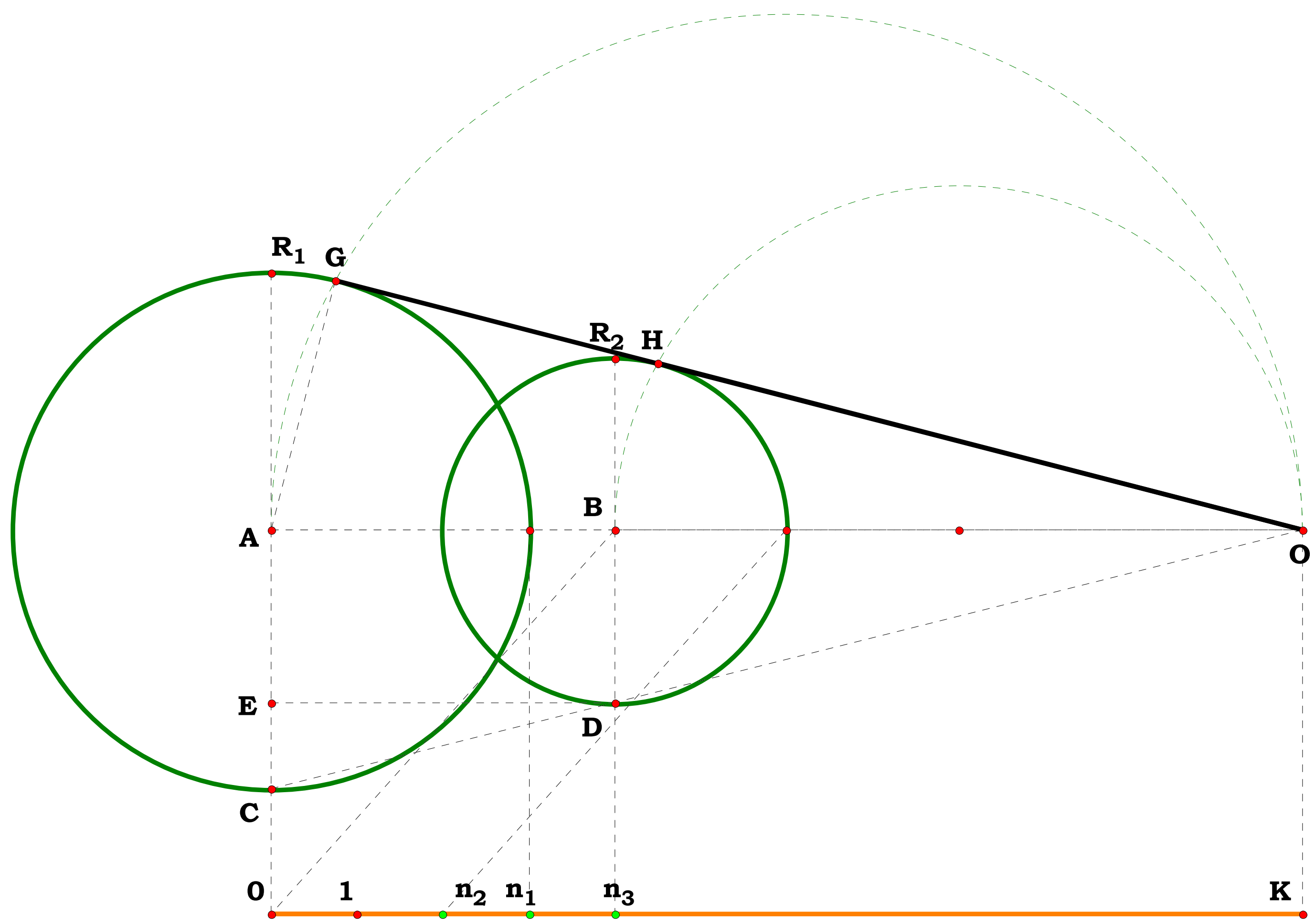
$$AG := R_1 \qquad GO := \sqrt{AO^2 - AG^2}$$

$$GO - \frac{R_1 \cdot \sqrt{N_3^{\,2} - R_1^{\,2} + 2 \cdot R_1 \cdot R_2 - R_2^{\,2}}}{\sqrt{(R_1 - R_2)^2}} = 0$$

What is the length of the tangent HO?

$$BH := R_2 \qquad HO := \sqrt{\left(\frac{N_3 \cdot R_1}{R_1 - R_2} - N_3\right)^2 - R_2^{\,2}}$$

$$HO - \frac{R_2 \cdot \sqrt{N_3^{\,2} - R_1^{\,2} + 2 \cdot R_1 \cdot R_2 - R_2^{\,2}}}{\sqrt{(R_1 - R_2)^2}} = 0$$



What is the length of line tangent to tangent of these circles?

$$GH := \frac{GO \cdot AB}{AO} \qquad GH - \sqrt{N_3^{\,2} - R_1^{\,2} + 2 \cdot R_1 \cdot R_2 - R_2^{\,2}} = 0$$

What are the names of the tangents AP and BP to the similarity point P?

$$\mathbf{AP} := \frac{\mathbf{N_3 \cdot R_1}}{\mathbf{R_1 + R_2}} \qquad \mathbf{BP} := \mathbf{N_3} - \frac{\mathbf{N_3 \cdot R_1}}{\mathbf{R_1 + R_2}} \qquad \mathbf{BP} - \frac{\mathbf{N_3 \cdot R_2}}{\mathbf{R_1 + R_2}} = \mathbf{0}$$

What is JP?

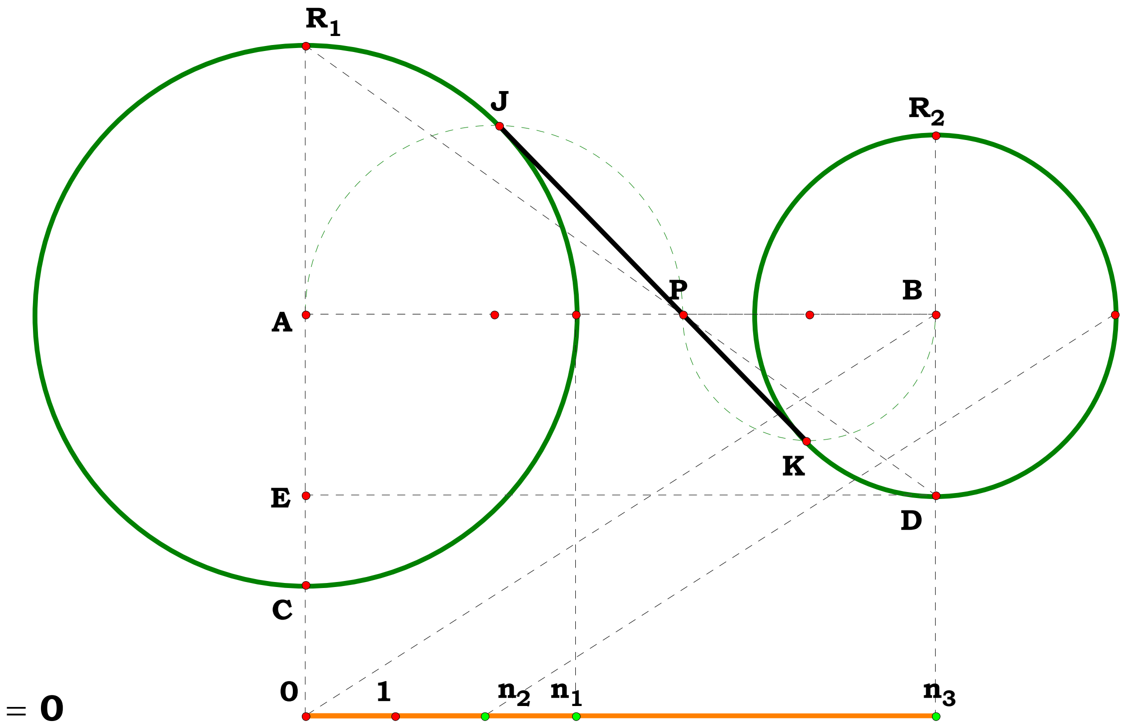
$$\mathbf{JP} := \sqrt{\mathbf{AP}^2 - \mathbf{R_1}^2} \qquad \mathbf{JP} - \frac{\mathbf{R_1} \cdot \sqrt{\mathbf{N_3}^2 - \mathbf{R_1}^2 - 2 \cdot \mathbf{R_1} \cdot \mathbf{R_2} - \mathbf{R_2}^2}}{\mathbf{R_1 + R_2}} = \mathbf{0}$$

What is KP?

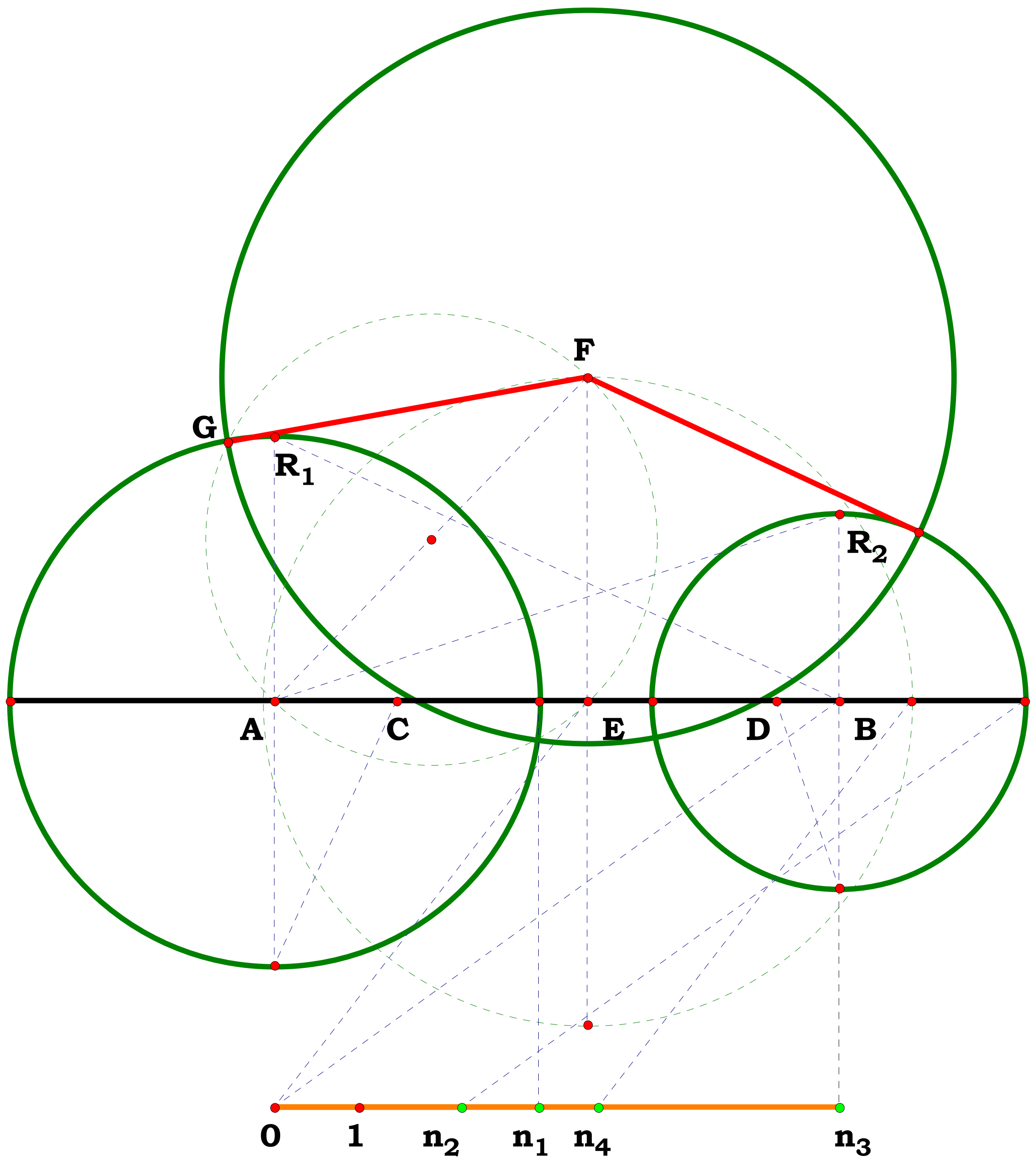
$$\mathbf{KP} := \frac{\mathbf{JP \cdot BP}}{\mathbf{AP}} \qquad \mathbf{KP} - \frac{\mathbf{R_2} \cdot \sqrt{\mathbf{N_3}^2 - \mathbf{R_1}^2 - 2 \cdot \mathbf{R_1} \cdot \mathbf{R_2} - \mathbf{R_2}^2}}{\mathbf{R_1 + R_2}} = \mathbf{0}$$

What is JK?

$$\mathbf{JK} := \frac{\mathbf{JP \cdot AB}}{\mathbf{AP}} \qquad \mathbf{JK} - \sqrt{\mathbf{N_3}^2 - \mathbf{R_1}^2 - 2 \cdot \mathbf{R_1} \cdot \mathbf{R_2} - \mathbf{R_2}^2} = \mathbf{0}$$



04/27/94 The Chordal or Power Line of two Circles



The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by **Heinrich Dörrie did not lend itself to an ordered process, so I took a couple of minuets (Bach) and developed my own method.**

One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.

Given two circles find their chordal or power line given just their radius and difference between their centers. Then, pick a spot on the power line and write the equation for the tangent circle.

$N_1 := 3 \qquad N_2 := 2 \qquad N_3 := 6 \qquad R_1 := \sqrt{N_1^{\,2}} \qquad R_2 := \sqrt{N_2^{\,2}} \qquad AB := N_3$

$AC := \dfrac{R_1^{\,2}}{AB} \qquad BD := \dfrac{R_2^{\,2}}{AB} \qquad CD := AB - (AC + BD) \qquad CE := \dfrac{CD}{2}$

$AE := AC + CE \qquad BE := AB - AE$

$$AE - \frac{N_3^{\,2} + R_1^{\,2} - R_2^{\,2}}{2 \cdot N_3} = 0$$

$$BE - \frac{N_3^{\,2} - R_1^{\,2} + R_2^{\,2}}{2 \cdot N_3} = 0$$

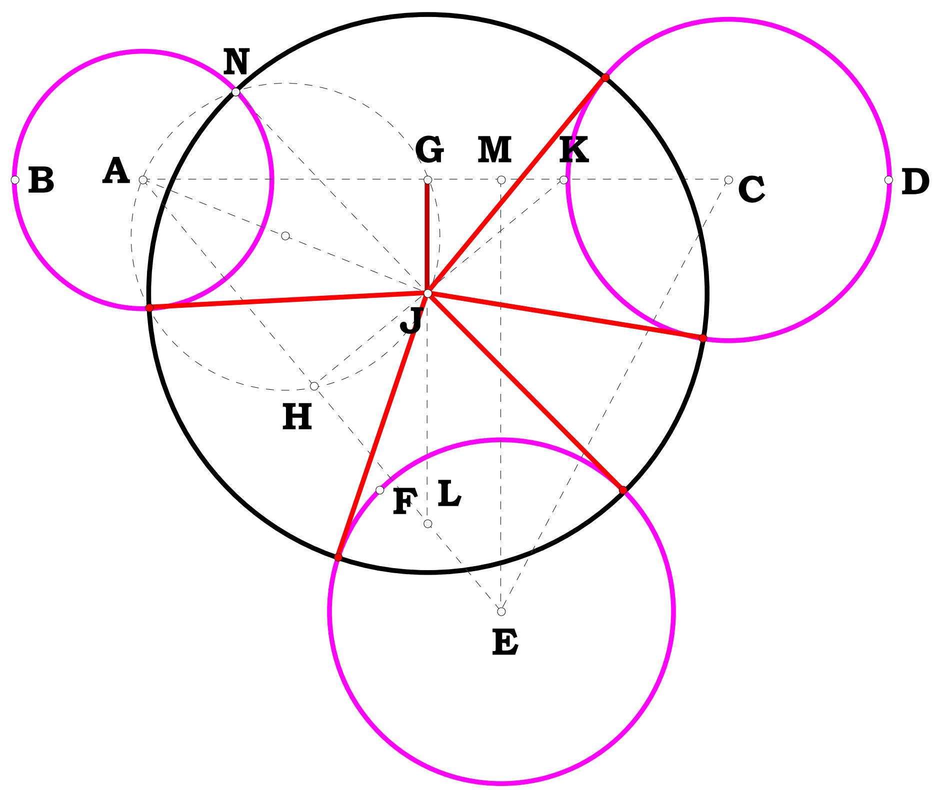
If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.

$N_4 := .698 \qquad AF := \sqrt{AE^2 + N_4^{\,2}} \qquad AG := R_1 \qquad FG := \sqrt{AF^2 - R_1^{\,2}}$

$$FG - \frac{\sqrt{N_3^{\,2} \cdot \left(N_3^{\,2} + 4 \cdot N_4^{\,2} - 2 \cdot R_1^{\,2} - 2 \cdot R_2^{\,2}\right) + \left(R_1 - R_2\right)^2 \cdot \left(R_1 + R_2\right)^2}}{2 \cdot \sqrt{N_3^{\,2}}} = 0$$

042894 Power Point

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate an Algebraic name for the power point and the length of the resultant tangent.



$N_1 := 3 \quad N_2 := 1 \quad N_3 := 2 \quad AC := 7.81447 \quad AE := 6.96686 \quad CE := 5.33279$

$R_1 := \sqrt{N_1^2} \quad R_2 := \sqrt{N_2^2} \quad R_3 := \sqrt{N_3^2} \quad D_1 := AC \quad D_2 := AE \quad D_3 := CE$

$$AG := \frac{\sqrt{\left(R_1^2 + D_1^2 - R_2^2\right)^2}}{2 \cdot D_1} \qquad AH := \frac{\sqrt{\left(R_1^2 + D_2^2 - R_3^2\right)^2}}{2 \cdot D_2}$$

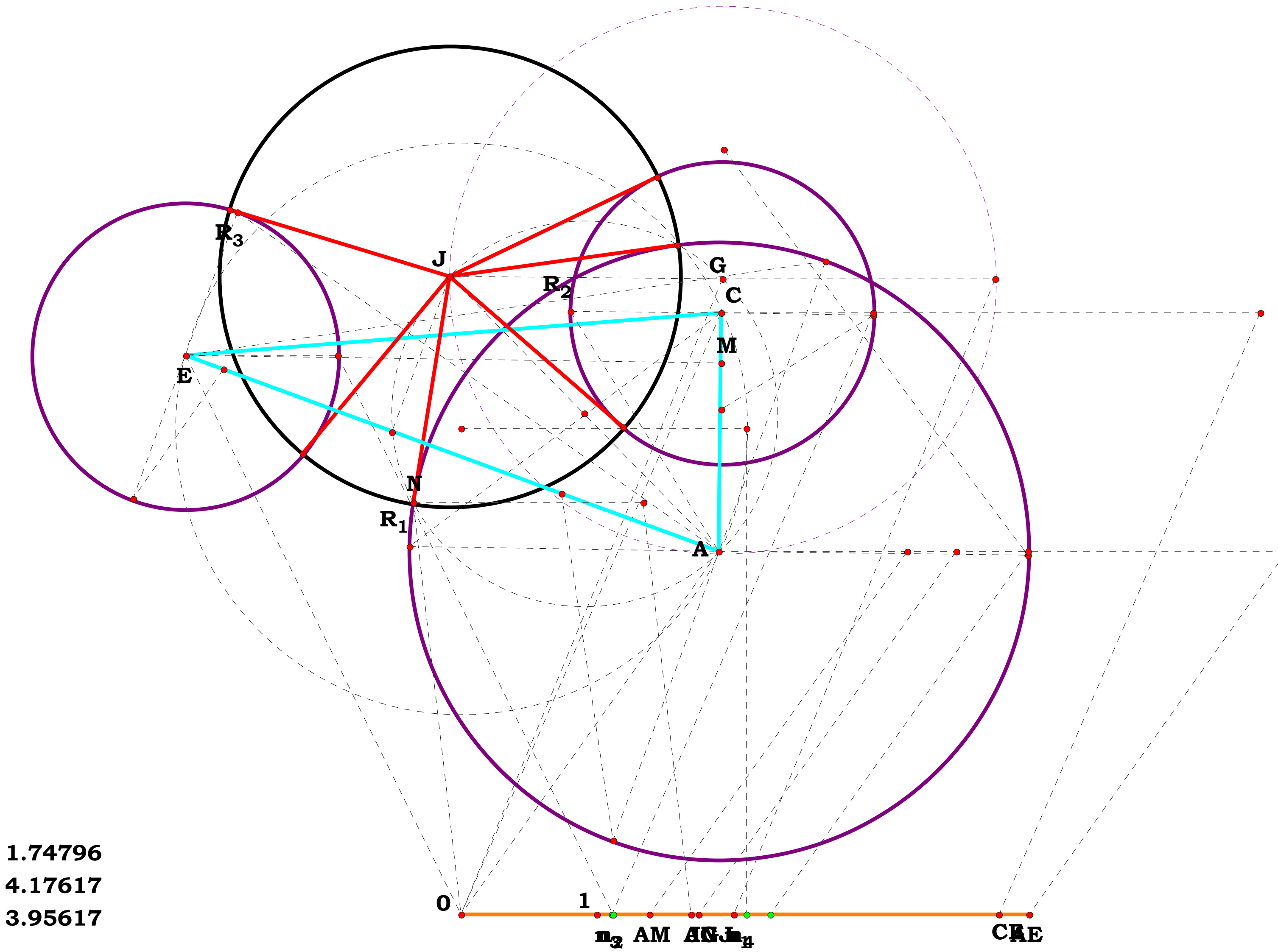
$$AM := \frac{\sqrt{\left(D_2^2 + D_1^2 - D_3^2\right)^2}}{2 \cdot D_1}$$

$$EM := \sqrt{AE^2 - AM^2} \quad AK := \frac{AE \cdot AH}{AM} \quad GK := AK - AG \quad GJ := \frac{AM \cdot GK}{EM}$$

$$GJ - \frac{2 \cdot D_1^2 \cdot \sqrt{\left(D_2^2 + R_1^2 - R_3^2\right)^2} - \sqrt{\left(D_1^2 + D_2^2 - D_3^2\right)^2} \cdot \sqrt{\left(D_1^2 + R_1^2 - R_2^2\right)^2}}{2 \cdot \sqrt{D_1^2} \cdot \sqrt{(D_1 + D_2 - D_3) \cdot (D_1 - D_2 + D_3) \cdot (D_2 - D_1 + D_3) \cdot (D_1 + D_2 + D_3)}} = 0$$

$AJ := \sqrt{AG^2 + GJ^2} \quad AN := R_1 \quad JN := \sqrt{AJ^2 - AN^2}$

$$JN - \sqrt{\frac{\begin{array}{l} D_1^2 \cdot R_3^2 \cdot \left(R_1^2 + R_2^2 - R_3^2 - D_1^2\right) + D_1^2 \cdot D_2^2 \cdot \left(R_2^2 + R_3^2 - D_3^2\right) \ldots \\ + D_2^2 \cdot R_1^2 \cdot \left(D_3^2 + R_2^2 - R_3^2\right) + D_1^2 \cdot D_3^2 \cdot \left(R_1^2 + R_3^2\right) + D_3^2 \cdot R_1^2 \cdot \left(R_2^2 + R_3^2 - D_3^2 - R_1^2\right) \ldots \\ + D_2^2 \cdot R_2^2 \cdot \left(R_3^2 - D_2^2 - R_2^2\right) + D_3^2 \cdot R_2^2 \cdot (D_2 - R_3) \cdot (D_2 + R_3) - D_1^2 \cdot R_1^2 \cdot R_2^2 \end{array}}{(D_1 + D_2 - D_3) \cdot (D_1 + D_2 + D_3) \cdot (D_1 - D_2 + D_3) \cdot (D_1 - D_2 - D_3)}} = 0$$



N_1 = 2.27017
N_2 = 1.11001
N_3 = 1.12235
N_4 = 2.09689

R_1 = 2.27017
R_2 = 1.11001
R_3 = 1.12235

AC = 1.74796
AE = 4.17617
CE = 3.95617
AM = 1.38576
GJ = 2.00571
JN = 1.68883

D_1 = 1.74796
D_2 = 4.17617
D_3 = 3.95617

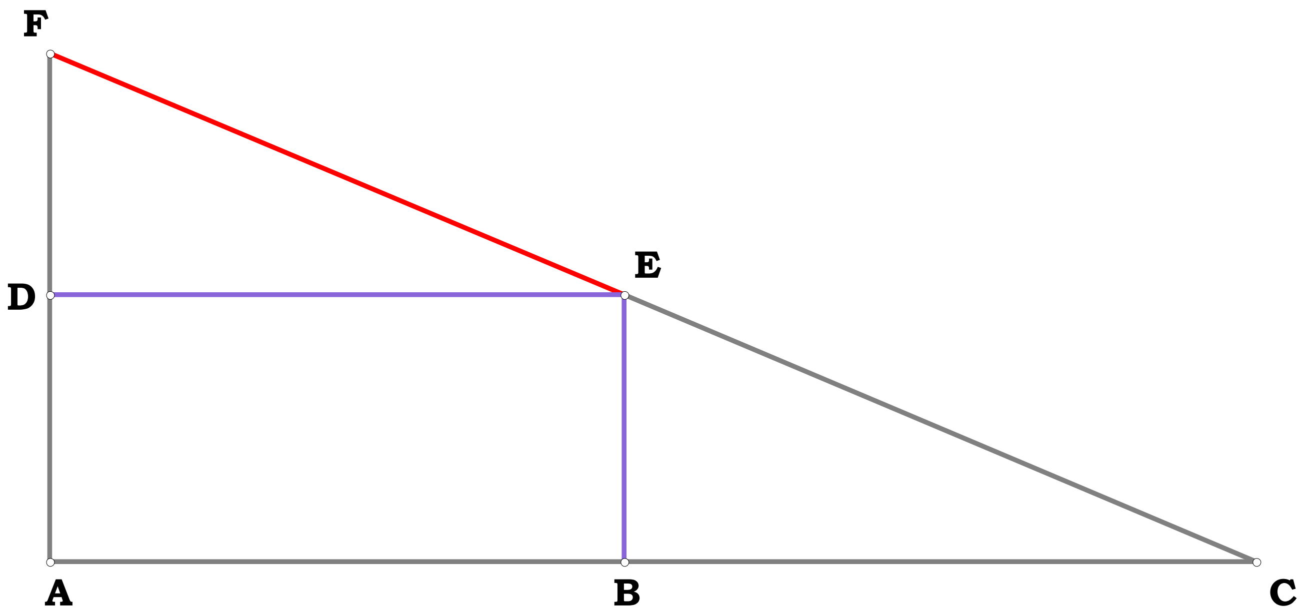
Animate Points

$$\frac{\sqrt{((D_1^2+D_2^2)-D_3^2)^2}}{2\cdot D_1}-AM = 0.00000$$

$$\frac{2\cdot D_1^2\cdot\sqrt{((D_2^2+R_1^2)-R_3^2)^2}-\sqrt{((D_1^2+D_2^2)-D_3^2)^2}\cdot\sqrt{((D_1^2+R_1^2)-R_2^2)^2}}{2\cdot\sqrt{D_1^2}\cdot\sqrt{((D_1+D_2)-D_3)\cdot((D_1-D_2)+D_3)\cdot((D_2-D_1)+D_3)\cdot(D_1+D_2+D_3)}}-GJ = 0.00000$$

$$\sqrt{\frac{(D_1^2\cdot R_3^2\cdot((R_1^2+R_2^2)-R_3^2-D_1^2)+D_1^2\cdot D_2^2\cdot((R_2^2+R_3^2)-D_3^2))+(D_2^2\cdot R_1^2\cdot((D_3^2+R_2^2)-R_3^2)+D_1^2\cdot D_3^2\cdot(R_1^2+R_3^2)+D_3^2\cdot R_1^2\cdot((R_2^2+R_3^2)-D_3^2-R_1^2))+((D_2^2\cdot R_2^2\cdot(R_3^2-D_2^2-R_2^2)+D_3^2\cdot R_2^2\cdot(D_2-R_3)\cdot(D_2+R_3))-D_1^2\cdot R_1^2\cdot R_2^2)}{(((D_1+D_2)-D_3)\cdot(D_1+D_2+D_3)\cdot((D_1-D_2)+D_3)\cdot(D_1-D_2-D_3))}}-JN = 0.00000$$

042994



Given AB, AF, BE, what is EF?

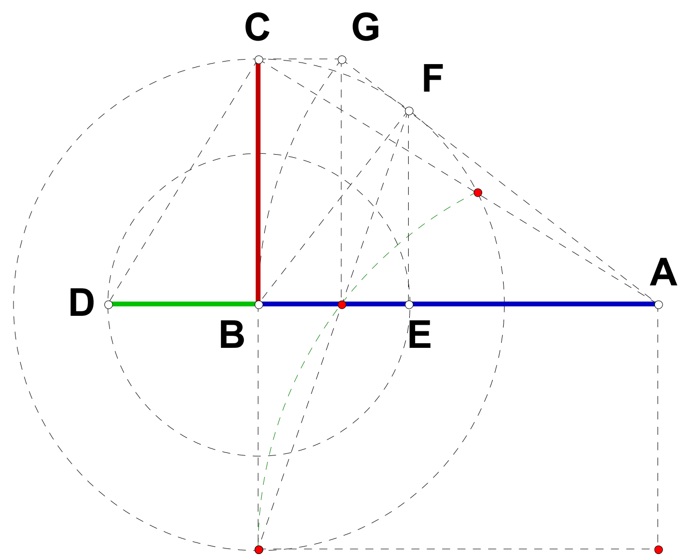
$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$EF - \sqrt{N_1{}^2 - 2 \cdot N_1 \cdot N_2 + N_2{}^2 + N_3{}^2} = 0$

$EF - \sqrt{N_1{}^2 - 2 \cdot N_1 \cdot N_2 + N_2{}^2 + N_3{}^2} = 0$

043094 Division N^2



$N_1 := 5 \qquad N_2 := 2$

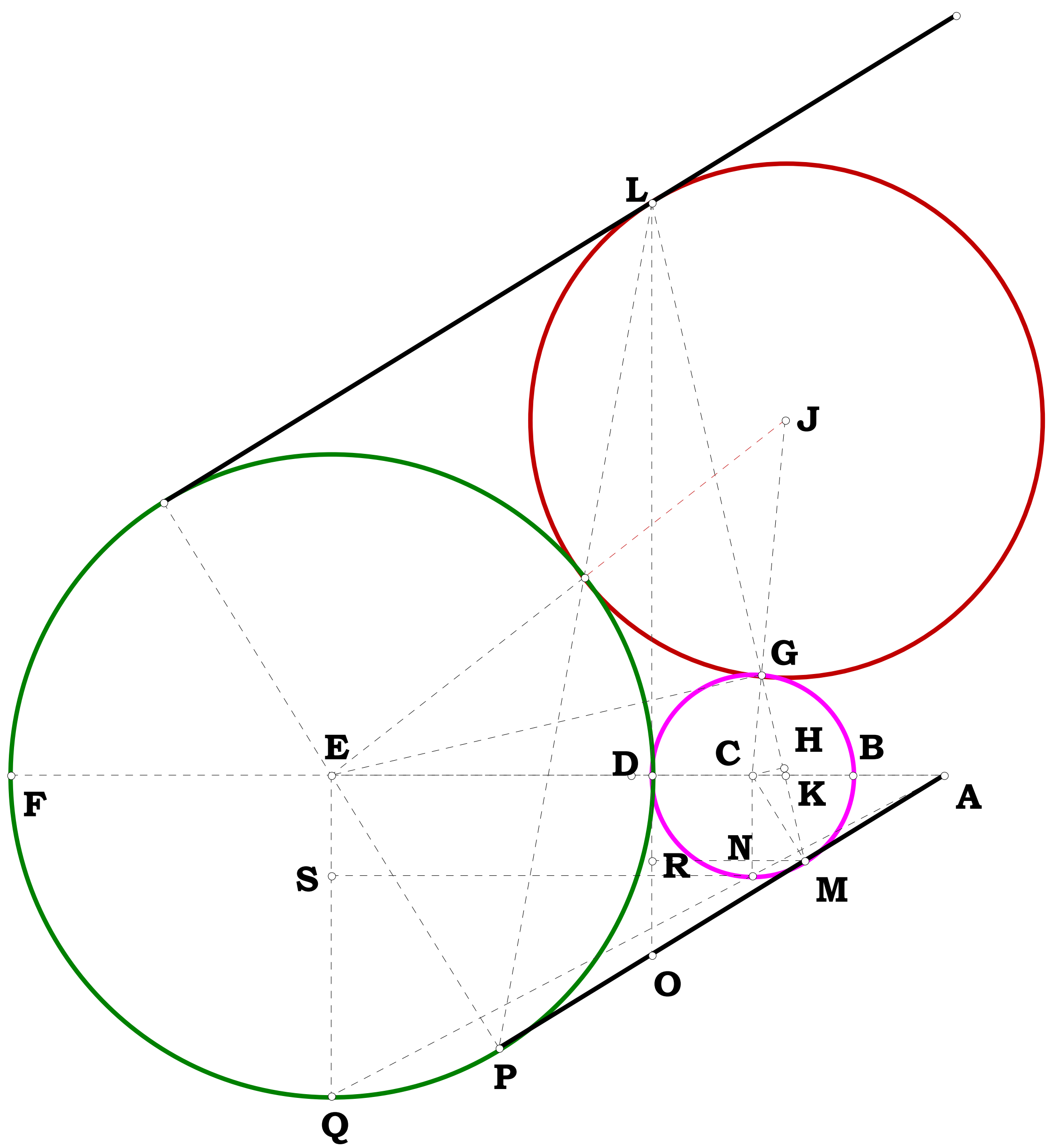
$AB := N_1 \qquad BC := N_2 \qquad AC := \sqrt{AB^2 + BC^2}$

$CD := \dfrac{BC \cdot AC}{AB} \qquad BD := \sqrt{CD^2 - BC^2}$

$$\frac{{N_2}^2}{N_1} - BD = 0$$

050194 Two Circles And A Parallel

Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.



$R_1 := 3 \quad R_2 := 2$

$DE := R_1 \quad BC := R_2 \quad CN := BC \quad EQ := DE \quad CD := BC \quad CE := CD + DE$

$ES := CN \quad NS := CE \quad SQ := EQ - ES \quad AE := \dfrac{NS \cdot EQ}{SQ} \quad AD := AE - DE \quad EP := DE$

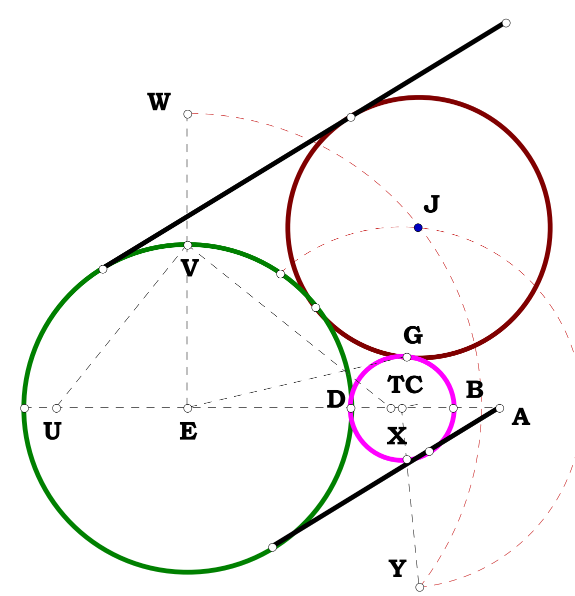
$AP := \sqrt{AE^2 - EP^2} \quad DO := \dfrac{EP \cdot AD}{AP} \quad DL := \dfrac{DO \cdot DE}{CD} \quad AC := AD - CD$

$AM := \dfrac{AP \cdot AC}{AE} \quad AO := \dfrac{AE \cdot AD}{AP} \quad MO := AO - AM \quad MR := \dfrac{AD \cdot MO}{AO} \quad RO := \dfrac{DO \cdot MR}{AD}$

$DR := DO - RO \quad LR := DR + DL \quad ML := \sqrt{MR^2 + LR^2} \quad DK := \dfrac{MR \cdot DL}{LR}$

$CK := DK - CD \quad CH := \dfrac{LR \cdot CK}{ML} \quad CM := BC \quad MH := \sqrt{CM^2 - CH^2}$

$MG := 2 \cdot MH \quad GL := ML - MG \quad GJ := \dfrac{CM \cdot GL}{MG}$



$R_3 := \dfrac{{R_1}^2}{4 \cdot R_2} \quad GJ - R_3 = 0$

$ET := 4 \cdot BC \quad EV := DE \quad EU := \dfrac{DE^2}{ET}$

$VW := EU \quad XY := EU \quad EV := DE \quad CX := BC$

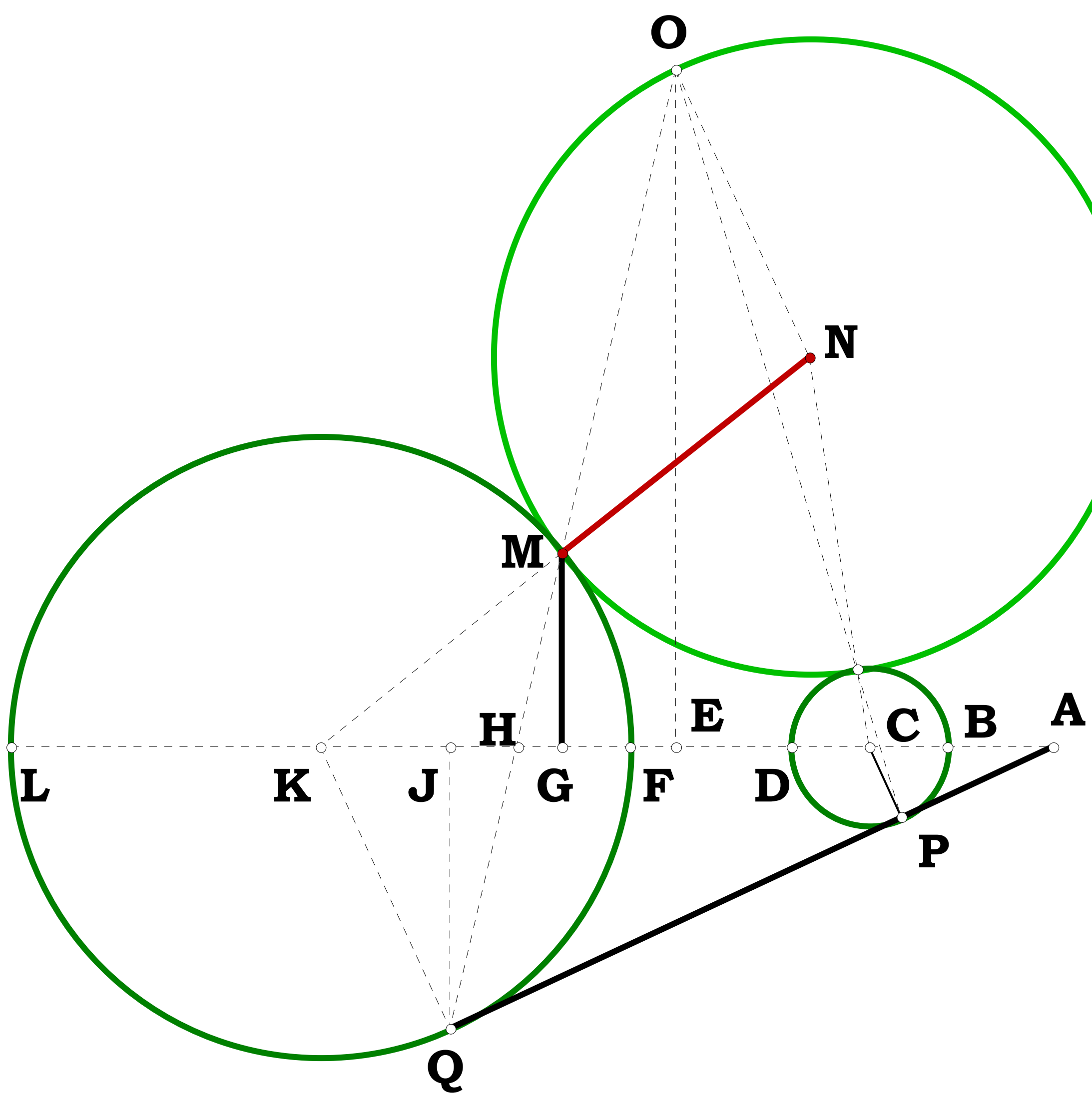
$EW := EV + VW \quad CY := CX + XY$

$GJ_2 := EU \quad GJ - GJ_2 = 0$

The Algebraic name for GJ suggests a simpler method of construction.

050494 Two Circles And A Tangent

Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.



$$R_1 := 3 \quad R_2 := 2 \quad D := 2 \quad dx := 2$$

$$FK := R_1 \quad BC := R_2 \quad CH := D \quad FL := 2 \cdot FK \quad AK := \frac{D \cdot R_1}{R_1 - R_2}$$

$$EK := \frac{{R_1}^2 + D^2 - {R_2}^2}{2 \cdot D} \quad AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2} \quad FG := \frac{FL}{dx}$$

$$GL := FL - FG \quad GM := \sqrt{FG \cdot GL} \quad AJ := \frac{AQ \cdot AQ}{AK} \quad AF := AK - FK$$

$$FJ := AJ - AF \quad JL := FL - FJ \quad JQ := \sqrt{FJ \cdot JL} \quad GJ := FJ - FG$$

$$QM := \sqrt{(JQ + GM)^2 + GJ^2} \quad GH := \frac{GJ \cdot GM}{JQ + GM} \quad HM := \frac{QM \cdot GM}{JQ + GM}$$

$$EF := EK - FK \quad EH := EF + FG + GH \quad HO := \frac{HM \cdot EH}{GH} \quad MO := HO - HM$$

$$KM := FK \quad MN := \frac{KM \cdot MO}{QM}$$

$$MN = -4.5$$

$$\sqrt{\left[\frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D}\right]^2} = 4.5$$

$$MN - \frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

$AB := 2.38542$

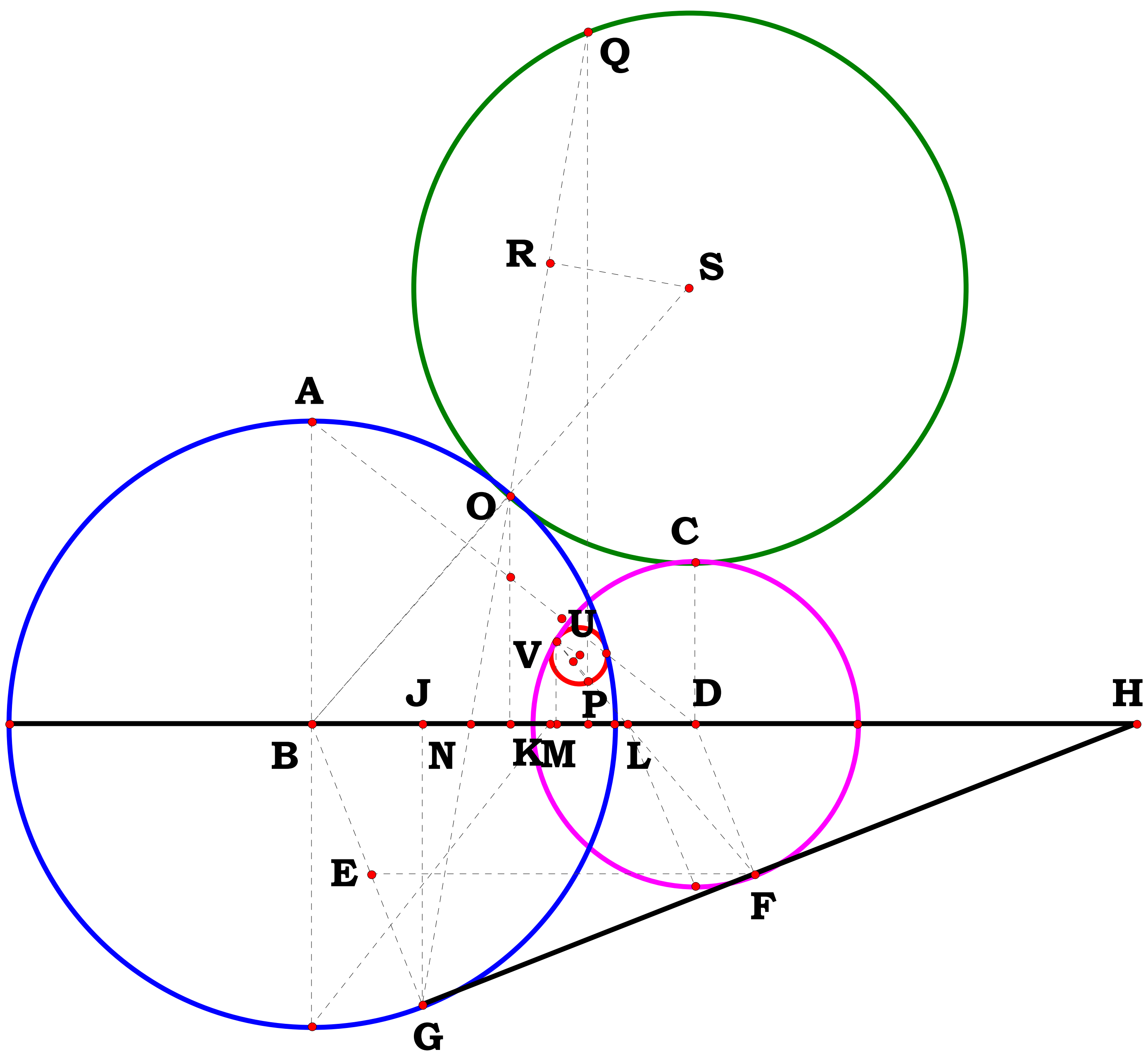
$CD := 1.72917$

$BD := 3.17708$

$dx := 1.22729$

$$\frac{N_3^{\,2} + R_1^{\,2} - R_2^{\,2}}{2 \cdot N_3}$$

From 042794 power line.



$EF := BD \qquad EG := AB - CD \qquad BH := \dfrac{EF \cdot AB}{EG} \qquad BJ := \dfrac{AB^2}{BH} \qquad KL := \dfrac{2 \cdot AB}{dx}$

$JL := AB - BJ \qquad JK := JL - KL \qquad GJ := \sqrt{(2 \cdot AB - JL) \cdot JL}$

$KO := \sqrt{(2 \cdot AB - KL) \cdot KL} \qquad KN := \dfrac{JK \cdot KO}{GJ + KO} \qquad BP := \dfrac{BD^2 + AB^2 - CD^2}{2 \cdot BD}$

$JN := \dfrac{JK \cdot GJ}{GJ + KO} \qquad NP := BP - (BJ + JN) \qquad NO := \sqrt{KO^2 + KN^2}$

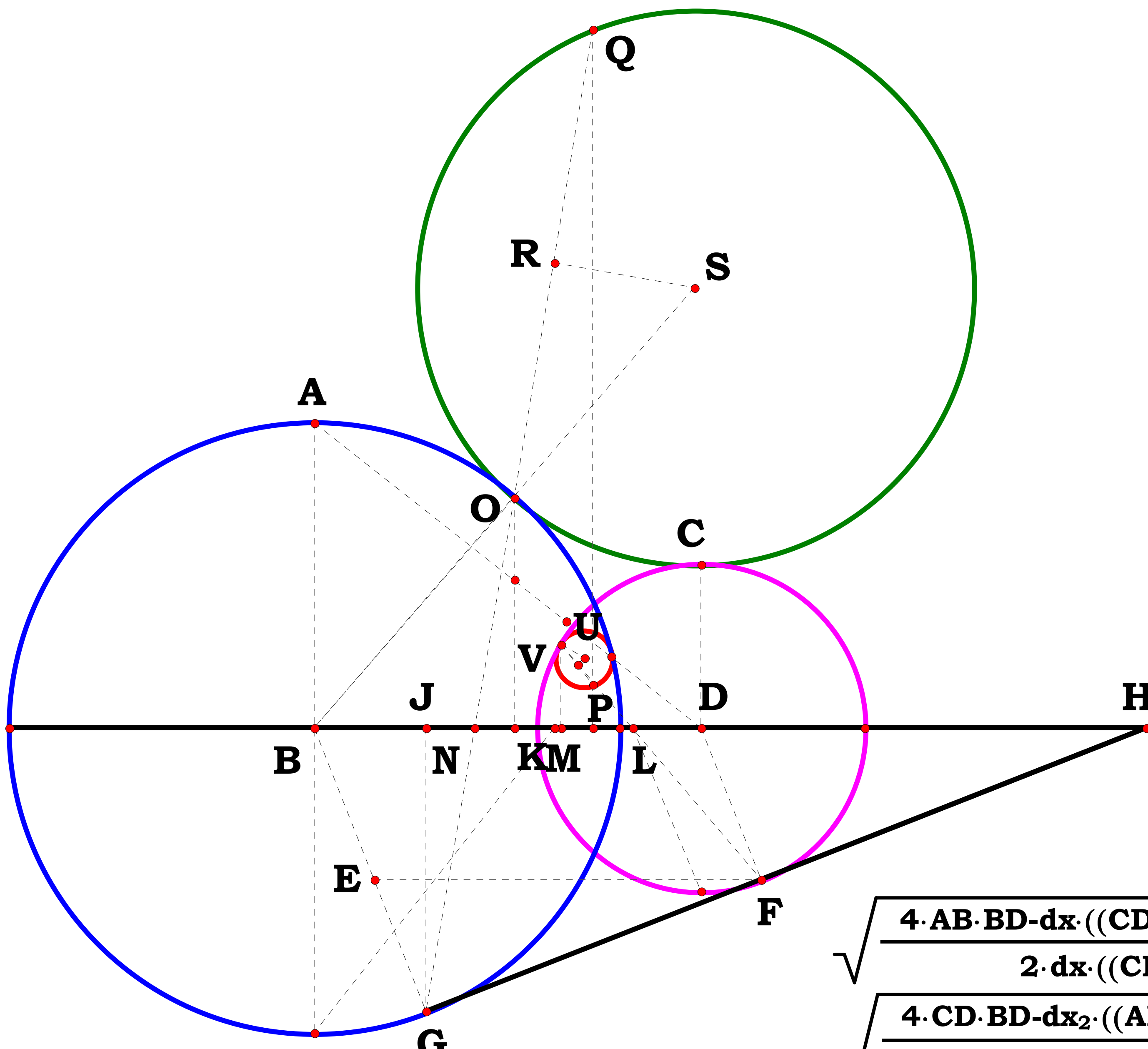
$QN := \dfrac{NO \cdot NP}{KN} \qquad OQ := QN - NO \qquad OR := \dfrac{OQ}{2} \qquad GO := \sqrt{(GJ + KO)^2 + JK^2}$

$SO := \dfrac{AB \cdot OR \cdot 2}{GO} \qquad SO = -4.204157$

$R_1 := AB \qquad R_2 := CD \qquad D := BD$

$$SO - \frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

$$\sqrt{\left[\frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D}\right]^2} = 4.204157$$



AB = 1.90625 in.
CD = 1.02083 in.
BD = 2.41667 in.
dx = 5.77984
SO = 1.73133 in.

KL = 0.65962 in.

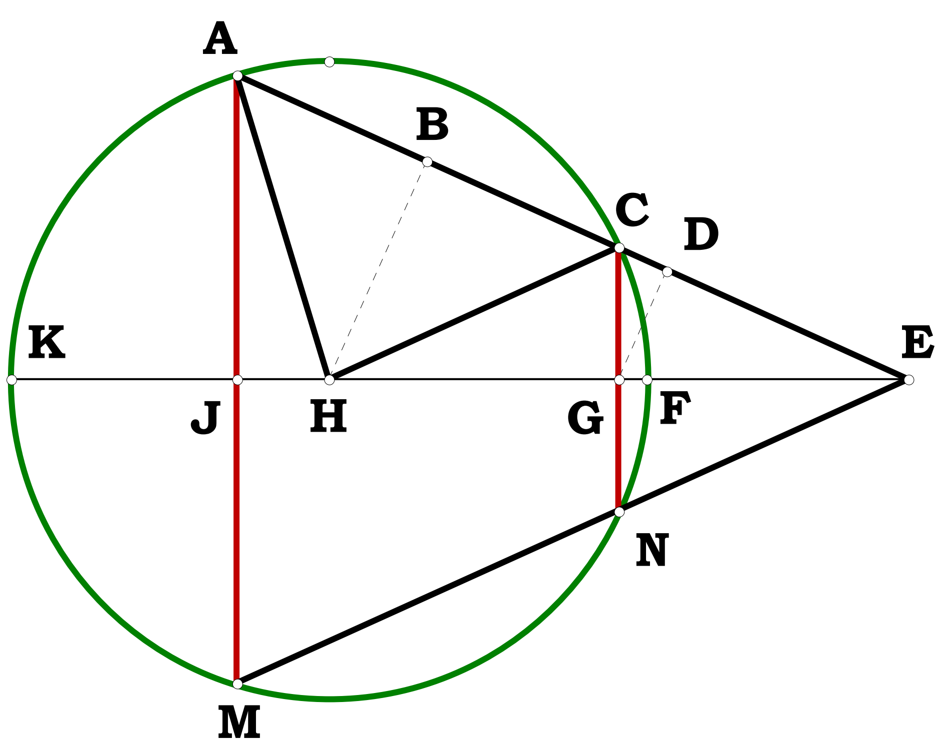
UV = 0.16880 in.
TM = 0.14146 in.
dx_2 = 14.43321

$$\sqrt{\frac{4\cdot AB\cdot BD-dx\cdot((CD+BD)-AB)\cdot((CD+AB)-BD)^{2}}{2\cdot dx\cdot((CD+BD)-AB)-4\cdot BD}}-SO = 0.00000\text{ in.}$$

$$\sqrt{\frac{4\cdot CD\cdot BD-dx_2\cdot((AB+BD)-CD)\cdot((AB+CD)-BD)^{2}}{2\cdot dx_2\cdot((AB+BD)-CD)-4\cdot BD}}-UV = 0.00000\text{ in.}$$

050694 A Ratio In Trisection

What is AJ to CG?



$N := 6$

$FH := 1$

$CE := FH$

$CG := \dfrac{FH}{N}$ $\quad EG := \sqrt{CE^2 - CG^2}$ $\quad CD := \dfrac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2}$ $\quad EH := 2 \cdot EG$ $\quad BH := \dfrac{DG \cdot EH}{EG}$ $\quad CH := FH$

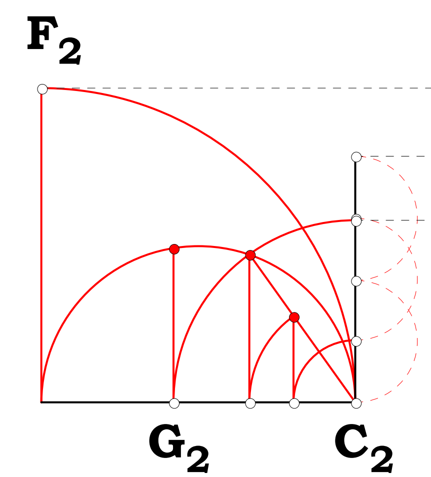
$BC := \sqrt{CH^2 - BH^2}$ $\quad AC := 2 \cdot BAE := AC + CE$ $\quad AJ := \dfrac{CG \cdot AE}{CE}$ $\quad 3 \cdot CG - \dfrac{4 \cdot CG^3}{CE^2} - AJ = 0$

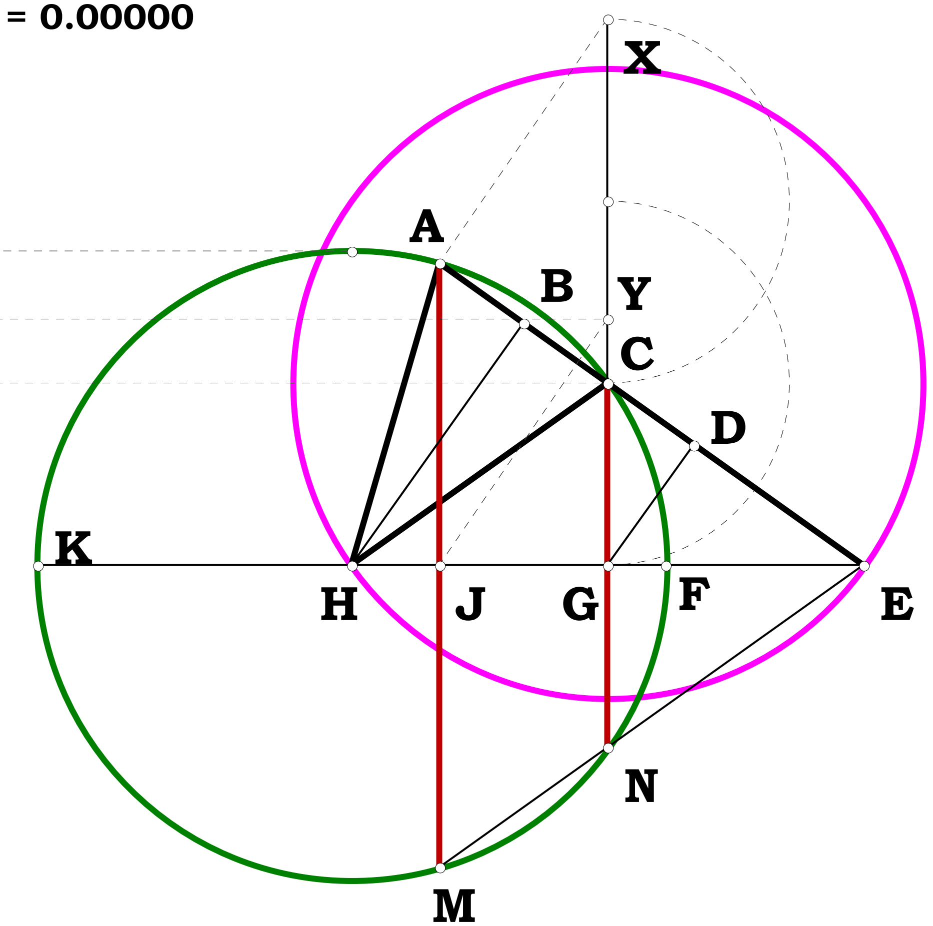
$3 \cdot CG - 4 \cdot CG^3 - AJ = 0$ $\quad AJ - \dfrac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$ $\quad AJ - \left(\dfrac{3}{N} - \dfrac{4}{N^3}\right) = 0$

The resultant equation suggests this construction.

$$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG^3}{FH}\right) - \frac{AJ}{FH} = 0.00000$$

AJ-XY = 0.000 in.

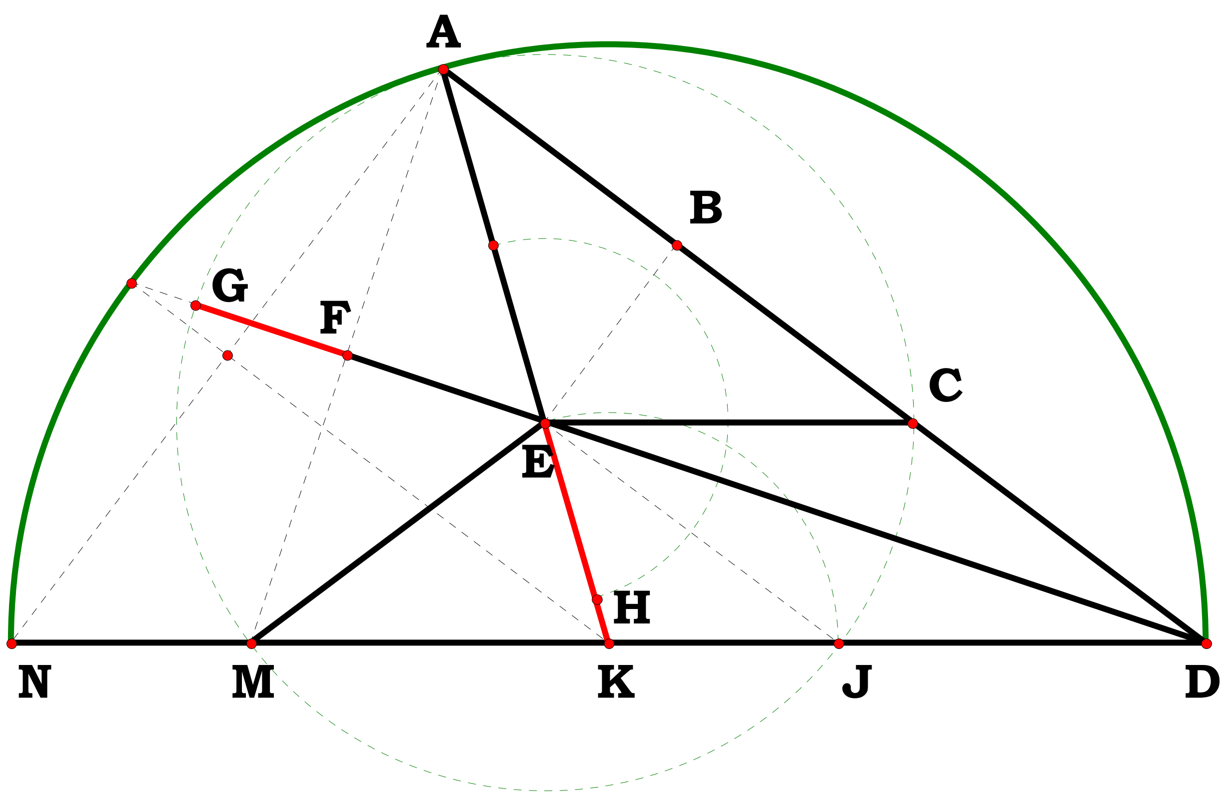




$$\frac{3 \cdot N^2 - 4}{N^3} - AJ = 0$$

050794 A Trisection Ratio

In trisection, what is the ratio of FG/EK?



Given any angle to trisect, one the simply solves for N.

$N := 6$

$AE := 1 \quad EH := \dfrac{AE}{2} \quad HK := \dfrac{AE}{N} \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \dfrac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \dfrac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \dfrac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

$EF := AE - FG \qquad DK := AE + EK$

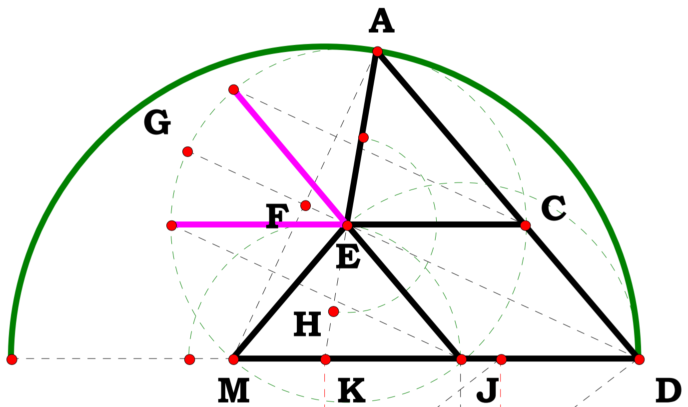
Algebraic Names.

$$\frac{FG}{EK} - \frac{2 \cdot N \cdot \left[\sqrt{N+1} \cdot (2-N) + (N+2)^{\frac{3}{2}}\right]}{(N+2)^{\frac{5}{2}}} = 0 \qquad \frac{EK}{EF} - \frac{(N+2)^{\frac{5}{2}}}{2 \cdot N \cdot \sqrt{N+1} \cdot (N-2)} = 0$$

The gist of the story is this. Once one knows the ratio between difference parts of a figure where one has been set up as a unit, for example, AE as unit and EF as a ratio of the figure, then one knows the multiple of the unit by which to project any of the remaining unknowns; i.e. EK/HK. -1 to 1.

$$EF - \frac{AE \cdot \sqrt{N+1} \cdot (N-2)}{(N+2)^{\frac{3}{2}}} = 0 \qquad EK - \frac{AE \cdot (N+2)}{2 \cdot N} = 0 \qquad \frac{EF}{EK} - \frac{2 \cdot N \cdot \sqrt{N+1} \cdot (N-2)}{(N+2)^{\frac{5}{2}}} = 0$$

$$FG - \frac{AE \cdot \left[\sqrt{N+1} \cdot (N+2)^2 + \sqrt{N+2} \cdot (N+1) \cdot (2-N)\right]}{\sqrt{N+1} \cdot (N+2)^2} = 0 \qquad \frac{AE}{EF} - \frac{\left(\sqrt{N+2}\right)^3}{\sqrt{N+1} \cdot (N-2)} = 0 \qquad \frac{EF}{AE} \cdot \frac{\left(\sqrt{N+2}\right)^3}{\sqrt{N+1} \cdot (N-2)} = 1 \qquad \frac{EK}{HK} - \frac{N+2}{2} = 0$$



$AE = 1.00000$

$EF = 0.26375$

$FG = 0.73625$

$EK = 0.77472$

$\sqrt{N_1+2}^{\,3} = 13.39433$

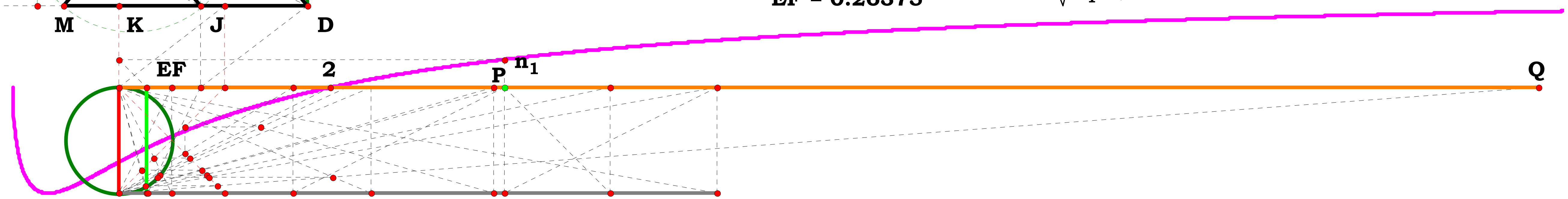
$Q = 13.39433$

$EF = 0.26375$

$\sqrt{N_1+1}\cdot(N_1-2) = 3.53272$

$P = 3.53272$

$$\frac{\left(\sqrt{N_1+1}\cdot(N_1-2)\right)}{\sqrt{N_1+2}^{\,3}} = 0.26375$$



$N_1 = 3.64002$

$$\frac{\left(2\cdot N_1\cdot\left(\sqrt{N_1+1}\cdot(2-N_1)+(N_1+2)^{\frac{3}{2}}\right)\right)}{(N_1+2)^{\frac{5}{2}}}-\frac{FG}{EK} = 0.00000$$

$$\frac{(N_1+2)^{\frac{5}{2}}}{\left(2\cdot N_1\cdot\sqrt{N_1+1}\cdot(N_1-2)\right)}-\frac{EK}{EF} = 0.00000$$

$$\frac{AE\cdot\sqrt{N_1+1}\cdot(N_1-2)}{\sqrt{N_1+2}^{\,3}}-EF = 0.00000$$

$$\frac{AE\cdot(N_1+2)}{2\cdot N_1}-EK = 0.00000$$

$$\frac{\left(2\cdot N_1\cdot\sqrt{N_1+1}\cdot(N_1-2)\right)}{(N_1+2)^{\frac{5}{2}}}-\frac{EF}{EK} = 0.00000$$

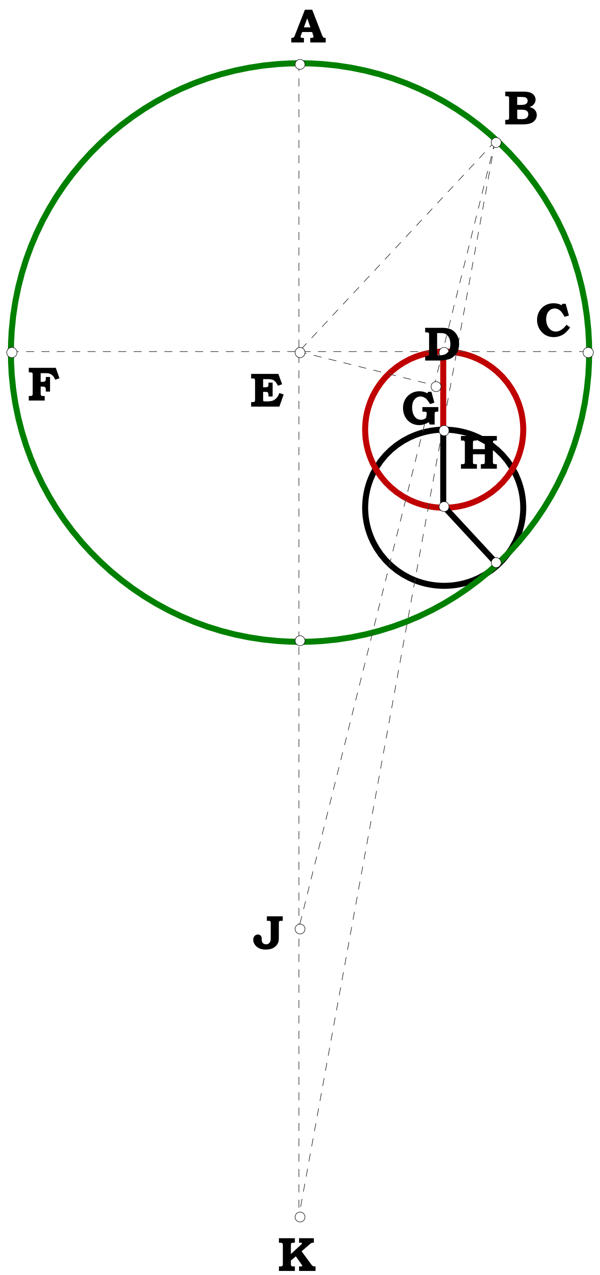
$$\frac{AE\cdot\left(\sqrt{N_1+1}\cdot(N_1+2)^2+\sqrt{N_1+2}\cdot(N_1+1)\cdot(2-N_1)\right)}{\sqrt{N_1+1}\cdot(N_1+2)^2}-FG = 0.00000$$

$$\frac{\sqrt{N_1+2}^{\,3}}{\left(\sqrt{N_1+1}\cdot(N_1-2)\right)}-\frac{AE}{EF} = 0.00000$$

$$\frac{\sqrt{N_1+2}^{\,3}}{\left(\sqrt{N_1+1}\cdot(N_1-2)\right)}\cdot\left(\frac{EF}{AE}\right) = 1.00000$$

051694A Tangent Diameter and Circles

Choose a point along CF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.



The first variable is just the chosen place for the tangent circles. The second is the number of circles to construct.

$$CF := 2 \quad CE := 1 \quad N_1 := 4 \quad N_2 := 2 \quad CD := \frac{CE}{N_1} \quad DE := CE - CD \quad EJ := CE \cdot N_2$$

$$DJ := \sqrt{DE^2 + EJ^2} \quad JG := \frac{EJ^2}{DJ} \quad BE := CE \quad EG := \sqrt{EJ^2 - JG^2} \quad BG := \sqrt{BE^2 - EG^2}$$

$$BJ := BG + JG \quad JK := CE \quad BD := BJ - DJ \quad DH := \frac{JK \cdot BD}{BJ}$$

Algebraic Names:

$$CD - \frac{1}{N_1} = 0 \quad DE - \frac{(N_1 - 1)}{N_1} = 0 \quad EJ - N_2 = 0 \quad DJ - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}{N_1} = 0$$

$$JG - \frac{N_1 \cdot N_2^2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0 \quad EG := \frac{(N_1 - 1) \cdot N_2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$

$$BG - \sqrt{\frac{\left(N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2\right)}{\left(N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2\right)}} = 0$$

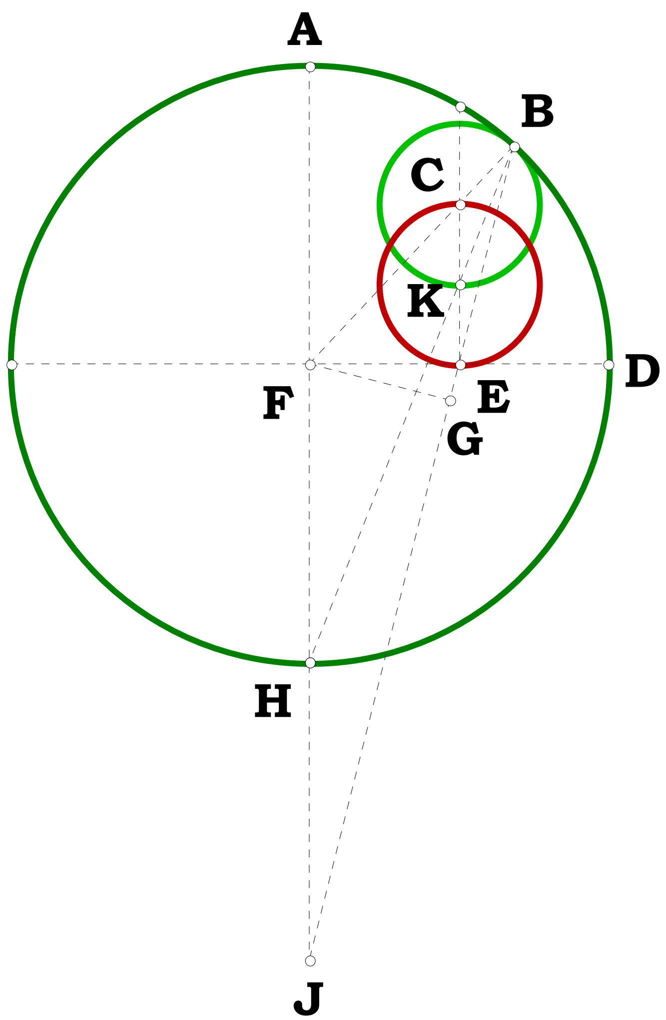
$$BJ - \frac{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0$$

$$BD - \frac{\left(2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 2 \cdot N_1 - N_2^2 + 1} - 1\right)}{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2} \cdot N_1\right)} = 0$$

$$DH - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} \cdot N_1 - N_1^2 + 2 \cdot N_1 - 1}{N_1 \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)} = 0$$

05/16/94B Tangent Diameter and Circles

Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.



$N_1 := 2 \quad N_2 := 2 \qquad AF := 1$

$DF := AF \quad DE := \dfrac{DF}{N_1} \quad AJ := AF \cdot N_2$

$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$

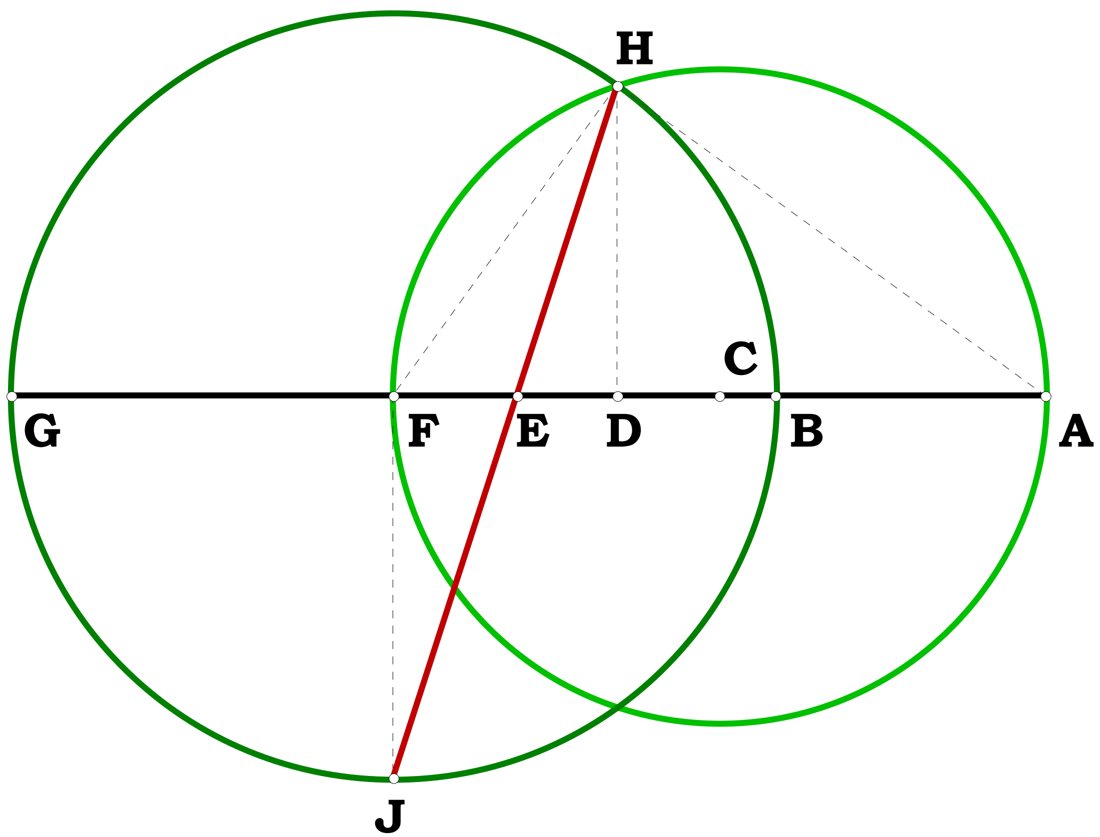
$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \dfrac{EF^2}{EJ} \quad BF := AF$

$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$

$BJ := BE + EJ \quad KE := \dfrac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\dfrac{AJ - AF}{AF}\right) \cdot KE\right]^2} \qquad BC - KE = 0$

$$BC - \frac{2 \cdot N_1 - N_1^{\,2} + N_1 \cdot \sqrt{N_1^{\,2} + 2 \cdot N_1 \cdot N_2^{\,2} - 4 \cdot N_1 \cdot N_2 - N_2^{\,2} + 2 \cdot N_2 - 1}}{N_1 \cdot \left(N_1 + N_1 \cdot N_2^{\,2} - 2 \cdot N_1 \cdot N_2 + \sqrt{N_1^{\,2} + 2 \cdot N_1 \cdot N_2^{\,2} - 4 \cdot N_1 \cdot N_2 - N_2^{\,2} + 2 \cdot N_2}\right)} = 0$$

102794 Trivial Method Square Root



AE is the square root of AB x AG.

$\mathbf{N} := 5 \quad \mathbf{AB} := 1$

$\mathbf{AG} := \mathbf{AB}\cdot\mathbf{N} \quad \mathbf{BG} := \mathbf{AG} - \mathbf{AB} \quad \mathbf{BF} := \dfrac{\mathbf{BG}}{2}$

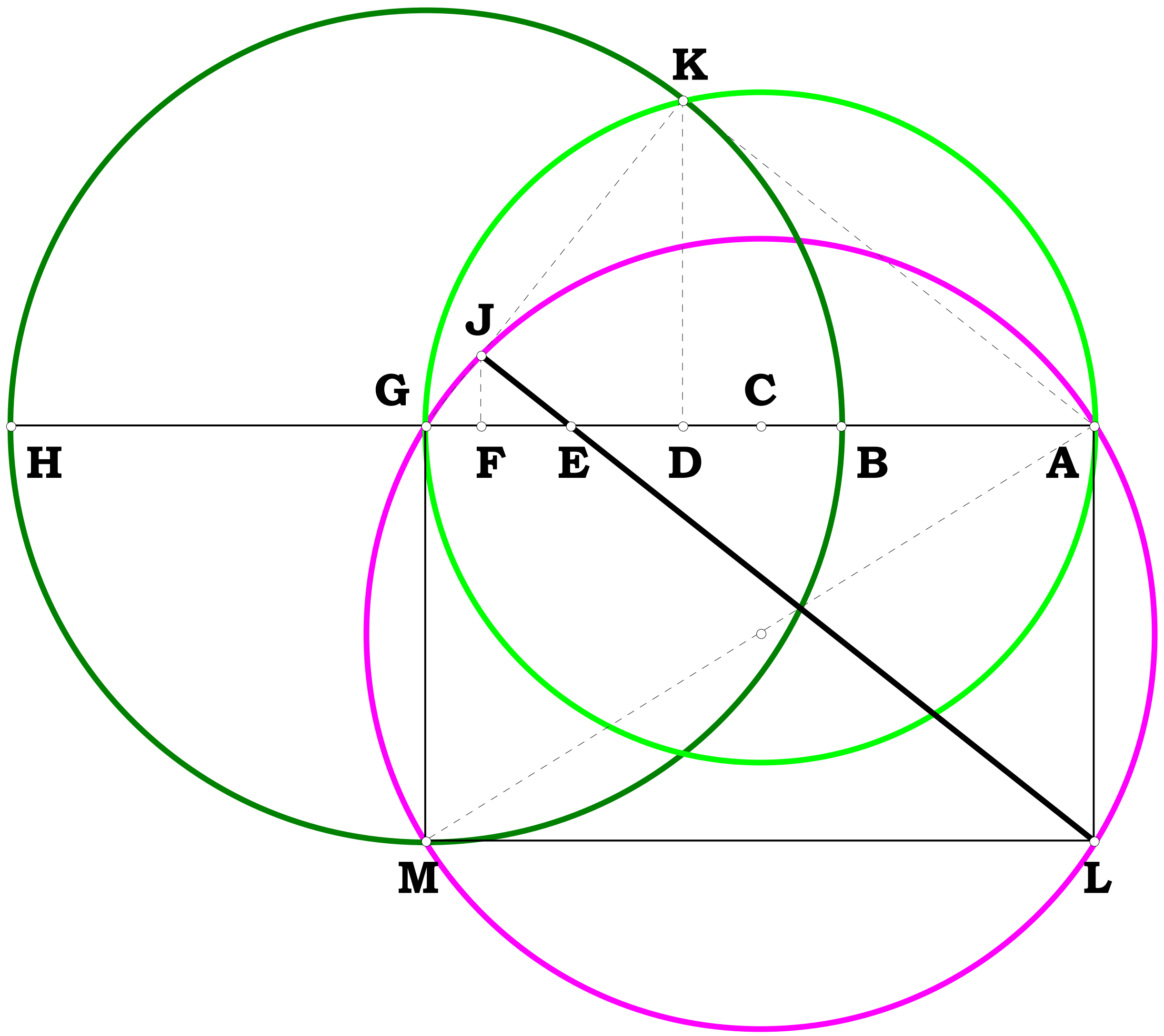
$\mathbf{AF} := \mathbf{AB} + \mathbf{BF} \quad \mathbf{FH} := \mathbf{BF} \quad \mathbf{DF} := \dfrac{\mathbf{FH}^2}{\mathbf{AF}} \quad \mathbf{AD} := \mathbf{AF} - \mathbf{DF}$

$\mathbf{BD} := \mathbf{AD} - \mathbf{AB} \quad \mathbf{DG} := \mathbf{BG} - \mathbf{BD} \quad \mathbf{FJ} := \mathbf{BF}$

$\mathbf{DH} := \sqrt{\mathbf{BD}\cdot\mathbf{DG}} \quad \mathbf{DE} := \dfrac{\mathbf{DF}\cdot\mathbf{DH}}{\mathbf{DH} + \mathbf{FJ}} \quad \mathbf{AE} := \mathbf{AB} + \mathbf{BD} + \mathbf{DE}$

$\sqrt{\mathbf{AB}\cdot\mathbf{AG}} - \mathbf{AE} = 0$

102894 Trivial Method Square Root



AE is the square root of AB x AH.

$N := 5 \quad AB := 1 \quad AH := AB \cdot N \quad BH := AH - AB$

$BG := \dfrac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \dfrac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

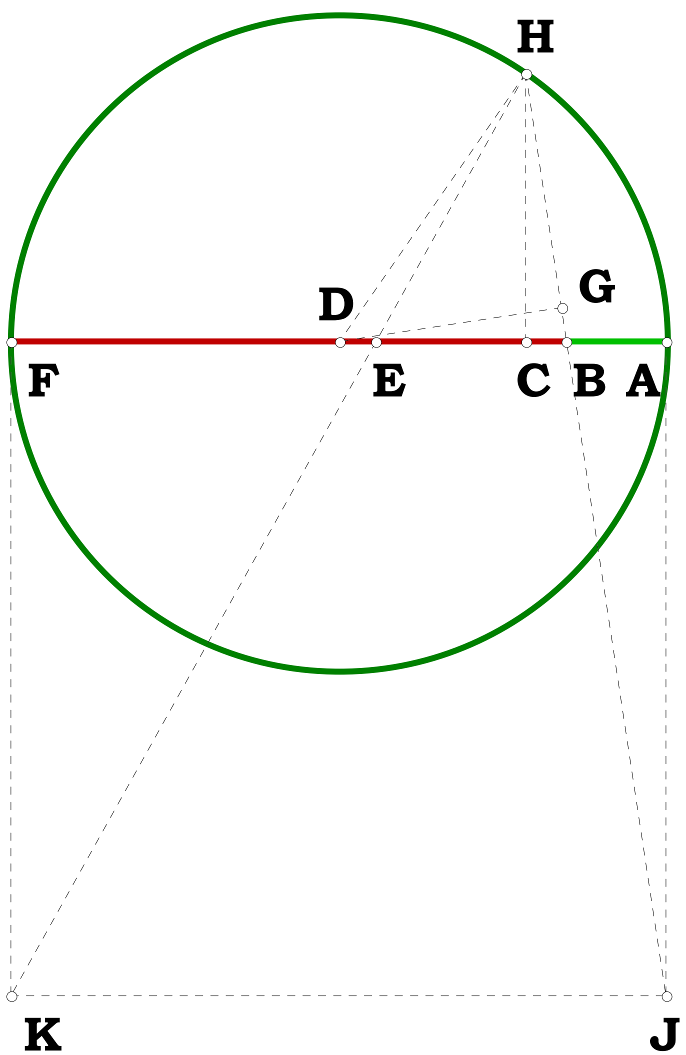
$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ := \dfrac{{S_2}^2 + {S_1}^2 - {S_3}^2}{2 \cdot S_1} \quad JL := \sqrt{GL^2 - GJ^2}$

$FG := \dfrac{DG \cdot GJ}{GK} \quad AF := AG - FG \quad FJ := \dfrac{DK \cdot GJ}{GK} \quad EF := \dfrac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0$

103194 Square Root of a Segment

Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square. Language begins with a naming convention.



$N := 22 \quad AB := 1$

$AF := AB \cdot N \quad BF := AF - AB \quad AD := \dfrac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \dfrac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \dfrac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \quad BC := \dfrac{AB \cdot (BG + GH)}{BJ} \quad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \quad CE := \dfrac{CF \cdot CH}{(CH + FK)}$

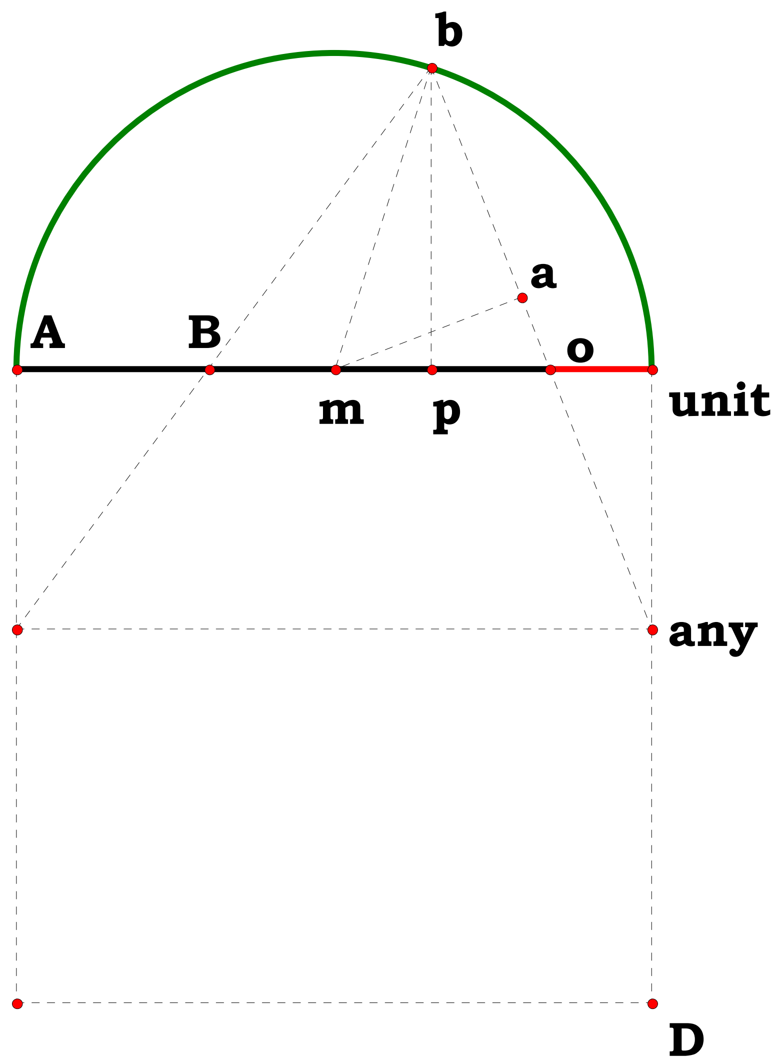
$EF := CF - CE \quad BE := BC + CE \quad DF := BF - {:}CD := AD - AC \quad BE^2 - EF = 0$

$$BE - \frac{N + N \cdot \sqrt{4 \cdot N - 3} - 2}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} = 0 \qquad EF - \left(\frac{2 \cdot N^2 - \sqrt{4 \cdot N - 3} - 2 \cdot N + 1}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} \right) = 0$$

$AE := AB + BE \qquad unit := AB$

$$\frac{AE}{unit} - \frac{BE}{unit} = 1 \quad \sqrt{\frac{EF}{unit}} - \frac{BE}{unit} = 0 \quad \left(\frac{BE}{unit}\right)^2 - \frac{EF}{unit} = 0$$

There is a difference between a unit, and the number 1. A unit is any difference, when it is called a standard, then it is given the name of 1.



Adding a multiplyer.

Let OA be any difference what so ever and let 1 be any difference what so ever and called a standard, a unit, or again a definition. It then follows that:

$$N_1 := 2.77083 \quad N_2 := .53125 \quad N_3 := N_1 + N_2 \quad N_4 := 1.35844$$

$$Ao := N_1 \quad unit := N_2 \quad m := \frac{N_3}{2} \quad any := N_4 \quad mo := m - unit \quad anyo := \sqrt{any^2 + unit^2}$$

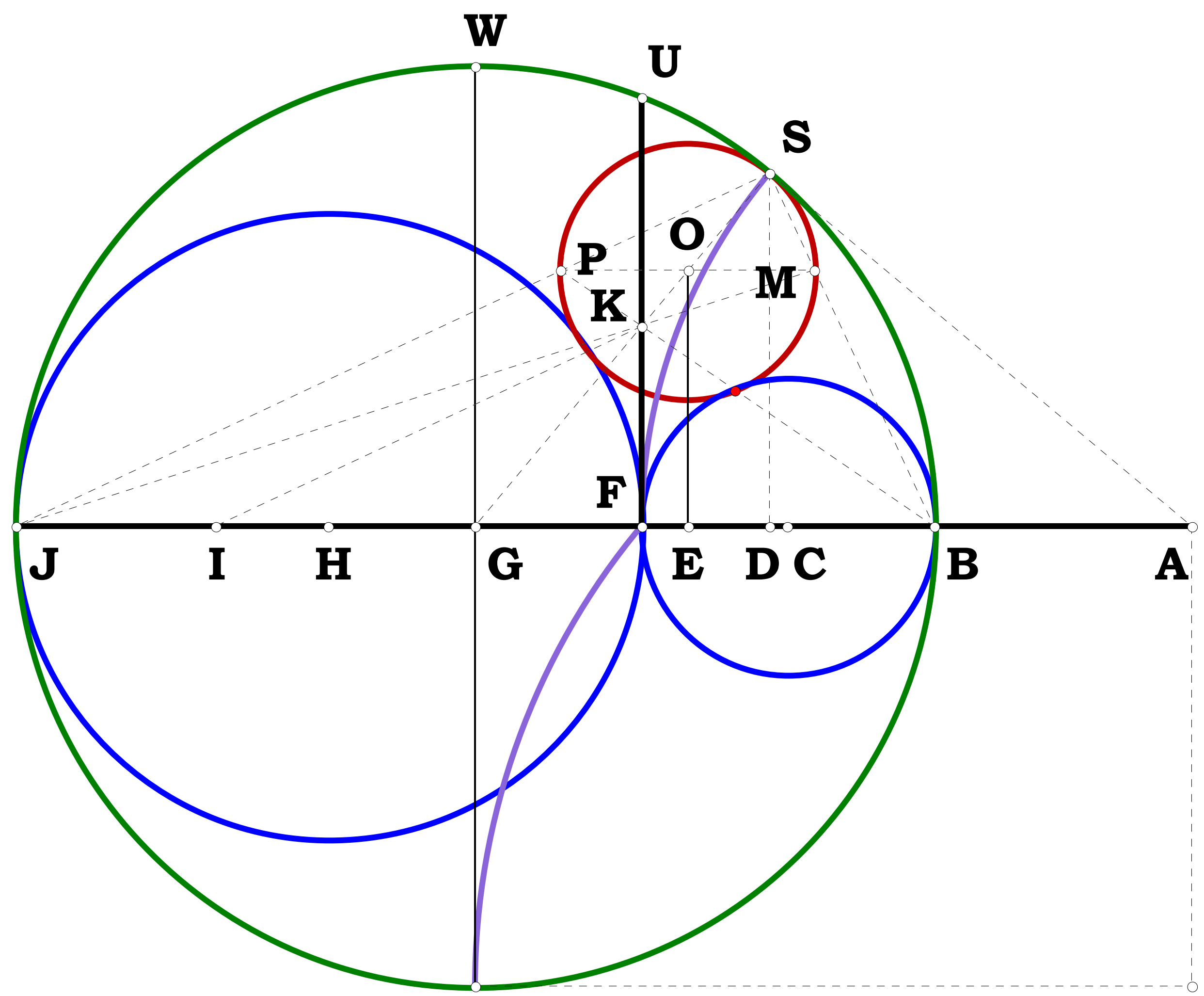
$$ao := \frac{unit \cdot mo}{anyo} \quad ma := \frac{any \cdot mo}{anyo} \quad ab := \sqrt{m^2 - ma^2}$$

$$po := \frac{unit \cdot (ab + ao)}{anyo} \quad Ap := N_3 - (po + unit) \quad bp := \sqrt{(po + unit) \cdot Ap}$$

$$AB := \frac{Ap \cdot any}{any + bp} \quad Bo := N_3 - (AB + unit)$$

$$\sqrt{\frac{AB}{unit} \cdot \frac{N_3}{any}} - \frac{Bo}{unit} = 0$$

122494 Power Line At Square Root



In this square root figure, what is the Algebraic name of the tangent circle OS?

$N := 5 \quad AB := 1 \quad AJ := AB \cdot N$

$AF := \sqrt{AB \cdot AJ} \quad BJ := AJ - AB \quad BG := \frac{BJ}{2}$

$AG := AB + BG \quad GS := BG \quad DG := \frac{GS^2}{AG}$

$FG := AG - AF \quad BD := BG - DG \quad DJ := BJ - BD$

$DS := \sqrt{BD \cdot DJ} \quad FK := \frac{DS \cdot FG}{DG} \quad BF := AF - AB$

$BK := \sqrt{BF^2 + FK^2} \quad FI := \frac{DJ \cdot FK}{DS} \quad BI := FI + BF$

$BP := \frac{BK \cdot BJ}{BI} \quad KP := BP - BK \quad MP := \frac{BJ \cdot KP}{BK} \quad OS := \frac{MP}{2}$

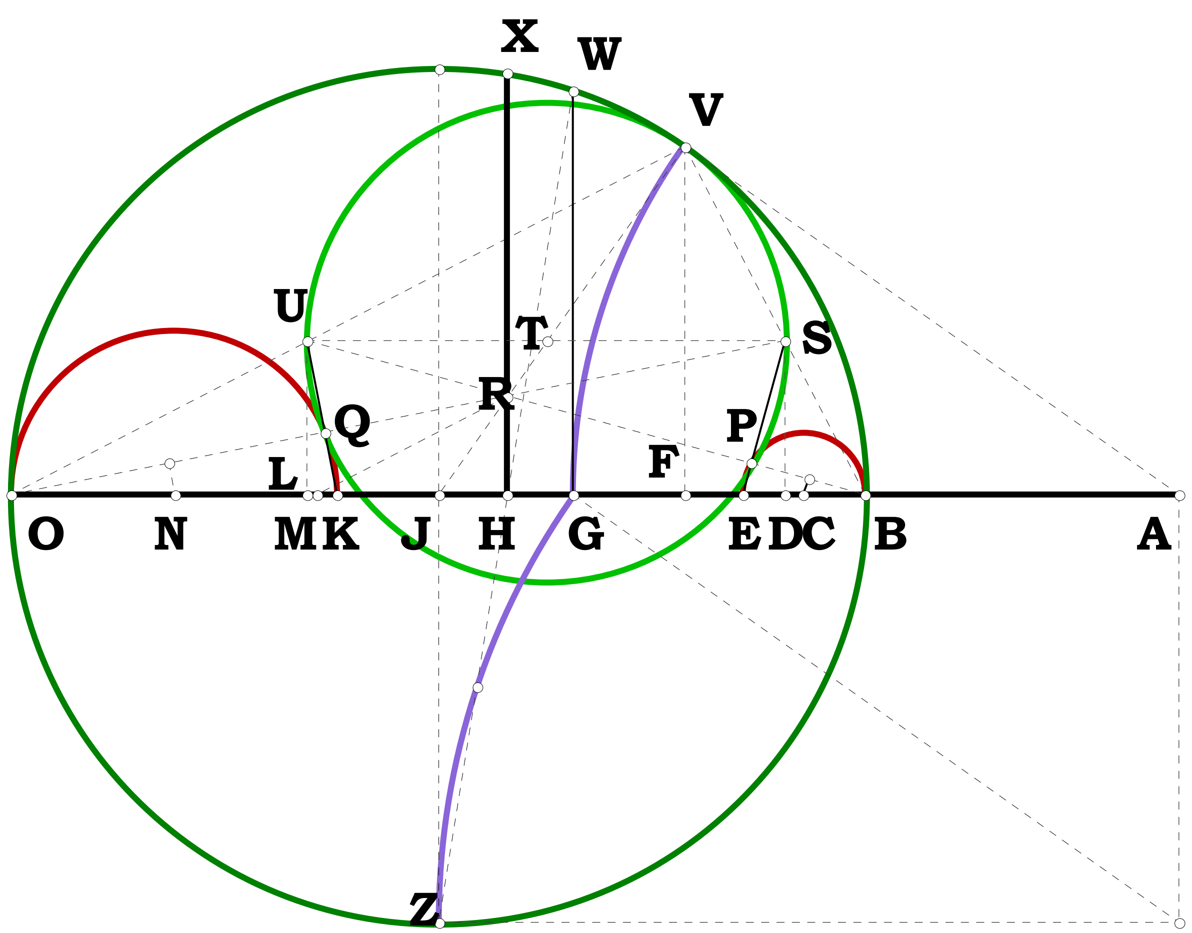
Algebraic Names

$$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0 \qquad OS - \frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(N + \sqrt{N} + 1\right)} = 0$$

122595 Two Prime Exponential Series
Developed Through The Powerline Progression

$\Delta := 8 \qquad \delta := 1 .. \Delta$

$N := 5 \qquad AB := 1$



$$AO := AB \cdot N \quad AG := \sqrt{AB \cdot AO} \quad BO := AO - AB \quad BJ := \frac{BO}{2} \quad JZ := BJ$$

$$JV := BJ \quad JO := BJ \quad BG_1 := AG - AB \quad GO_1 := BO - BG_1$$

$$GW_1 := \sqrt{BG_1 \cdot GO_1} \quad GJ_1 := BJ - BG_1 \quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$$

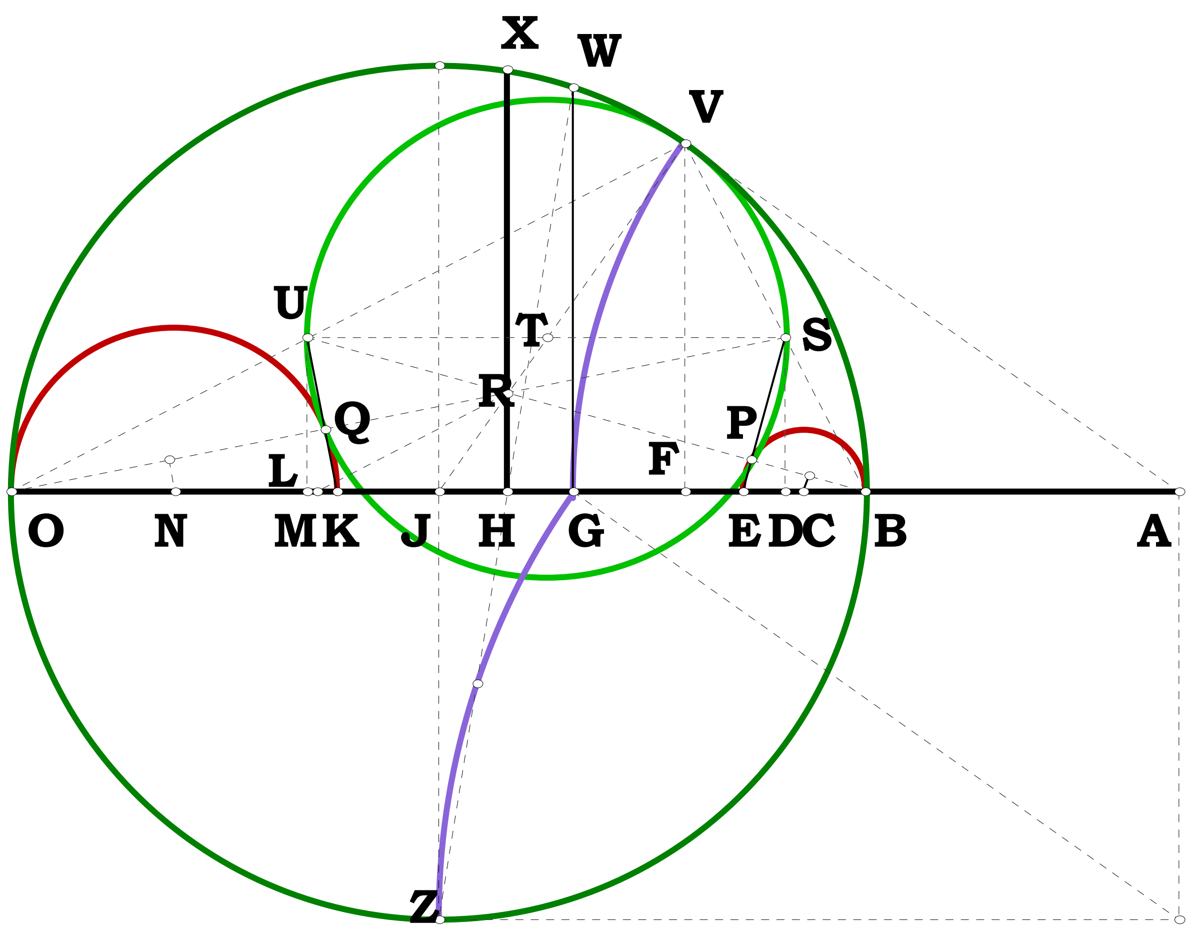
$$\begin{pmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{pmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - (BG_\delta + GH_\delta) \\ \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]} \\ BJ - (BG_\delta + GH_\delta) \\ \dfrac{[BJ - (BG_\delta + GH_\delta)] \cdot \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}}{JZ + \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}} \end{bmatrix}$$

$$HJ := BJ - BG_\Delta \quad FJ := \frac{(N-1)^2}{2 \cdot (N+1)} \quad BF := BJ - FJ \quad FO := FJ + JO$$

$$FV := \sqrt{BF \cdot FO} \quad HR := \frac{FV \cdot HJ}{FJ} \quad BH := BJ - HJ \quad BR := \sqrt{HR^2 + BH^2}$$

$$HM := \frac{FO \cdot HR}{FV} \quad BU := \frac{BR \cdot BO}{BH + HM} \quad RU := BU - BR \quad SU := \frac{BO \cdot RU}{BR} \quad TV := \frac{SU}{2} \quad PU := \frac{BH \cdot SU}{BR} \quad BP := BU - PU \quad BE := \frac{BR \cdot BP}{BH} \quad AE := AB + BE$$

$$N^{\frac{1}{2^\Delta}} - AE = 0$$



$$LU := \frac{HR \cdot BU}{BR} \quad BL := \frac{BH \cdot BU}{BR} \quad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \quad DS := LU \quad OS := \frac{OR \cdot DS}{HR}$$

$$DO := \frac{HO \cdot DS}{HR} \quad QS := \frac{DO \cdot SU}{OS} \quad OQ := OS - QS$$

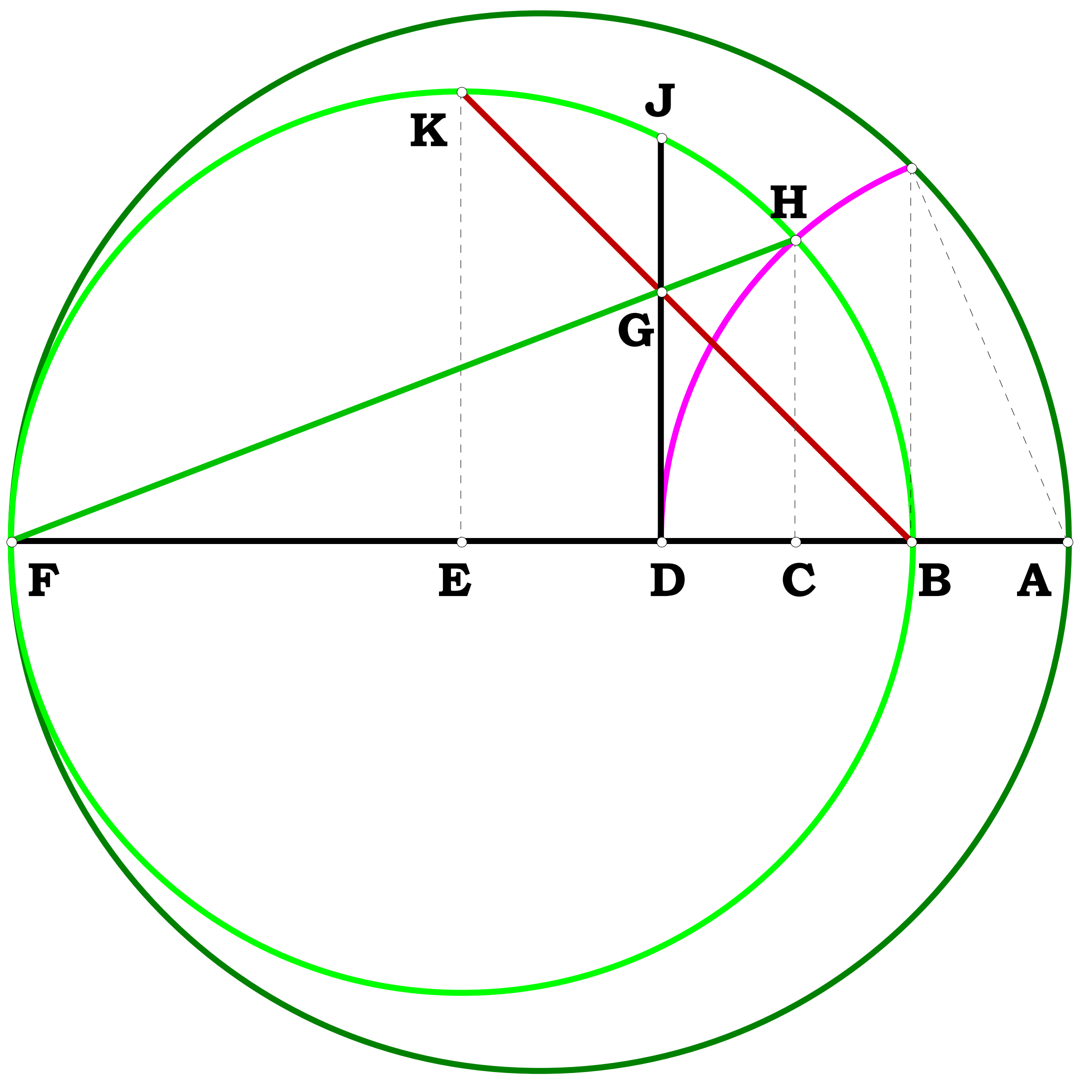
$$KO := \frac{OS \cdot OQ}{DO} \quad AK := AO - KO$$

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$

122694 Is Point G on DJ?

Is G, the intersection of FH and BK, on DJ?

$$\frac{(N-1)^2}{2\cdot(N+1)}$$ From 122494



$N := 5 \quad AB := 1 \quad AF := AB\cdot N \quad BF := AF - AB$

$BE := \frac{BF}{2} \quad EF := BE \quad EK := BE$

$AD := \sqrt{AB\cdot AF} \quad CE := \frac{(N-1)^2}{2\cdot(N+1)} \quad CF := CE + EF$

$DF := AF - AD \quad BC := BF - CF \quad CH := \sqrt{BC\cdot CF}$

$DG_1 := \frac{CH\cdot DF}{CF}$

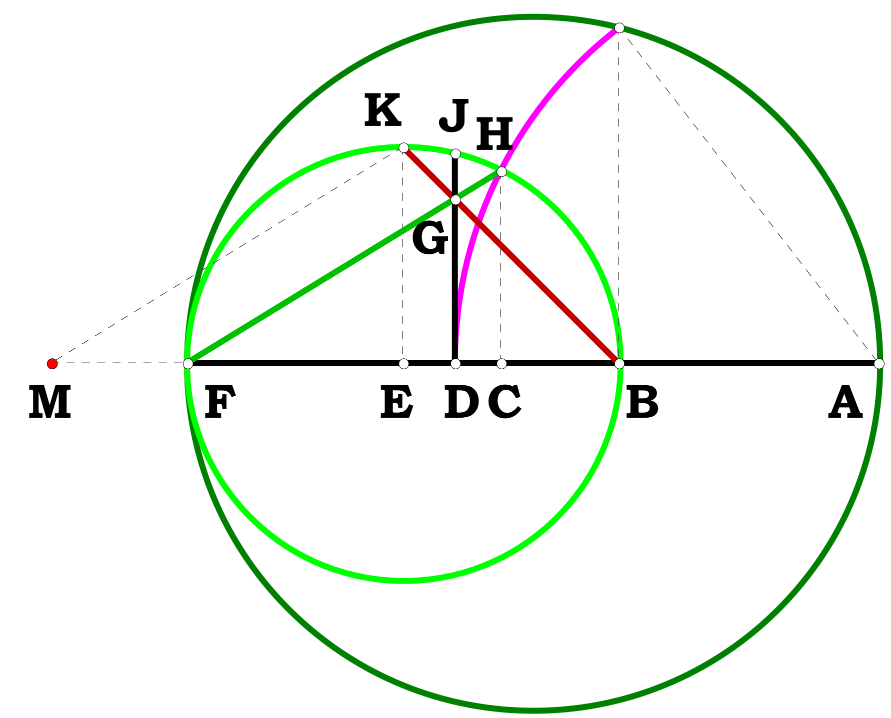
$BD := AD - AB \quad DG_2 := \frac{EK\cdot BD}{BE}$

$DG_1 - DG_2 = 0$

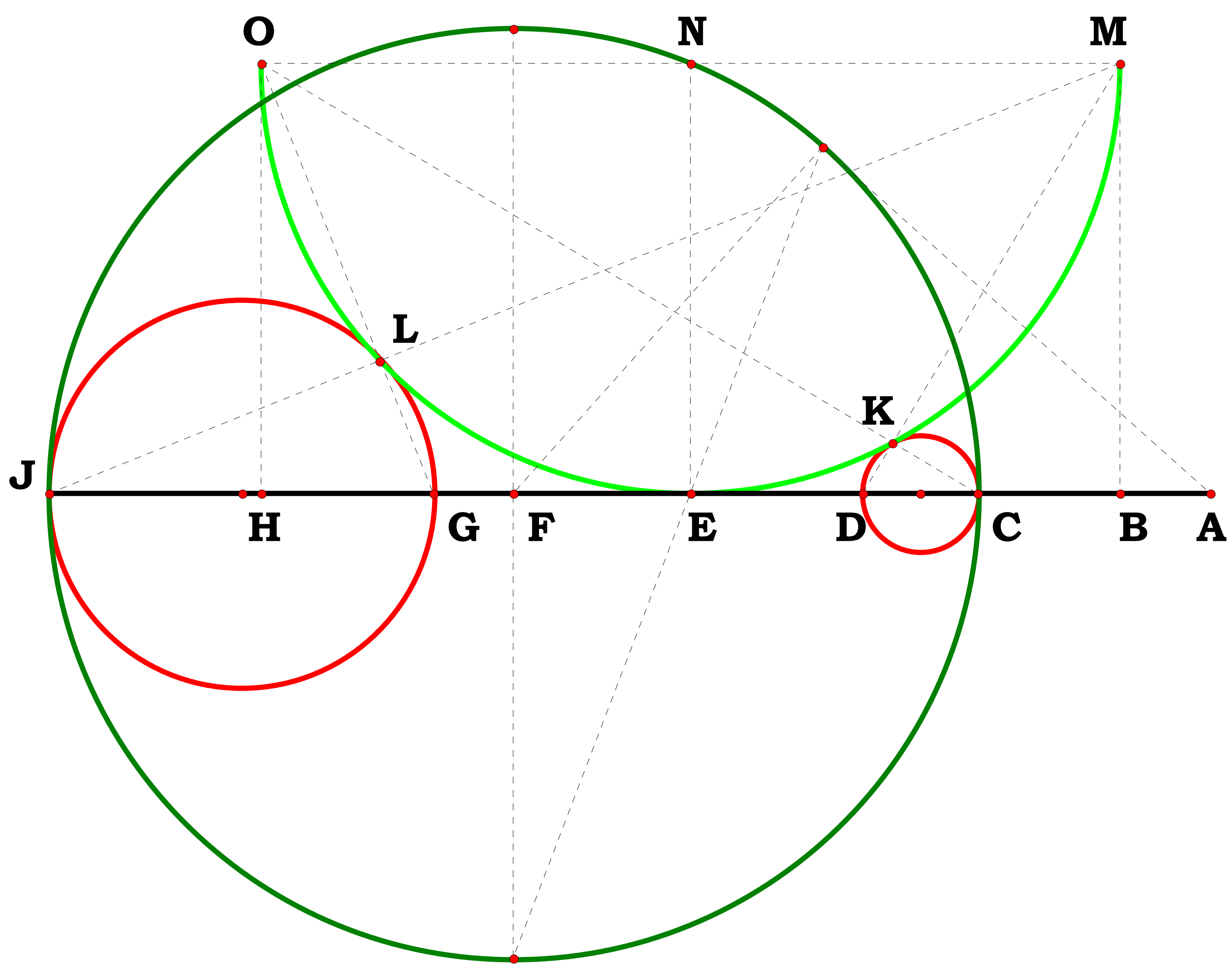
Alternately;

$EM := \frac{CF\cdot BE}{CH} \quad DG_3 := \frac{BE\cdot BF}{EM + BE}$

$DG_3 - DG_1 = 0$



010695 Alternate Method Quad Roots



$N := 5 \quad AC := 1 \quad AJ := AC \cdot N$

$CJ := AJ - AC \quad AE := \sqrt{AC \cdot AJ} \quad CE := AE - AC$

$EJ := CJ - CE \quad EN := \sqrt{CE \cdot EJ} \quad BM := EN \quad HO := EN$

$MN := EN \quad NO := EN \quad BE := EN \quad EH := EN$

$BJ := BE + EJ \quad MJ := \sqrt{BJ^2 + BM^2} \quad MO := MN + NO$

$ML := \dfrac{BJ \cdot MO}{MJ} \quad JL := MJ - ML \quad GJ := \dfrac{MJ \cdot JL}{BJ} \quad AG := AJ - GJ$

$$\left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$$

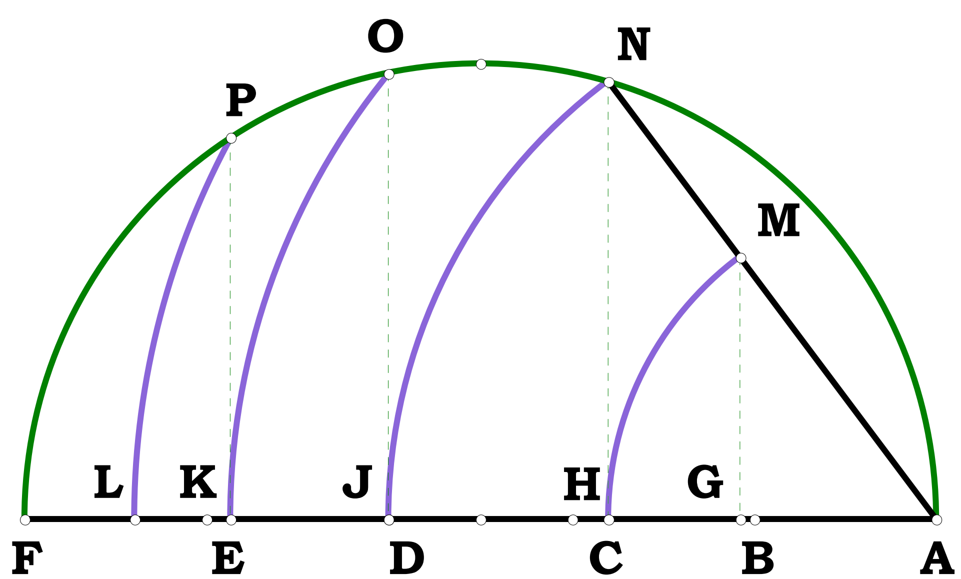
$CH := CE + EH \quad CO := \sqrt{CH^2 + HO^2} \quad KO := \dfrac{CH \cdot MO}{CO} \quad CK := CO - KO$

$CD := \dfrac{CO \cdot CK}{CH} \quad AD := AC + CD$

$$\left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0 \qquad N^{\frac{3}{4}} - AG = 0 \qquad N^{\frac{1}{4}} - AD = 0$$

040195 Exponential Series-Roots and Powers

I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.



$$N_1 := 5 \quad AB := 1 \quad AF := AB \cdot N_1$$

$$\delta := 0 .. 3$$

$$N_2 := 3 \quad AJ_0 := AB \cdot N_2 \quad AN := AJ_0 \quad AJ_1 := \frac{AN^2}{AF} \quad AJ_{\delta+1} := \frac{AJ_\delta \cdot AN}{AF}$$

$$AD_0 := AJ_0 \quad DF_0 := AF - AD_0 \quad DO_0 := \sqrt{AD_0 \cdot DF_0} \quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$$

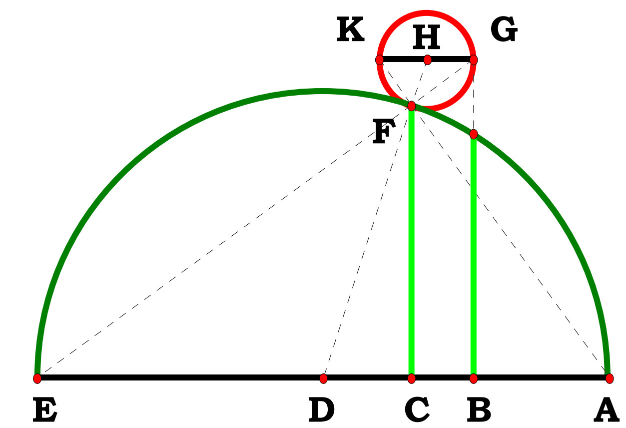
$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AF - AO_\delta \\ \sqrt{AO_\delta \cdot (AF - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AF - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

$$\sum_\delta \left[\frac{AF}{AJ_\delta} - \left(\frac{N_1}{N_2} \right)^{\delta+1} \right] = 0$$

$$\sum_\delta \left[\frac{AF}{AD_\delta} - \left(\frac{N_1}{N_2} \right)^{\frac{1}{2^\delta}} \right] = 0$$

091395 A Study In Placement

Given AE, AB, AC what is GH?

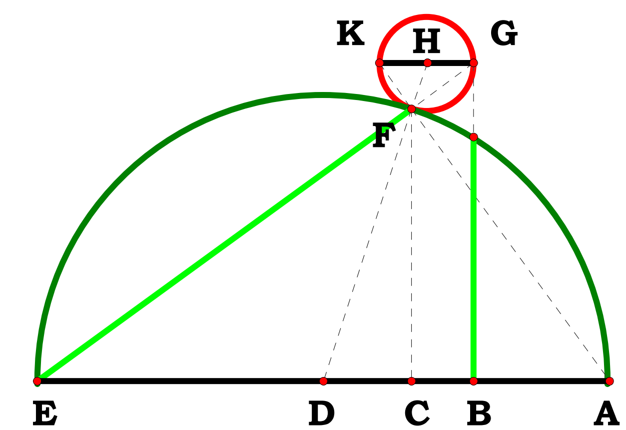


$$N_1 := 8.028 \quad N_2 := 3.044 \quad AE := 1 \quad AB := \frac{AE}{N_1} \quad AC := \frac{AE}{N_2}$$

$$CE := AE - AC \quad BC := AC - AB \quad GK := \frac{AE \cdot BC}{CE} \quad GH_1 := \frac{GK}{2}$$

$$GH_1 - \frac{(N_1 - N_2)}{2 \cdot N_1 \cdot (N_2 - 1)} = 0$$

Given AE, AB, EF what is GH?

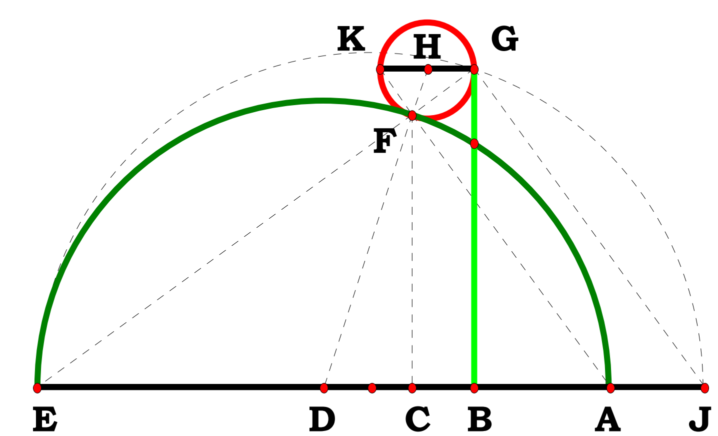


$$N_1 := 4.22537 \quad N_2 := 1.23589 \quad AD := \frac{AE}{2} \quad AB := \frac{AE}{N_1} \quad DE := AD \quad EF := \frac{AE}{N_2} \quad CE := \frac{EF^2}{AE}$$

$$BD := AD - AB \quad BE := BD + DE \quad EG := \frac{EF \cdot BE}{CE} \quad FG := EG - EF \quad GH_2 := \frac{AD \cdot FG}{EF}$$

$$\frac{N_1 \cdot N_2^{\,2} - N_1 - N_2^{\,2}}{2 \cdot N_1} - GH_2 = 0$$

Given AE, AB, BG what is GH?



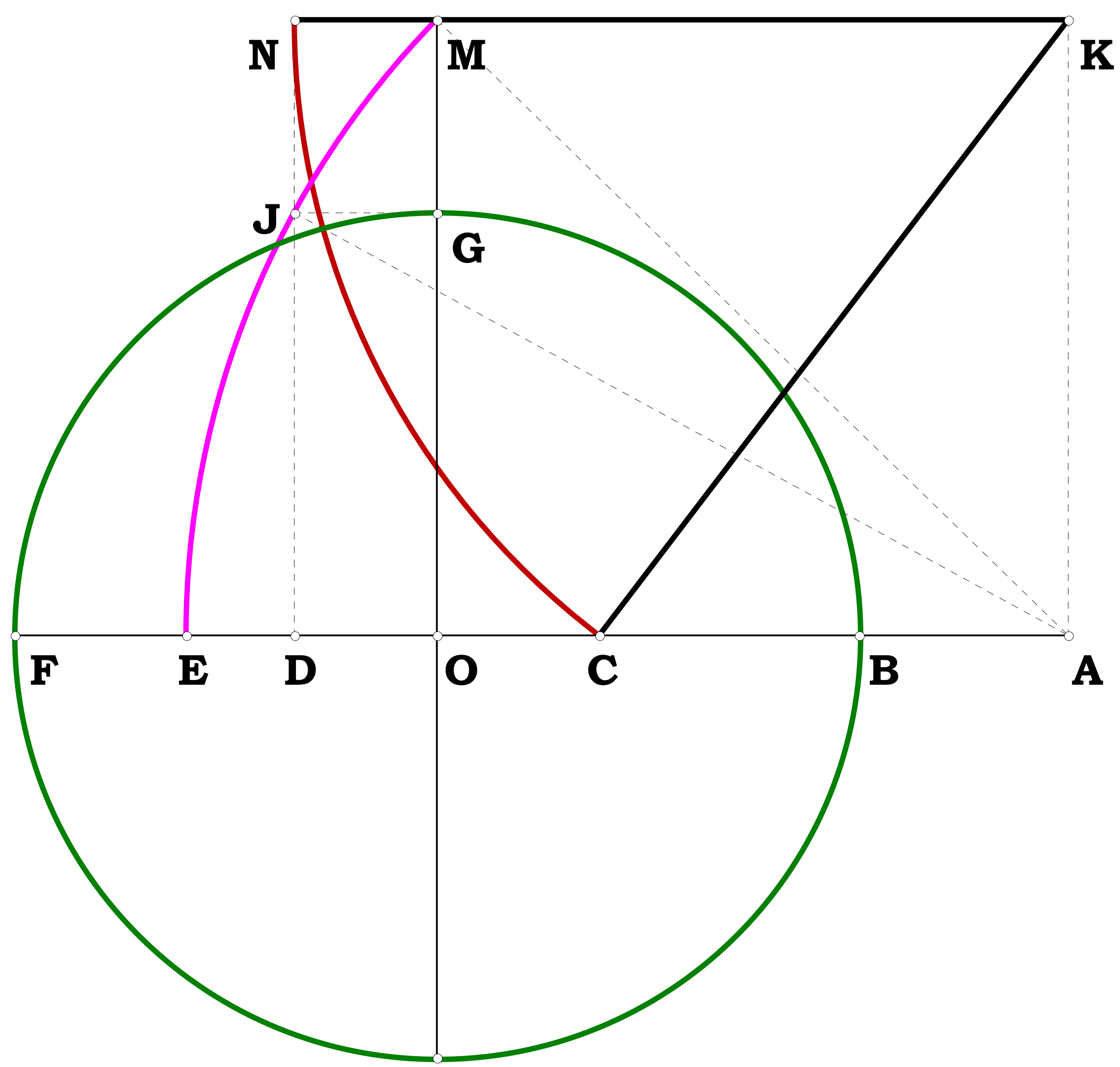
$$N_1 := 4.22537 \quad N_2 := 1.80385 \quad AB := \frac{AE}{N_1} \quad BG := \frac{AE}{N_2} \quad BE := AE - AB$$

$$BJ := \frac{BG^2}{BE} \quad CE := \frac{BE \cdot AE}{BE + BJ} \quad BC := BE - CE \quad GK := \frac{AE \cdot BC}{CE} \quad GH_3 := \frac{GK}{2}$$

$$GH_3 - \frac{\left(N_1^{\,2} - N_1 \cdot N_2^{\,2} + N_2^{\,2}\right)}{2 \cdot N_1 \cdot N_2^{\,2} \cdot (N_1 - 1)} = 0$$

101495 Alternate Method Square Root

For any AK is AC the root of AB x AF?



$N_1 := 5 \quad N_2 := 4$

$AB := 1 \quad AF := AB \cdot N_1 \quad AK := AB \cdot N_2 \quad BF := AF - AB$

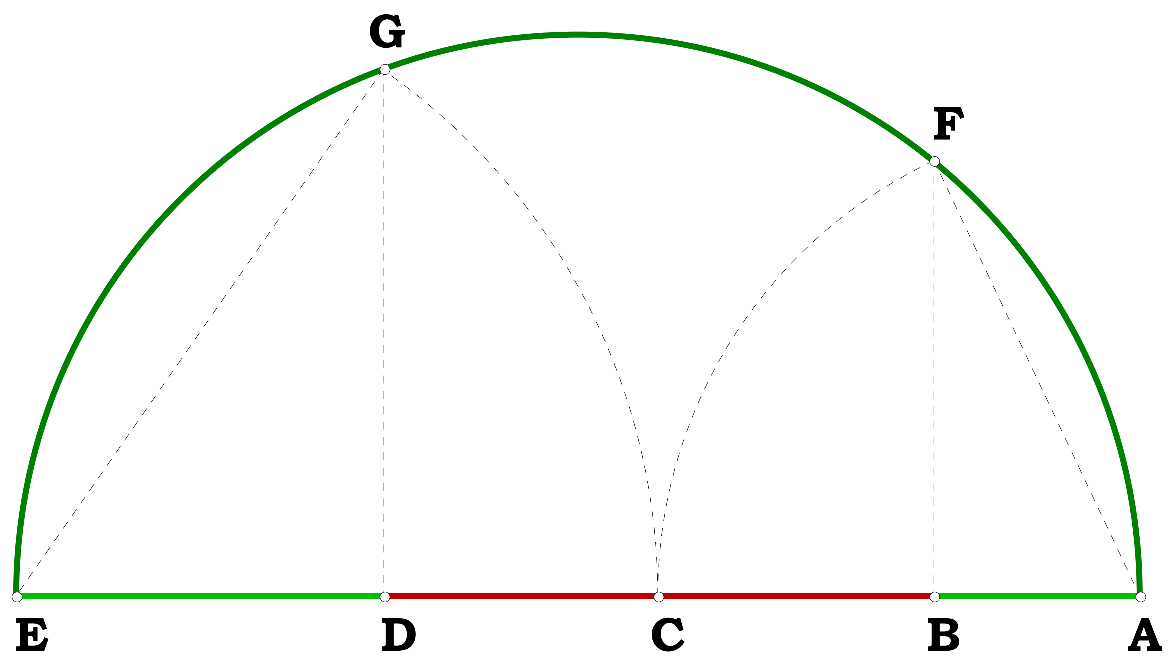
$BO := \dfrac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$GO := BO \quad DJ := B(AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$KN := AD \quad CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

$\sqrt{AB \cdot AF} - AC = 0$

102095 Four Times The Square



$N_1 := 2 \quad AE := 1 \quad AB := \dfrac{AE}{N_1}$

$BE := AE - AB \quad BF := \sqrt{AB \cdot BE} \quad AF := \sqrt{AB^2 + BF^2}$

$AC := AF \quad CE := AE - AC \quad EG := CE \quad DE := \dfrac{EG^2}{AE}$

$AD := AE - DE \quad DG := \sqrt{AD \cdot DE} \qquad BD := AE - (AB + DE) \quad \dfrac{BD^2}{4 \cdot (AB \cdot DE)} = 1$

Algebraic Names:

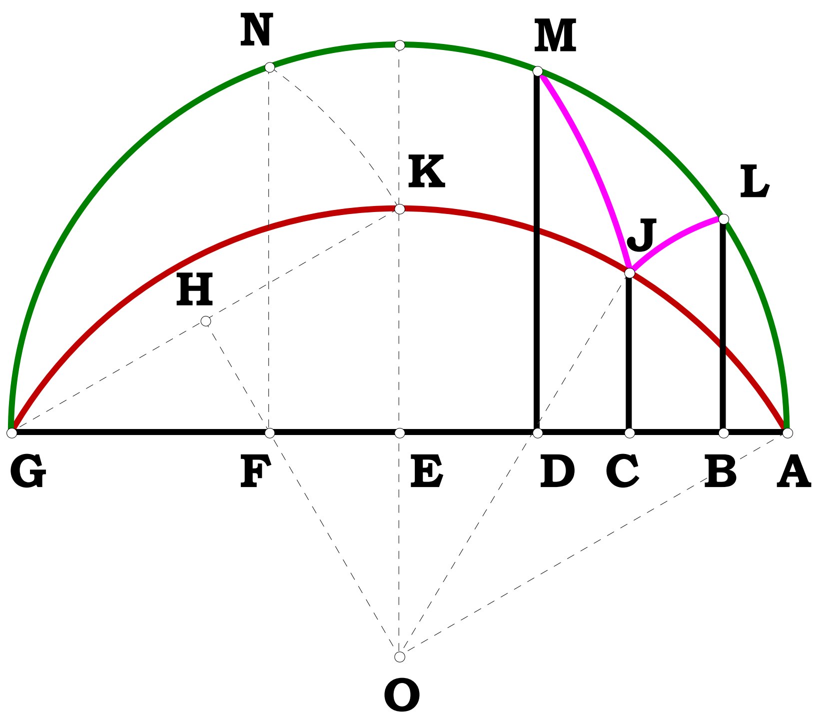
$\dfrac{1}{N_1} - AB = 0 \quad 1 - \dfrac{1}{N_1} - BE = 0 \quad \sqrt{\dfrac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0 \quad \sqrt{\dfrac{N_1}{N_1^{\,2}}} - AF = 0$

$1 - \sqrt{\dfrac{N_1}{N_1^{\,2}}} - CE = 0 \quad 1 - 2 \cdot \sqrt{\dfrac{1}{N_1}} + \dfrac{1}{N_1} - DE = 0 \quad 2 \cdot \sqrt{\dfrac{1}{N_1}} - \dfrac{2}{N_1} - BD = 0$

$2 \cdot \sqrt{\dfrac{1}{N_1}} - \dfrac{1}{N_1} - AD = 0 \qquad \dfrac{\sqrt{\left[N_1^{\,2} \cdot \left(\sqrt{\dfrac{1}{N_1}} - 1\right)^2 \cdot \left(2 - \sqrt{\dfrac{1}{N_1}}\right) \cdot \sqrt{\dfrac{1}{N_1}}\right]}}{N_1} - DG = 0$

110195 A Modification Of The Square Root Figure, Gemini Roots

On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.



$$N_1 := 5 \quad N_2 := 2 \qquad AG := 1$$

$$AE := \frac{AG}{2} \quad EG := AE \quad EF := \frac{AG}{2 \cdot N_1} \quad AF := AE + EF$$

$$FG := EG - EF \quad FN := \sqrt{AF \cdot FG} \quad GN := \sqrt{FN^2 + FG^2} \quad GK := GN$$

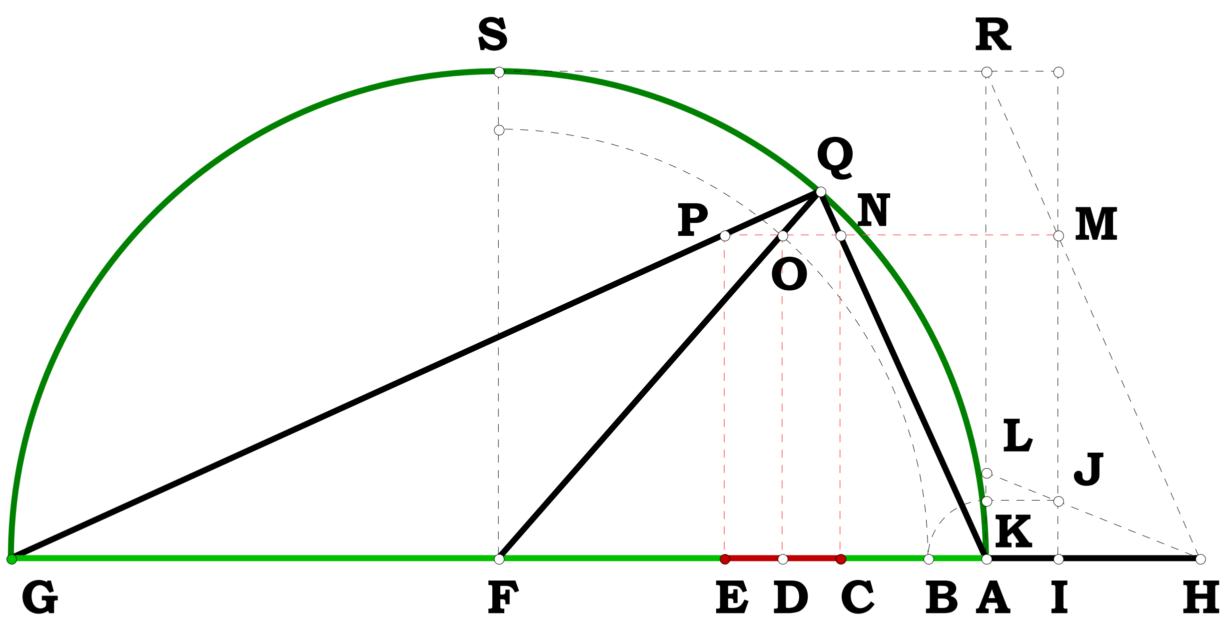
$$EK := \sqrt{GK^2 - EG^2} \quad EO := \frac{EG \cdot EF}{EK} \quad OK := EO + EK \quad DE := \frac{AE}{N_2} \quad DO := \sqrt{DE^2 + EO^2}$$

$$DJ := OK - DO \quad CD := \frac{DE \cdot DJ}{DO} \quad CE := CD + DE \quad CJ := \frac{EO \cdot DJ}{DO} \quad AC := AE - CE \quad AJ := \sqrt{AC^2 + CJ^2}$$

$$AL := AJ \quad AB := \frac{AL^2}{AG} \quad CG := AG - AC \quad GJ := \sqrt{CG^2 + CJ^2} \quad GM := GJ \quad DG := \frac{GM^2}{AG}$$

$$BD := AG - (AB + DG) \qquad \frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$$

110595 Alternate Method Gemini Roots



$$N_1 := 6 \quad N_2 := 7$$

$$AG := 1 \quad AF := \frac{AG}{2} \quad AR := AF \quad FQ := AF \quad FG := AF$$

$$AL := \frac{AR}{N_1} \quad IM := \frac{AR}{N_2} \quad AK := \frac{AL \cdot IM}{AR}$$

$$DO := IM \quad AB := AK \quad BF := AF - AB \quad FO := BF$$

$$OQ := FQ - FO \quad NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO} \quad NP - 2 \cdot AK = CD := AK \quad DE := AK$$

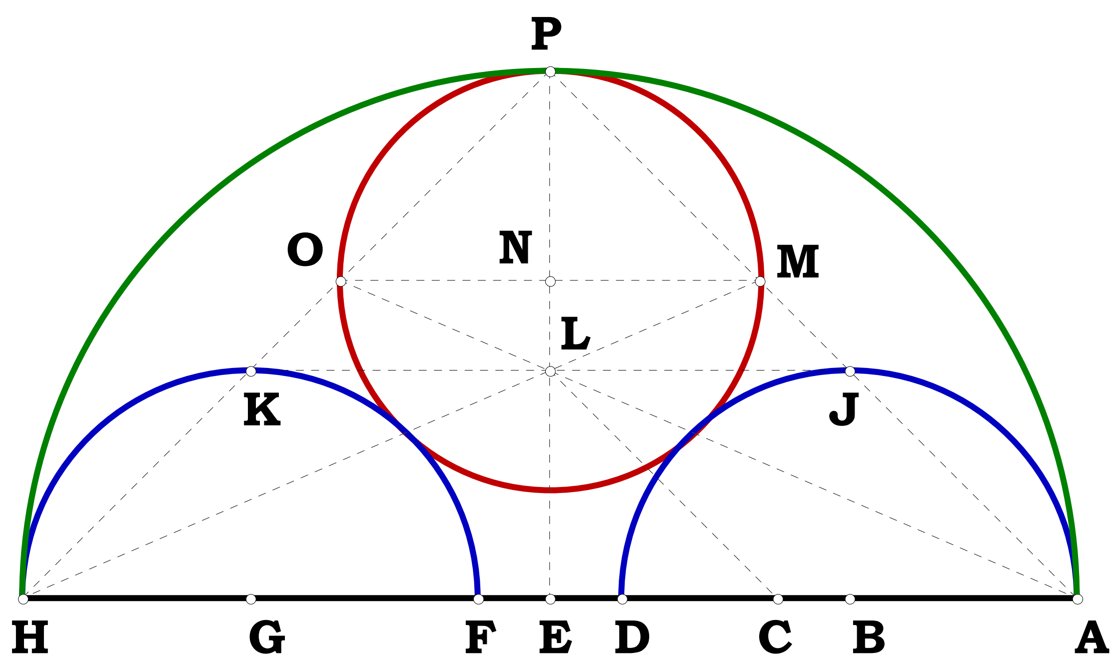
$$DF := \sqrt{FO^2 - DO^2} \quad AD := AF - DF \quad AC := AD - CD \quad EG := FG + DF - DE$$

$$CE := NP \quad \frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$$

$$\frac{\sqrt{AC \cdot EG}}{CE} = 3$$

120195 Method For Equals

Given AB find NP.



$N_1 := 5$

$AH := 1 \quad AE := \dfrac{AH}{2} \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$

$AB := \dfrac{AE}{N_1} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$

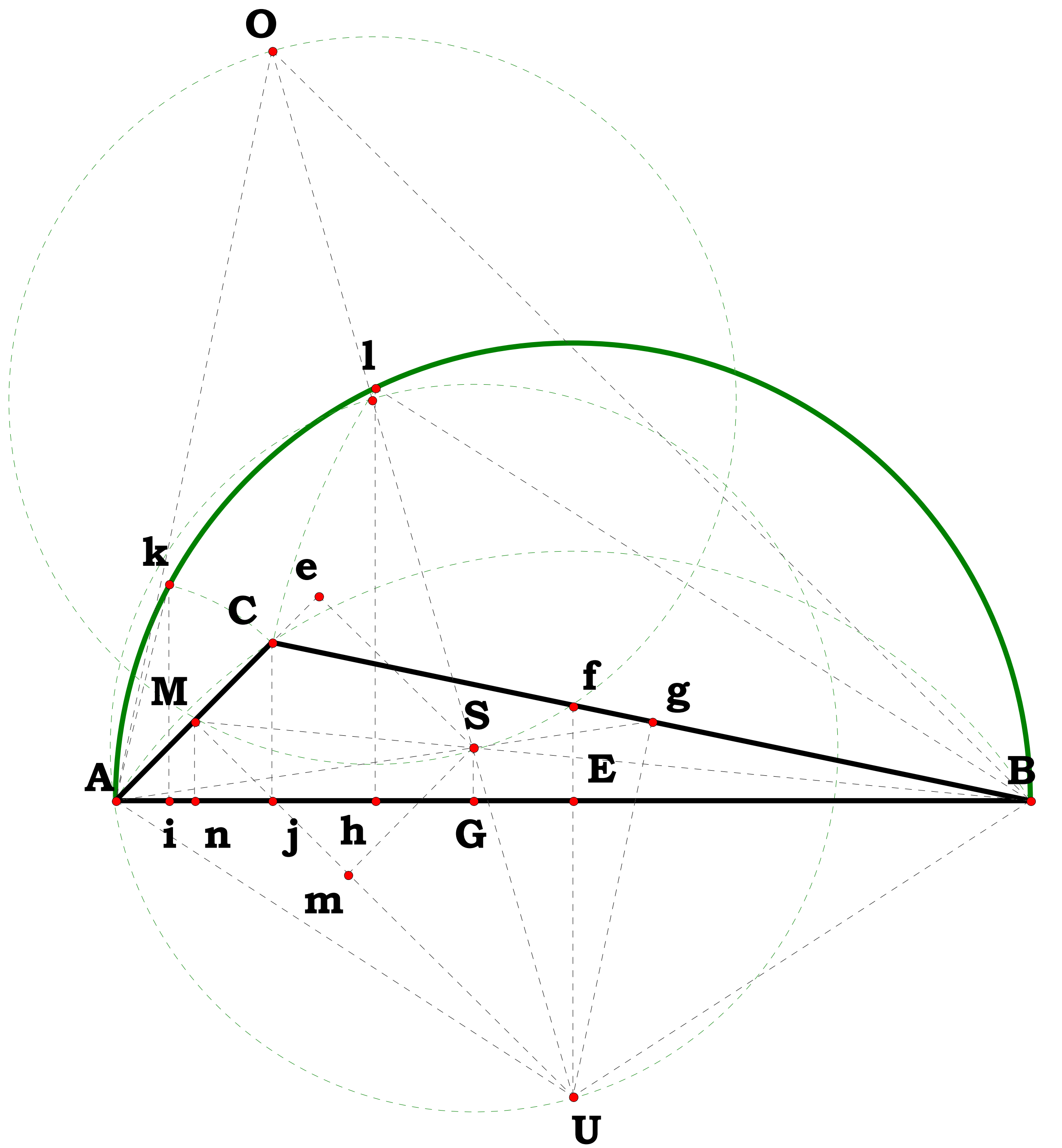
$AM := \dfrac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \dfrac{EP \cdot MP}{AP}$

$\dfrac{1}{2} \cdot \dfrac{(N_1 - 1)}{(N_1 + 1)} - NP = 0 \qquad \dfrac{(N_1 - 1)}{(N_1 + 1)} - 2 \cdot NP = 0$

12_07_95.MCD

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

Scholia: ***100 Great Problems of Elementary Mathematics*** **H. Dorrie Problem 27 Euler's Line**



$$\delta := 0 .. 2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$$

$$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta \quad Ai_\delta := \frac{\left(Ak_\delta\right)^2}{AB_\delta} \quad Bh_\delta := \frac{\left(Bl_\delta\right)^2}{AB_\delta}$$

$$Ah_\delta := AB_\delta - Bh_\delta \quad hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$$

$$Cj_\delta := \sqrt{\left(AC_\delta\right)^2 - \left(Aj_\delta\right)^2} \quad BE_\delta := AE_\delta \quad Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2}$$

$$Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta} \quad fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := \mathrm{if}\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$$

$$BU_\delta := \mathrm{if}\left[Ug_\delta, \sqrt{\left(Ug_\delta\right)^2 + \left(Bg_\delta\right)^2}, \infty\right] \quad AM_\delta := \frac{AC_\delta}{2} \quad An_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta}$$

$$Bn_\delta := AB_\delta - An_\delta \quad nM_\delta := \sqrt{\left(AM_\delta\right)^2 - \left(An_\delta\right)^2} \quad BM_\delta := \sqrt{\left(nM_\delta\right)^2 + \left(Bn_\delta\right)^2}$$

$$BS_\delta := \frac{2 \cdot BM_\delta}{3} \quad BG_\delta := \frac{Bn_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{nM_\delta \cdot BS_\delta}{BM_\delta}$$

$$\text{TRIANGLE} := (\text{Side\_1} + \text{Side\_2} > \text{Side\_3}) \cdot (\text{Side\_1} + \text{Side\_3} > \text{Side\_2}) \cdot (\text{Side\_2} + \text{Side\_3} > \text{Side\_1})$$

$$AG_\delta := AB_\delta - BG_\delta \qquad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2} \qquad MS_\delta := BM_\delta - BS_\delta$$

$$AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \qquad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$$

The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$$eM_\delta := Ae_\delta - AM_\delta \qquad Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$$

$$Um_\delta := \text{if}\left[AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2},\ MU_\delta - Mm_\delta,\ MU_\delta + Mm_\delta\right]$$

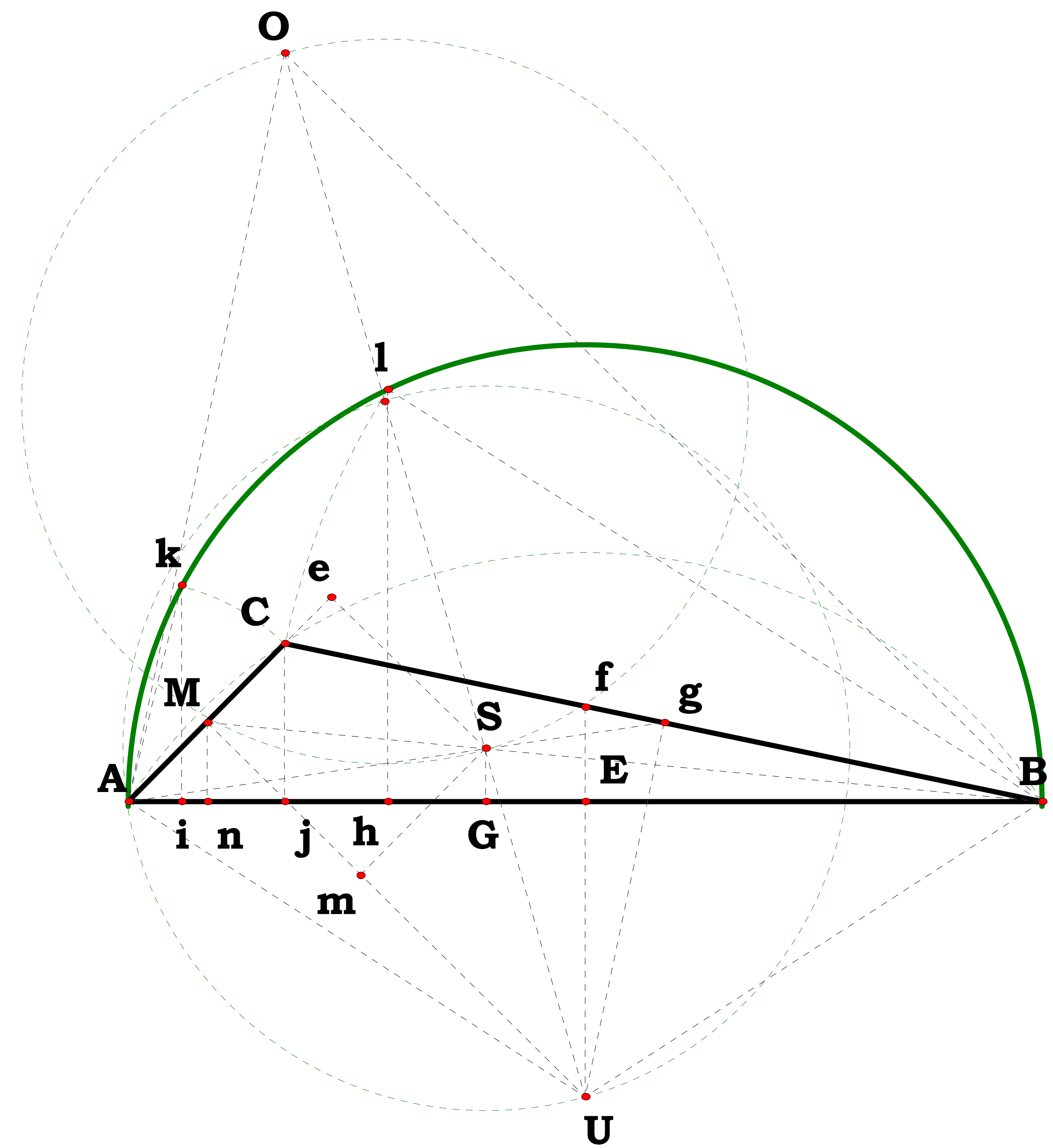
$$SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} \qquad UO_\delta := 3\cdot SU_\delta$$

Due to the way in which certain lines lay, the above switch was needed.

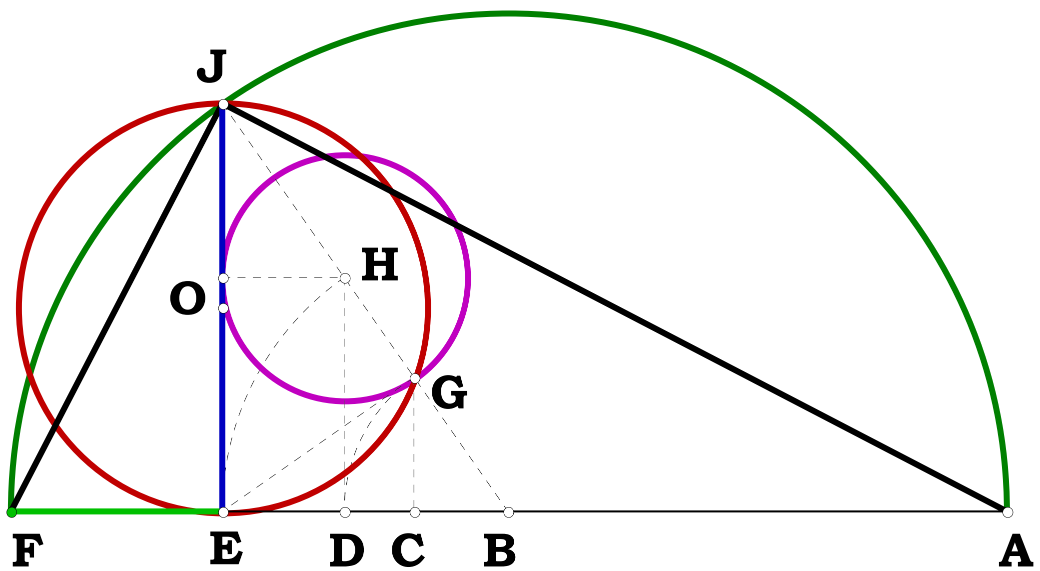
Is this a $\text{TRIANGLE} = 1$ **?**

$\text{Side\_1} \equiv 14.07583 \quad \text{Side\_2} \equiv 3.42377 \quad \text{Side\_3} \equiv 11.91932$

$SU_\delta =$	$UO_\delta =$	$AU_\delta =$
5.58256	16.74768	8.382562
5.58256	16.74768	8.382562
5.58256	16.74768	8.382562



122095 Just For Fun



$$N := 3$$

$$EF := 1 \quad EJ := EF \cdot N \quad AE := \frac{EJ^2}{EF} \quad AF := AE + EF$$

$$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF \quad BH := BE \quad BJ := \sqrt{EJ^2 + BE^2}$$

$$BD := \frac{BE \cdot BH}{BJ} \quad BG := BD \quad BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD$$

$$HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG \quad EG_2 := \sqrt{EJ^2 - GJ^2}$$

$$\frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1 \quad \left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$$

$$BC - \frac{(N+1)^3 \cdot (N-1)^3}{2 \cdot \left(N^2 + 1\right)^2} = 0$$

122195 Pascal's Triangle With Exponential Division

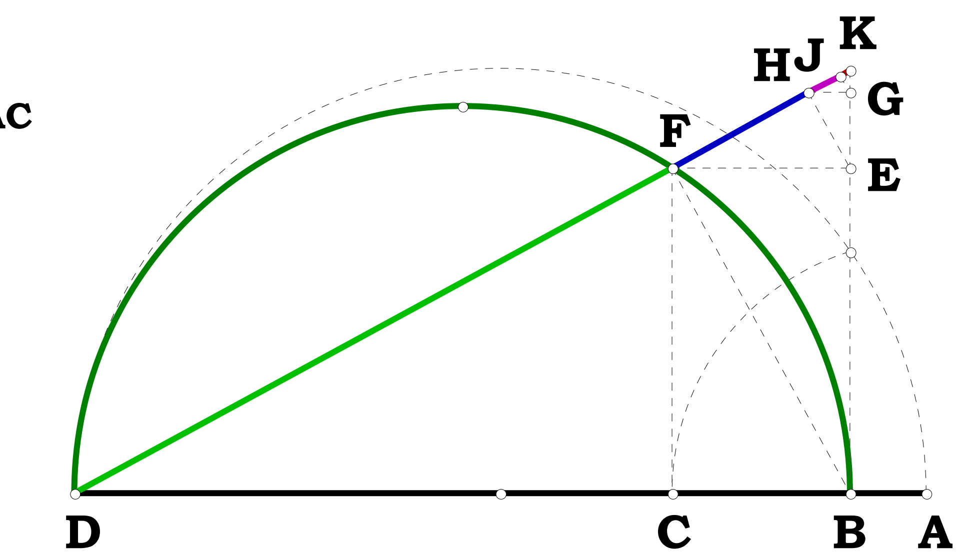
$N := 7$

$AB := 1 \quad AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD} \quad BD := AD - AB \quad CD := AD - AC$

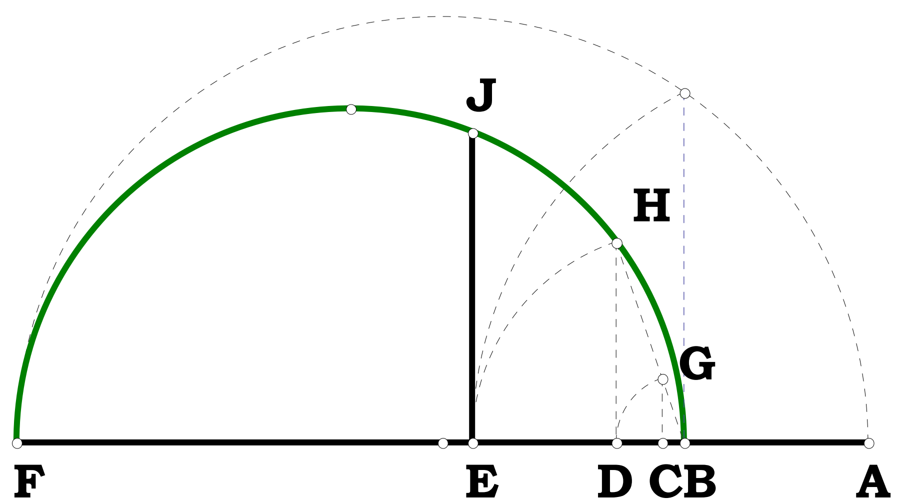
$BC := BD - CD \quad CF := \sqrt{BC \cdot CD} \quad CF := \sqrt{BC \cdot CD} \quad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \quad FK := \frac{DK \cdot BC}{BD} \quad HK := \frac{FK \cdot FK}{DK} \quad JK := \frac{HK \cdot HK}{FK}$

$\frac{DK}{FK} - \left(\sqrt{N} + 1\right) = 0 \qquad \frac{DK}{HK} - \left(\sqrt{N} + 1\right)^2 = 0 \qquad \frac{DK}{JK} - \left(\sqrt{N} + 1\right)^3 = 0$



A more civil figure.



$AB := 1 \quad AF := N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF} \quad BE := AE - AB \quad BH := BE$

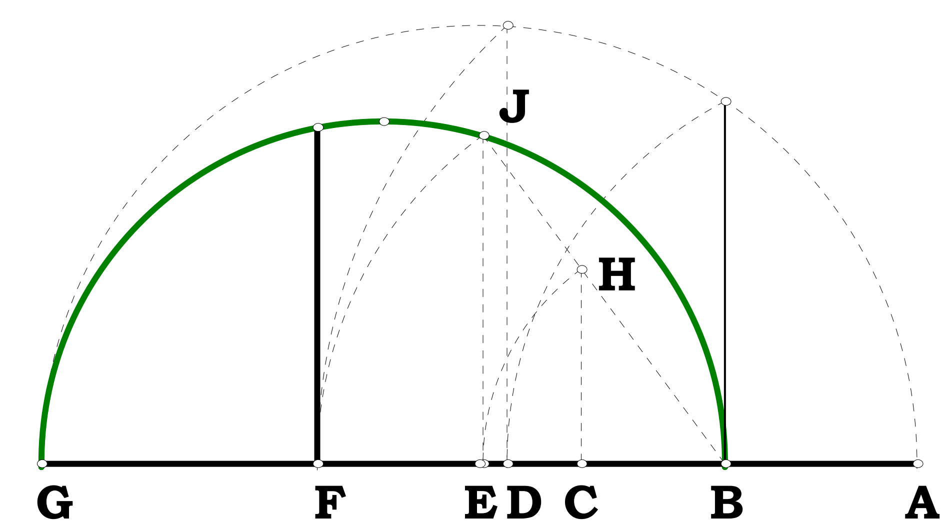
$BD := \frac{BE \cdot BH}{BF} \quad BG := BD \quad BC := \frac{BD \cdot BG}{BE}$

$\frac{BF}{BE} - \left(\sqrt{N} + 1\right) = 0 \qquad \frac{BF}{BD} - \left(\sqrt{N} + 1\right)^2 = 0 \qquad \frac{BF}{BC} - \left(\sqrt{N} + 1\right)^3 = 0$

$AB := 1 \quad AG := N \quad BG := AG - AB \quad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BF := AF - AB$

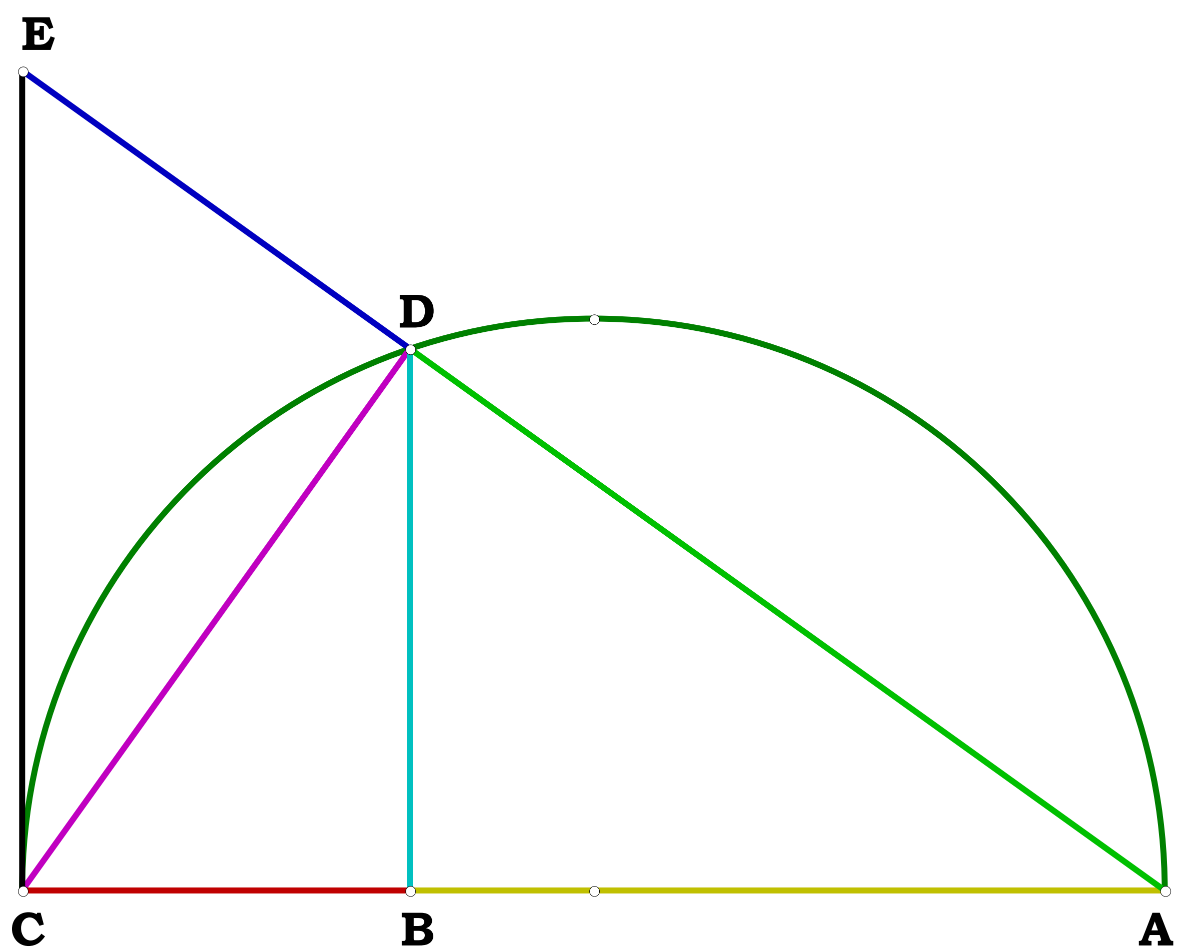
$BJ := BF \quad BE := \frac{BJ \cdot BF}{BG} \quad BH := BE \quad BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N - 1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{(N - 1)^2}{\left(N^{\frac{3}{4}} - 1\right)^2} = 0 \qquad \frac{BG}{BC} - \frac{(N - 1)^3}{\left(N^{\frac{3}{4}} - 1\right)^3} = 0$



122995

Given AC and CE find BC.



$$N_1 := 4 \quad N_2 := 9$$

$$AC := N_1 \quad CE := N_2 \quad AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$$

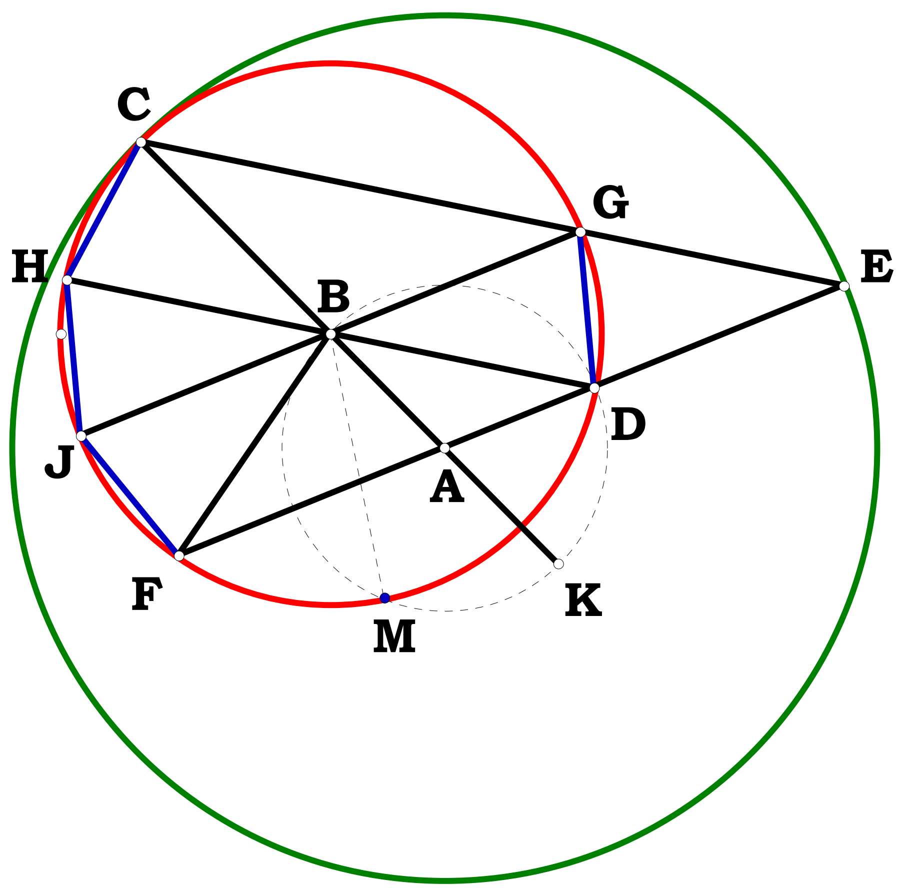
$$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$$

$$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$$

$$BC - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} = 0 \quad BD - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2} = 0 \quad AB - \left(N_1 - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2}\right) = 0$$

$$AD - \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} = 0 \quad DE - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$$

010496 The Archamedian Paper Trisector- Without the Numbers.



Given any circle AB.

Given any circle BC such that BC ≤ 2AB.

Construct AE such that AE = AC.

As AC = AB + BC and AD = AB so too DE = BC.

Construct DH parallel to BD. Construct CE.

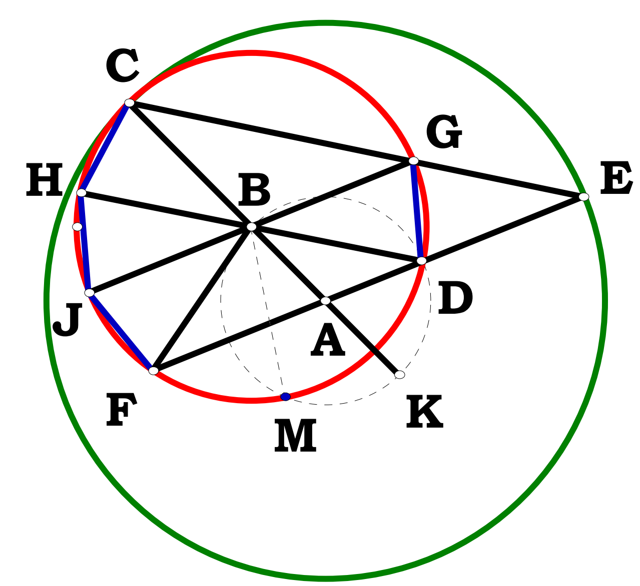
As AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD.

From here one can take two paths.

Construct GJ parallel to EF.

As CE is parallel to DH, DG = CH.

As GJ is parallel to EF, DG = FJ.



As ∠ HBJ is opposite and equal to ∠ GBD, DG = HJ, therefore ∠ DG is $\frac{1}{3}$ CF.

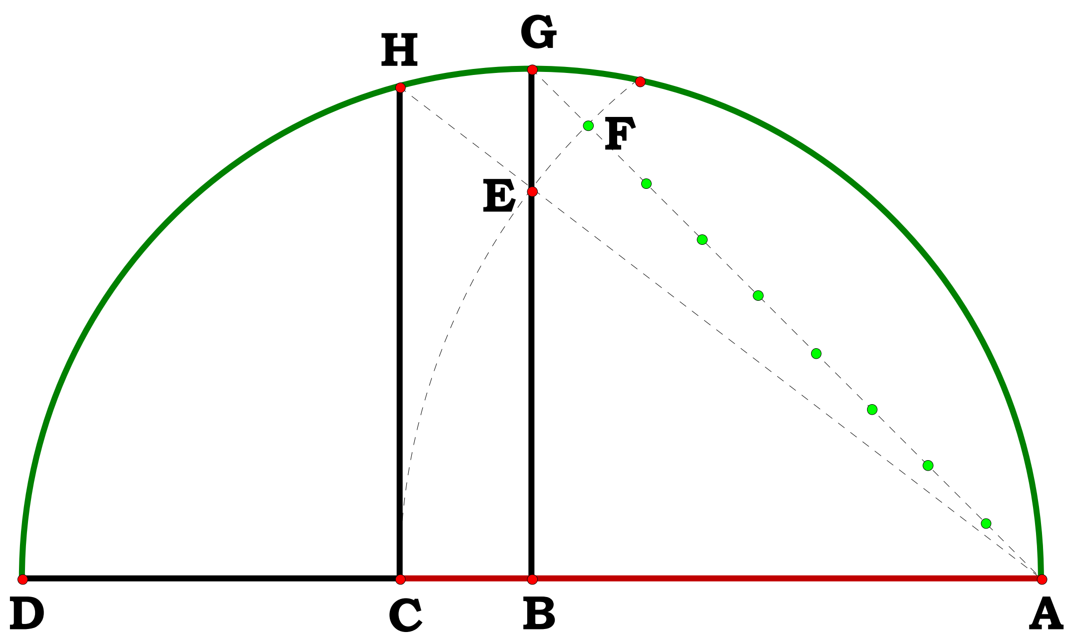
As CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.

By construction DK = KM.

As DH is parallel to CE, CH = DG.

As DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.

010796 Rusty Cubes



A rusty Compass construction for the duplication of the cube.

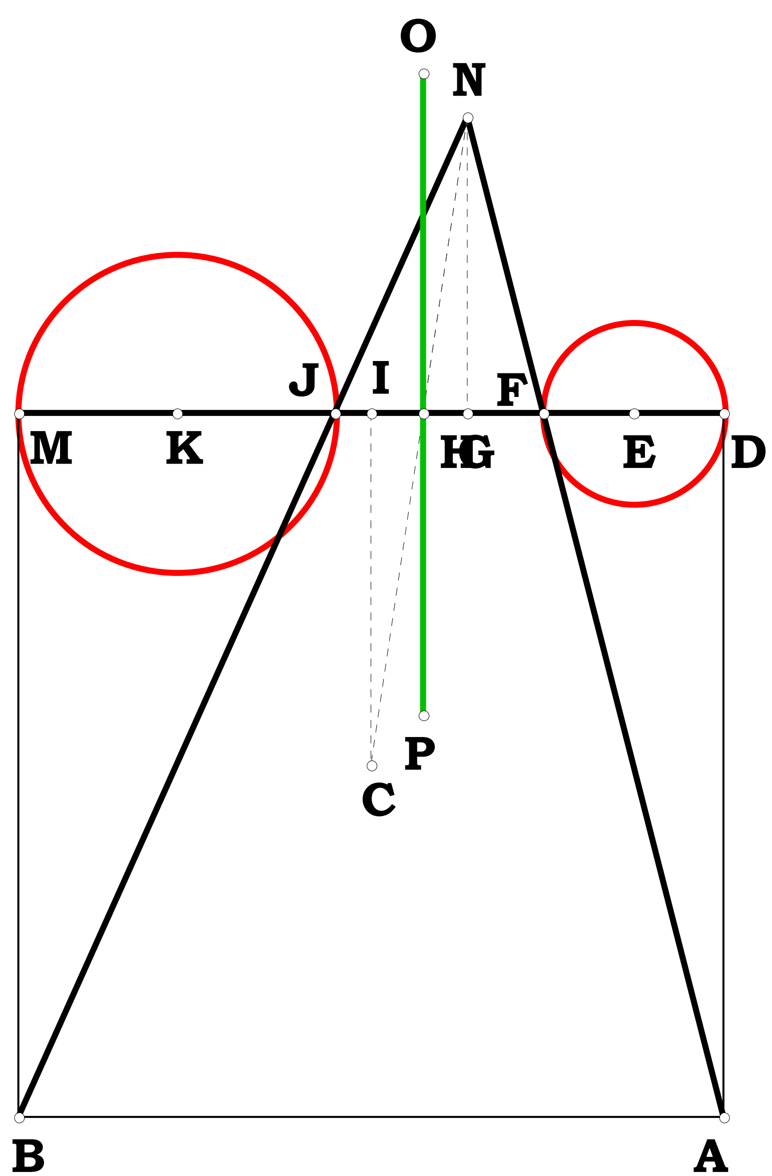
$$AD := 2 \quad AB := \frac{AD}{2} \quad AG := \sqrt{2 \cdot AB^2} \quad AF := \frac{AG}{9} \cdot 8$$

$$AC := AF \quad AC = 1.257079$$

$$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.259921 \quad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002261$$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.

010896a Alternate Method Power Line



$R_1 := 2 \quad R_2 := 5 \quad D := 4$

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$$FG := \frac{DF \cdot FJ}{DF + JM} \quad GJ := FJ - FG \quad DI := \frac{DM}{2}$$

$$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \frac{AD \cdot FG}{DF}$$

$$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$$

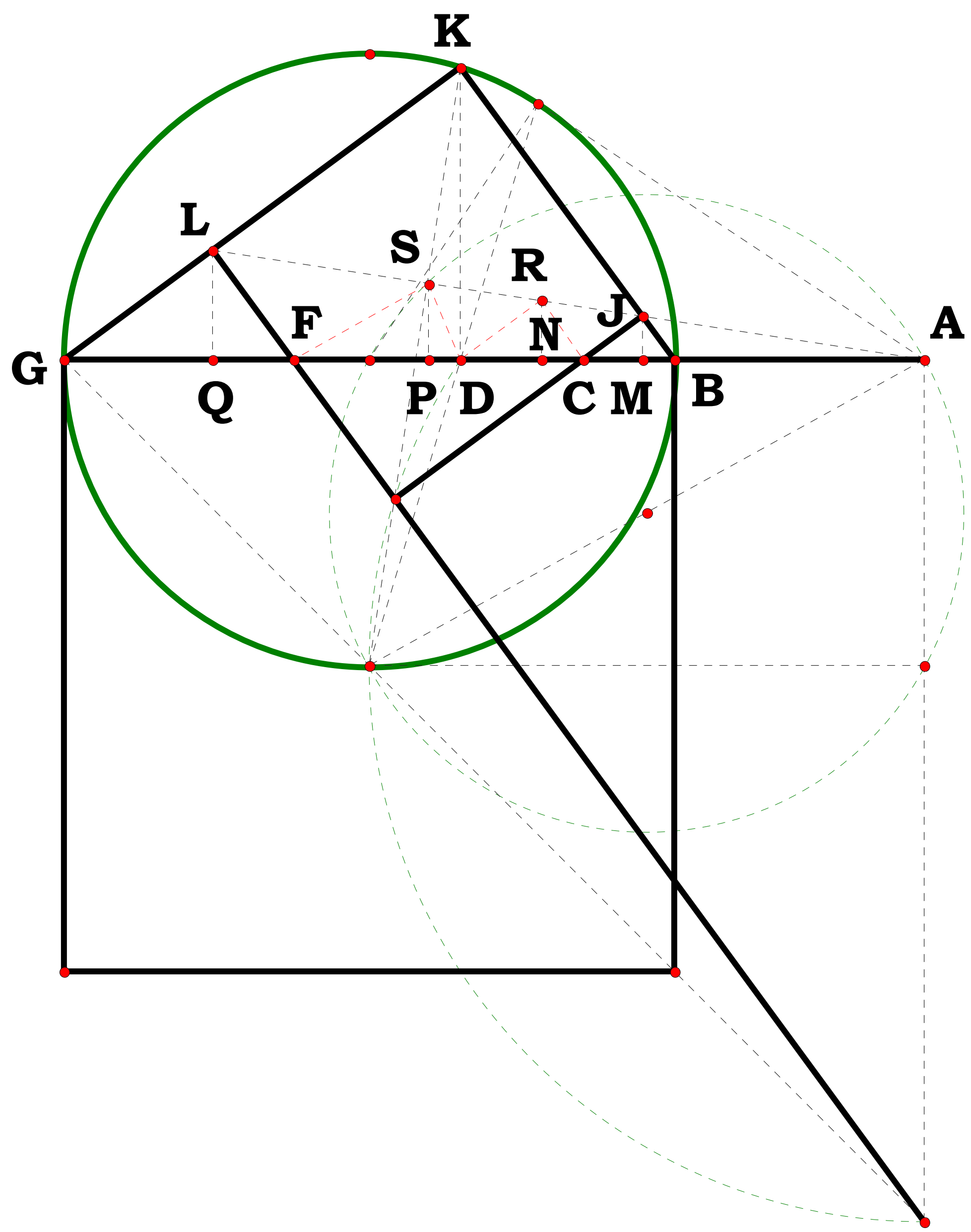
$HM := DM - DH$

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$

01_08_96.MCD

The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.



$N = 5 \quad AG := N \quad AB := \dfrac{AG}{N} \quad BG := AG - AB$

$BO := \dfrac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

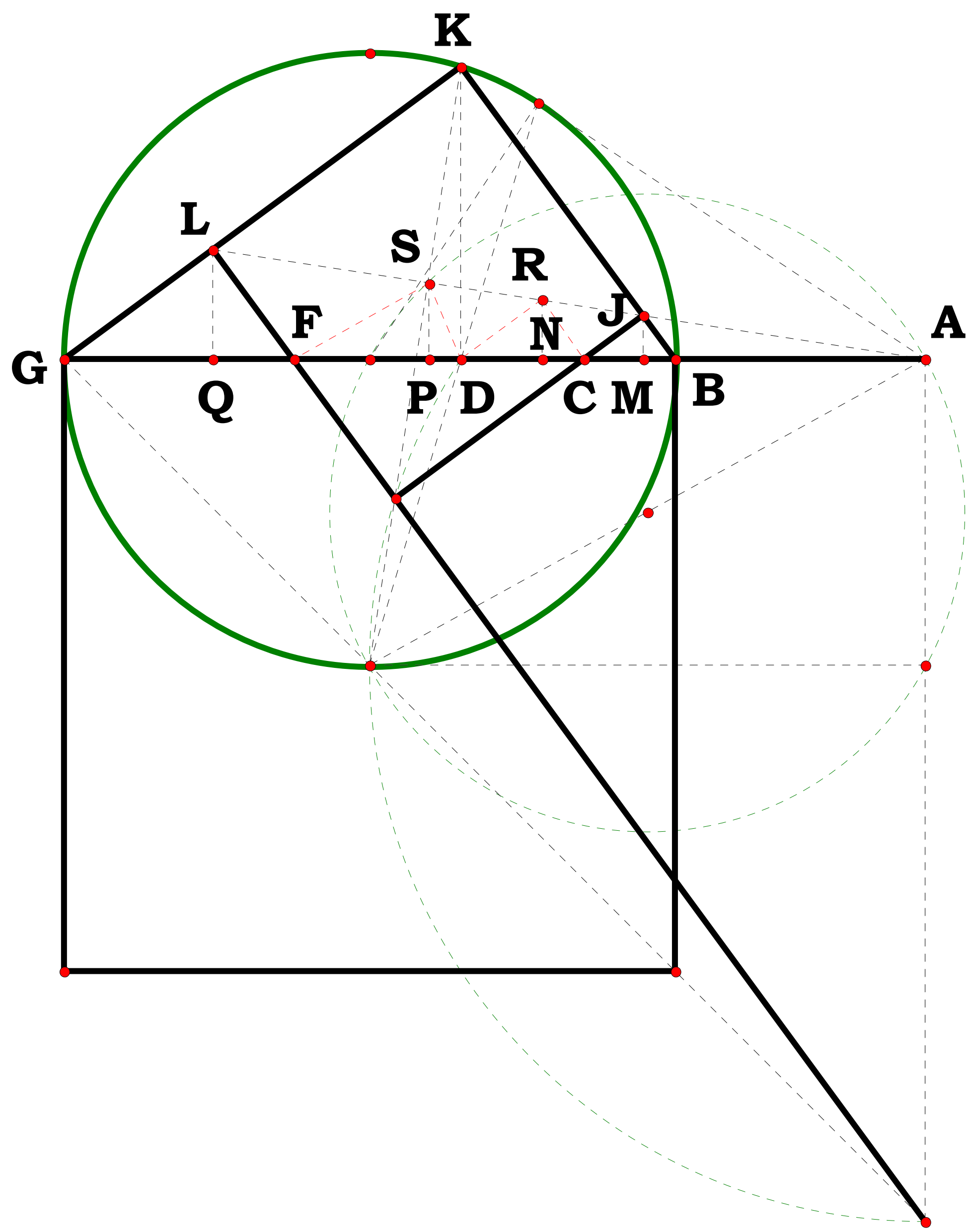
$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \dfrac{BK \cdot BC}{BG} \quad GL := \dfrac{GK \cdot FG}{BG}$

$\dfrac{GL}{BJ} = 5 \qquad \dfrac{AG}{AB} = 5$

Plug in AG here. AB will become "1".

$N \equiv 5$



$$\frac{GK}{GL} - \frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 0 \qquad \frac{GK}{GL} - \left(\frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}}\right) = 0$$

$$\frac{BK}{BJ} - \left[\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}\right] = 0 \qquad \frac{BK}{BJ} - \left(N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}\right) = 0$$

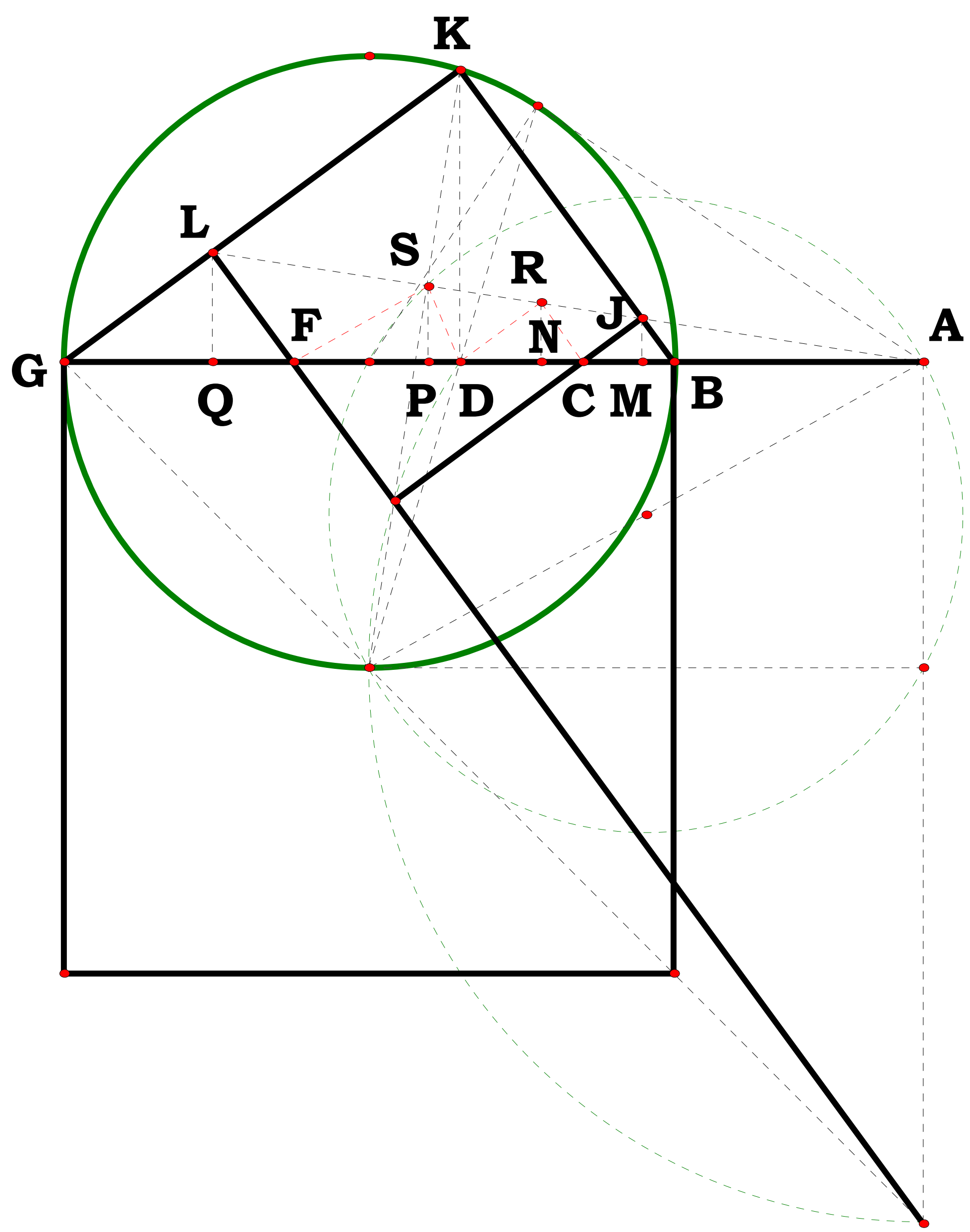
$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG} \quad CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{BG}{BM} - \left[\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}\right] = 0$$

$$\frac{BG}{BM} - \left(N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}\right) = 0$$

$$\frac{BG}{CN} - \left[\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}}\right] = 0$$

$$\frac{BG}{CN} - \left(N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}}\right) = 0$$



$$\frac{BG}{DP}-\left[\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}+\left(\frac{AB}{AG}\right)^{\frac{2}{4}}\right]=0$$

$$\frac{BG}{DP}-\left(N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}+N^{\frac{-2}{4}}\right)=0$$

$$\frac{BG}{FQ}-\left[\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}+\left(\frac{AB}{AG}\right)^{\frac{2}{4}}+\left(\frac{AB}{AG}\right)^{\frac{3}{4}}\right]=0$$

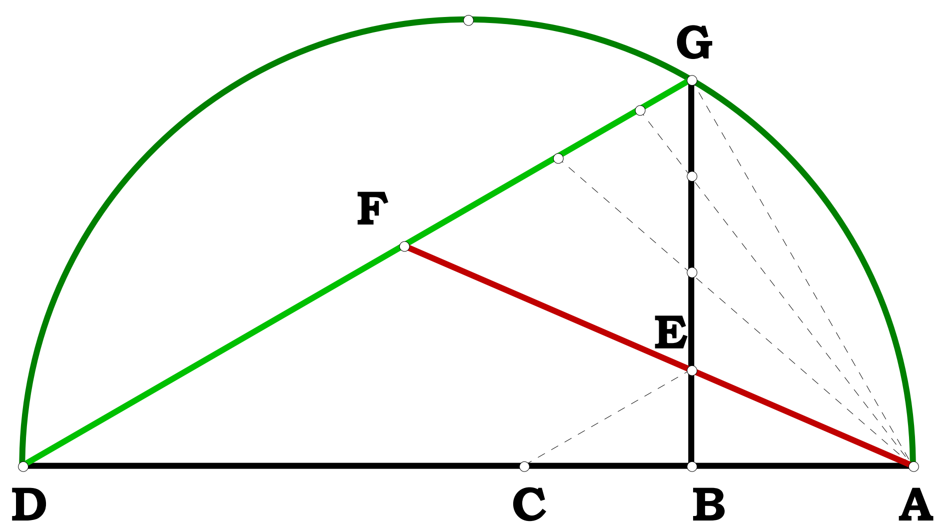
$$\frac{BG}{FQ}-\left(N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}+N^{\frac{-1}{4}}+N^{\frac{-2}{4}}+N^{\frac{-3}{4}}\right)=0$$

$$\frac{AG}{BM}-\left(\frac{AG^{\frac{6}{4}}+AG^{\frac{4}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{5}{4}}-AB^{\frac{6}{4}}}\right)=0 \qquad \frac{AG}{BM}-\left(\frac{N^{\frac{3}{2}}+N}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0 \qquad \frac{AG}{CN}-\left(\frac{AG^{\frac{5}{4}}+AG^{\frac{3}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{4}{4}}-AB^{\frac{5}{4}}}\right)=0$$

$$\frac{AG}{CN}-\left(\frac{N^{\frac{5}{4}}+N^{\frac{3}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0 \qquad \frac{AG}{DP}-\left(\frac{AG^{\frac{4}{4}}+AG^{\frac{2}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{3}{4}}-AB^{\frac{4}{4}}}\right)=0 \qquad \frac{AG}{DP}-\left(\frac{N+N^{\frac{2}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0$$

$$\frac{AG}{FQ}-\left(\frac{AG^{\frac{3}{4}}+AG^{\frac{1}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{2}{4}}-AB^{\frac{3}{4}}}\right)=0 \qquad \frac{AG}{FQ}-\left(\frac{N^{\frac{3}{4}}+N^{\frac{1}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0$$

011396 Pyramid of Ratios VI, Moving the Point



$N_1 := 5 \quad N_2 := 7 \qquad N_1 := N_1 \quad N_2 := N_2$

$AD := 1 \quad AB := \frac{AD}{N_1} \quad BD := AD - AB \quad BG := \sqrt{AB \cdot BD} \quad BE := \frac{BG}{N_2}$

$BC := \frac{BD \cdot BE}{BG} \quad AE := \sqrt{AB^2 + BE^2} \quad AC := AB + BC \quad AF := \frac{AE \cdot AD}{AC}$

$EF := AF - AE \quad \frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1) \cdot (N_2 - 1)} = 0$

$\Delta := 2 .. N_1 \quad \delta := 2 .. N_2 \quad SeriesAF_{\Delta, \delta} := \frac{\Delta \cdot \delta}{(\Delta - 1) \cdot (\delta - 1)}$

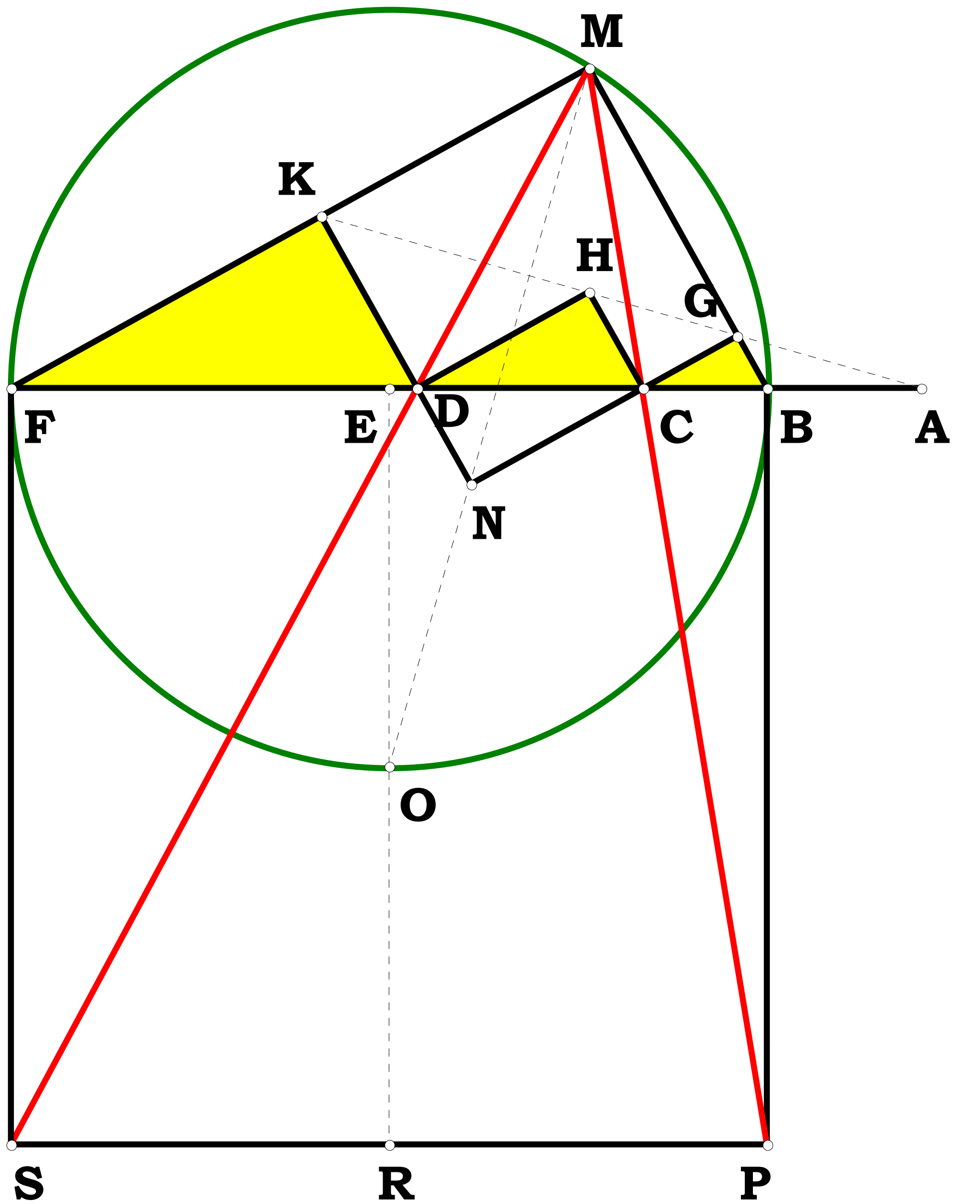
$$SeriesAF = \begin{pmatrix} 4 & 3 & 2.666667 & 2.5 & 2.4 & 2.333333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.666667 & 2 & 1.777778 & 1.666667 & 1.6 & 1.555556 \\ 2.5 & 1.875 & 1.666667 & 1.5625 & 1.5 & 1.458333 \end{pmatrix}$$

$DG := \sqrt{BD^2 + BG^2} \quad CE := \sqrt{BC^2 + BE^2} \quad DF := \frac{CE \cdot AD}{AC} \quad GF := DG - DF \quad \frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0$

$$SeriesDG_{\Delta, \delta} := \frac{(\delta + \Delta - 1)}{(\delta - 1)} \quad SeriesDG = \begin{pmatrix} 3 & 2 & 1.666667 & 1.5 & 1.4 & 1.333333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333333 & 2 & 1.8 & 1.666667 \\ 6 & 3.5 & 2.666667 & 2.25 & 2 & 1.833333 \end{pmatrix}$$

011696

The figure cuts the base in Cube Roots and provides some interesting ratios.



$$N := 10$$

$$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$$

$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$$

$$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$$

$$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$$

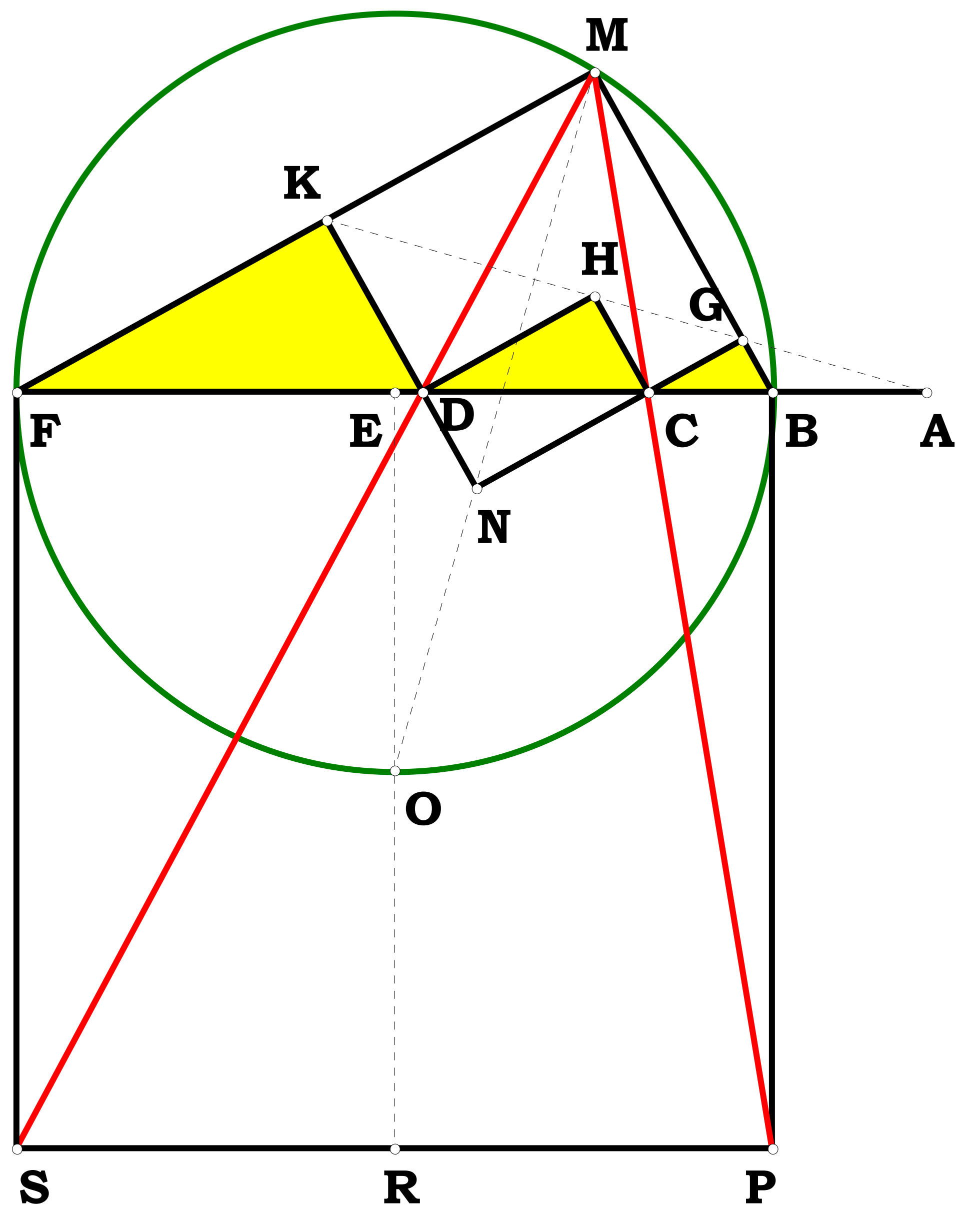
$$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$$

$$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$$

$$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$$

$$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.679602 \qquad \frac{GJ}{GN} = 1.679602 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.679602$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796024 \qquad \frac{BJ}{BM} = 7.796024 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796024$$



$$CF := AF - AC \quad BP := \frac{BD \cdot BC}{BG} \quad CD := \frac{BD \cdot CF}{BG} \quad FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.981959$$

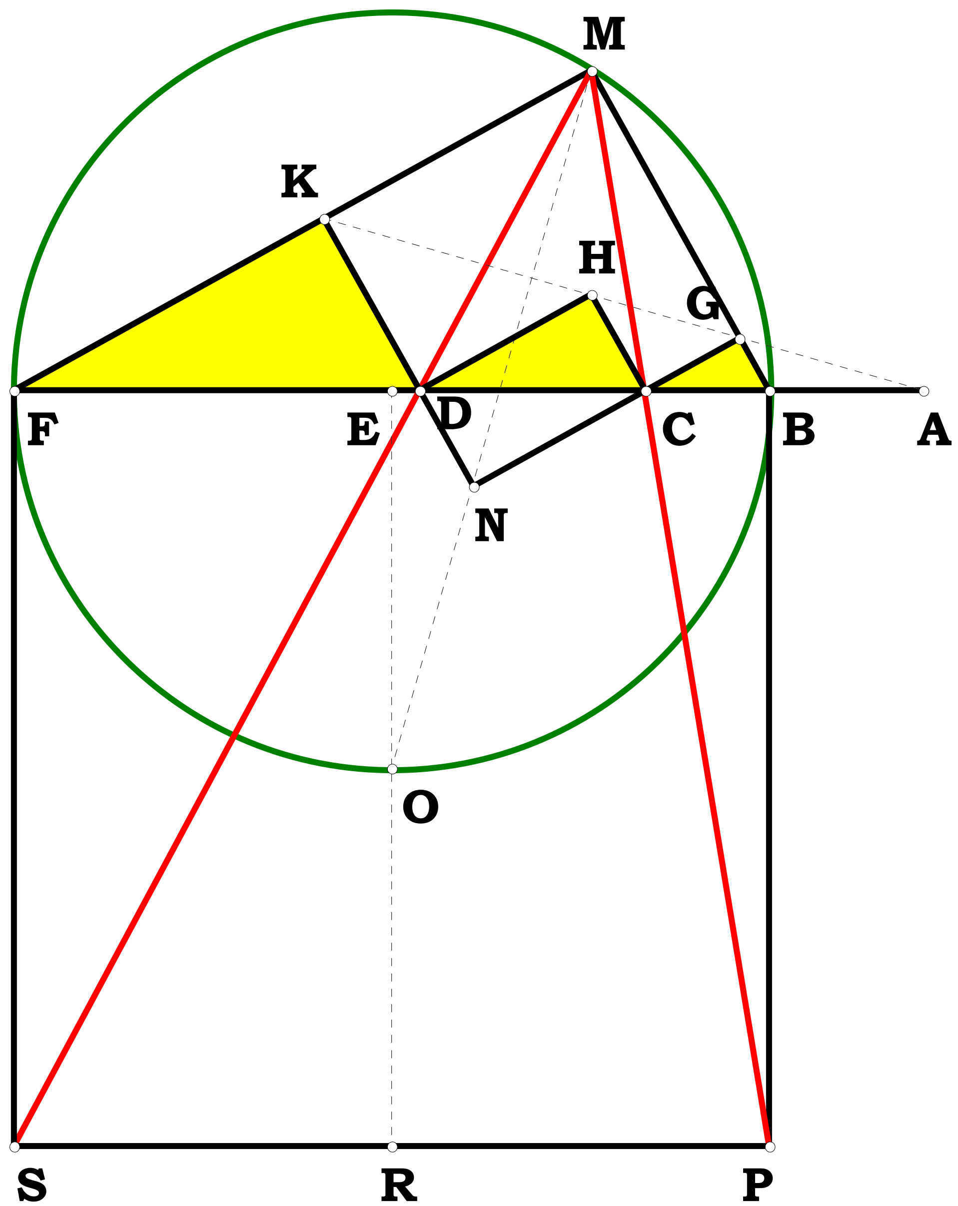
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.981959 \qquad \frac{BG}{BP} = 43.981959$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.414617$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.414617 \qquad \frac{BG}{CD} = 20.414617$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.475626$$

$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.475626 \qquad \frac{BG}{FR} = 9.475626$$



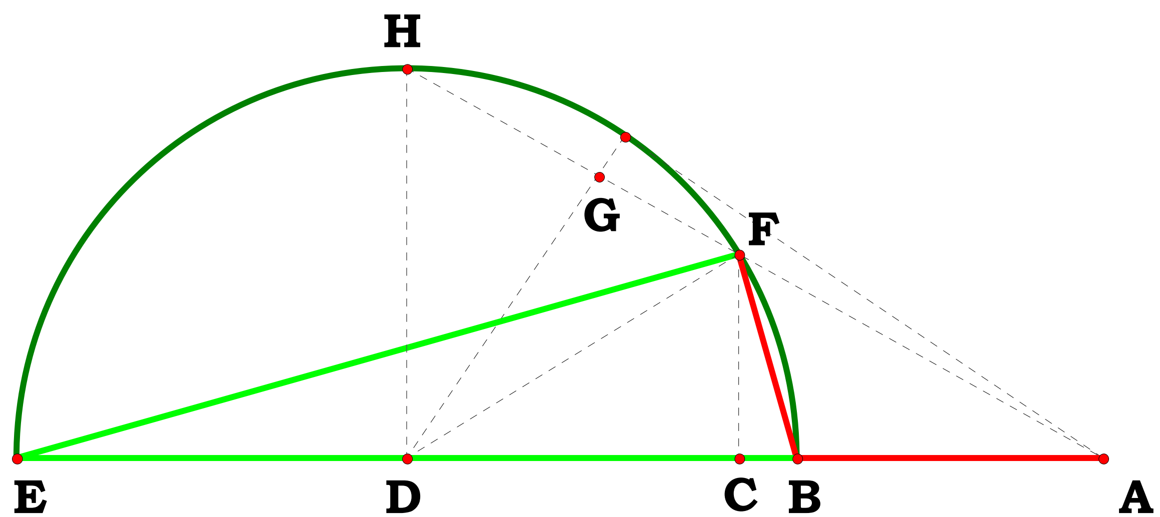
$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.868844 \qquad \frac{AG}{BP} = 48.868844 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.868844$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.682908 \qquad \frac{AG}{CD} = 22.682908 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.682908$$

$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528473 \qquad \frac{AG}{FR} = 10.528473 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528473$$

011796A Right Triangle In A Given Ratio

Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.



$N := 3$

$AB := 1 \quad AE := AB \cdot N \quad BE := AE - AB$

$BD := \dfrac{BE}{2} \quad DF := BD \quad DE := BD \quad AD := AB + BD$

$DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \dfrac{AD \cdot AD}{AH}$

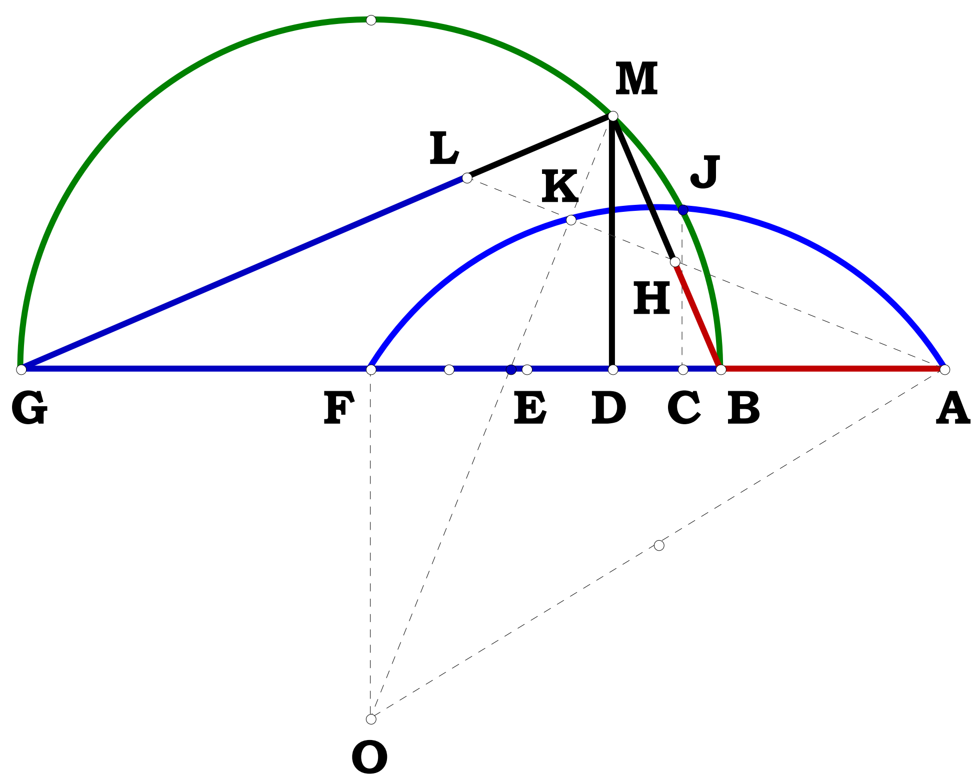
$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$S_1 := AD \quad S_2 := AF \quad S_3 := DF \quad CD := \dfrac{S_3{}^2 + S_1{}^2 - S_2{}^2}{2 \cdot S_1} \quad BC := BD - CD$

$CE := CD + DE \quad CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2} \quad AC := AD - CD$

$\dfrac{AE}{AB} - \dfrac{EF}{BF} = 0 \quad AC - \dfrac{N^2 + N}{N^2 + 1} = 0 \quad BF - \dfrac{N - 1}{\sqrt{N^2 + 1}} = 0 \quad EF - \dfrac{N^2 - N}{\sqrt{N^2 + 1}} = 0$

011796B Divide The Sides Of A Triangle In A Given Ratio



Given AG and AB on AG and a right triangle on BG divide the sides of the triangle such that a section on one side is to the other as AB is to AG.

$N_1 := 5 \quad N_2 := 9$

$AB := 1 \quad AG := AB \cdot N_1 \quad BG := AG - AB \quad BF := \dfrac{BG}{2}$

$AC := \dfrac{N_1{}^2 + N_1}{N_1{}^2 + 1} \quad AF := AB + BF \quad CF := AF - AC$

$BC := AC - AB \quad BD := \dfrac{CF}{N_2} + BC \quad DF := BF - BD$

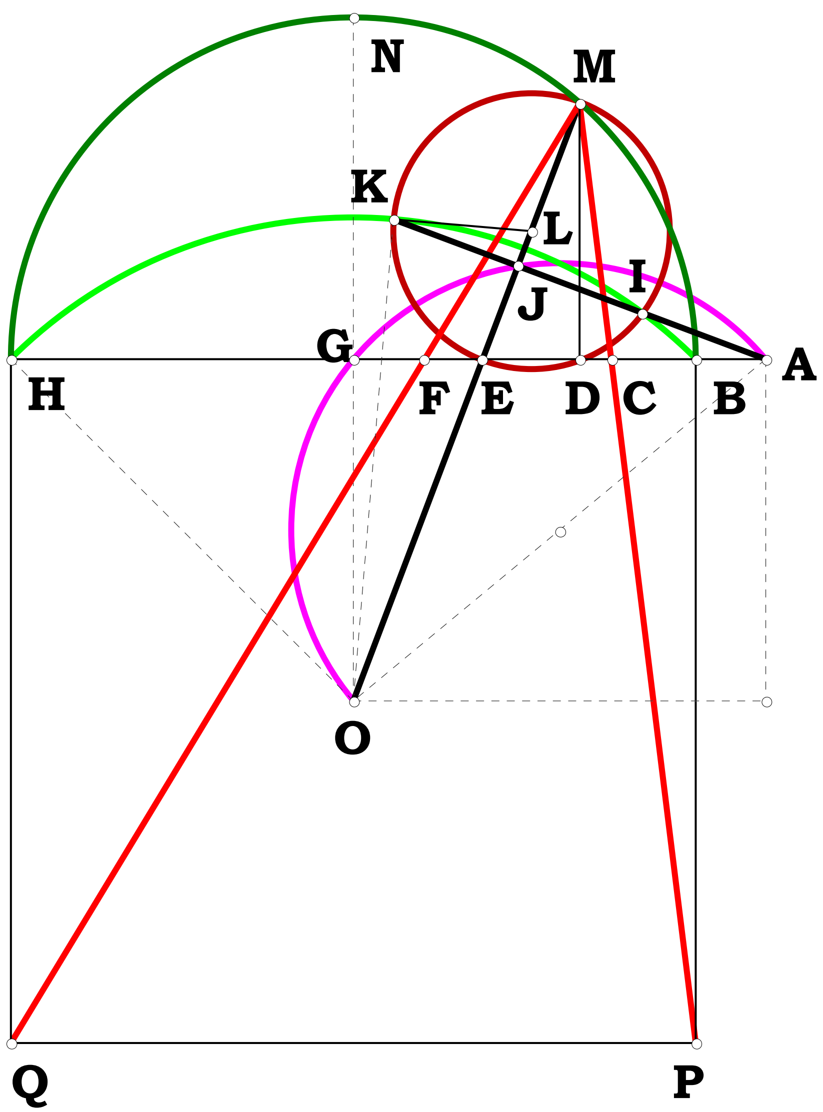
$DG := BG - BD \quad DM := \sqrt{BD \cdot DG} \quad FO := BF \quad EF := \dfrac{DF \cdot FO}{FO + DM} \quad AE := AF - EF \quad EO := \sqrt{EF^2 + FO^2}$

$MO := \dfrac{EO \cdot (DM + FO)}{FO} \quad EK := \dfrac{EF \cdot AE}{EO} \quad KM := MO - (EK + EO) \quad HK := KM \quad HM := \sqrt{2 \cdot KM^2}$

$BM := \sqrt{BD^2 + DM^2} \quad BH := BM - HM \quad GM := \sqrt{DG^2 + DM^2} \quad LM := HM \quad GL := GM - LM$

$\dfrac{AG}{AB} - \dfrac{GL}{BH} = 0$

012196 More On Cube Roots



$N := 5 \qquad BH := 1$

$BP := BH \quad HQ := BH$

$BG := \dfrac{BH}{2} \quad GO := BG \quad GN := BG \quad NO := BH \quad GH := BG$

$BE := \dfrac{BG}{N} \quad EG := BG - BE \quad EO := \sqrt{EG^2 + GO^2}$

$MO := \dfrac{GO \cdot NO}{EO} \quad EM := MO - EO \quad EL := \dfrac{EM}{2} \quad LK := EL$

$LO := EO + EL \quad LJ := \dfrac{LK^2}{LO} \quad EJ := EL - LJ$

$AE := \dfrac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH \quad AB := AH - BH$

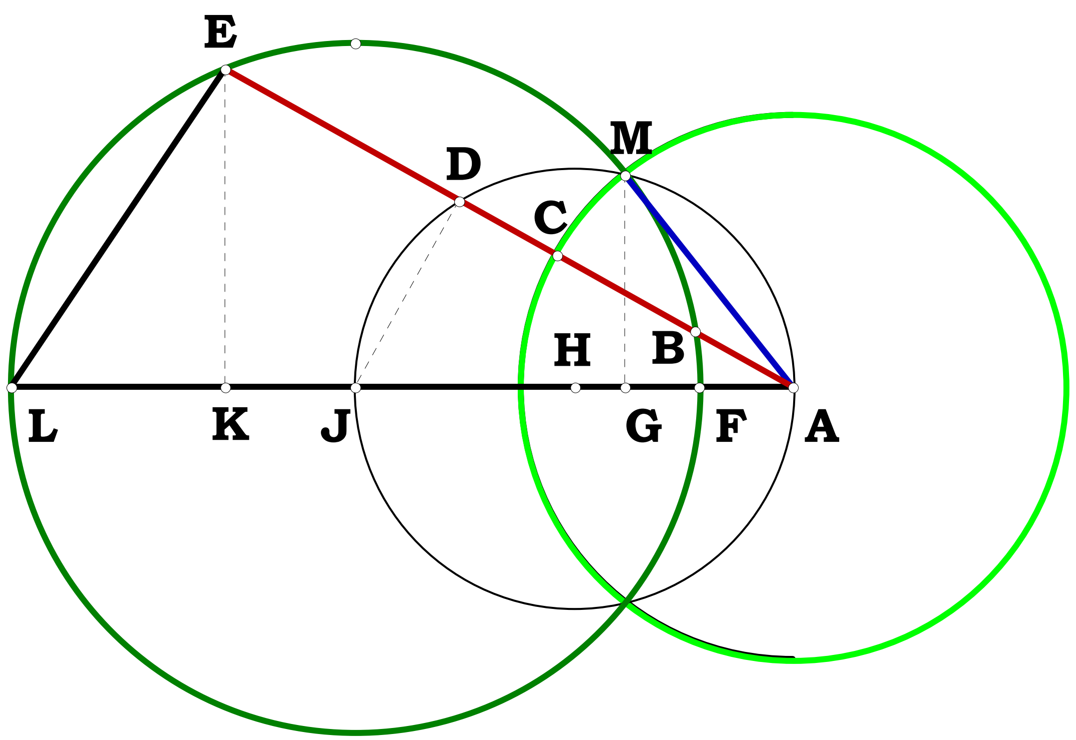
$DE := \dfrac{EG \cdot EM}{EO} \quad DM := \dfrac{GO \cdot EM}{EO} \quad BD := BG - (EG + DE)$

$BC := \dfrac{BD \cdot BP}{BP + DM} \quad DH := BH - BD \quad DF := \dfrac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$$

012296 Trivial Method Square Root

For any E between M and L, AM is the square root of AB x AE.



$$N_1 := 6 \quad N_2 := 3$$

$$AF := 1 \quad AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$$

$$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$$

$$GL := AL - AG \quad GK := \frac{GL}{N_2} \quad FG := AG - AF$$

$$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$$

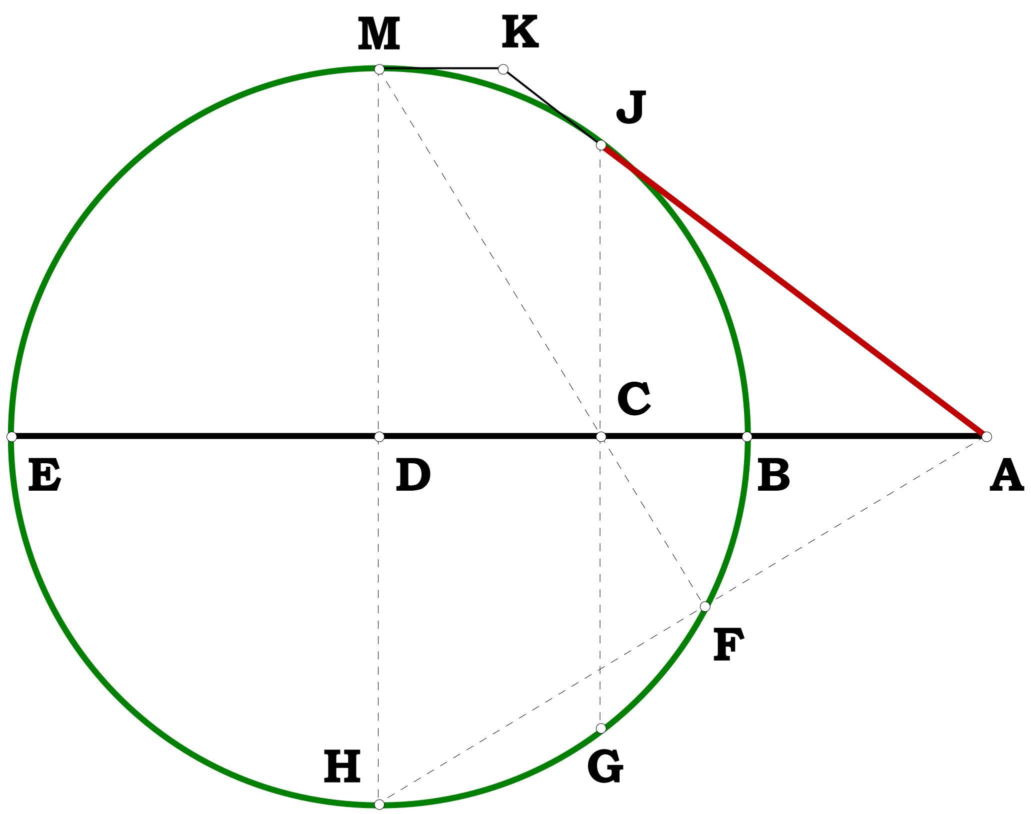
$$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$$

$$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$$

$$\sqrt{AB \cdot AE} - AM = 0$$

012496 Tangent

The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.



$$N := 5 \quad AB := 1 \quad AE := AB \cdot N$$

$$BE := AE - AB \quad BD := \frac{BE}{2} \quad AD := AB + BD$$

$$DM := BD \quad DH := BD \quad CD := \frac{DH \cdot DM}{AD}$$

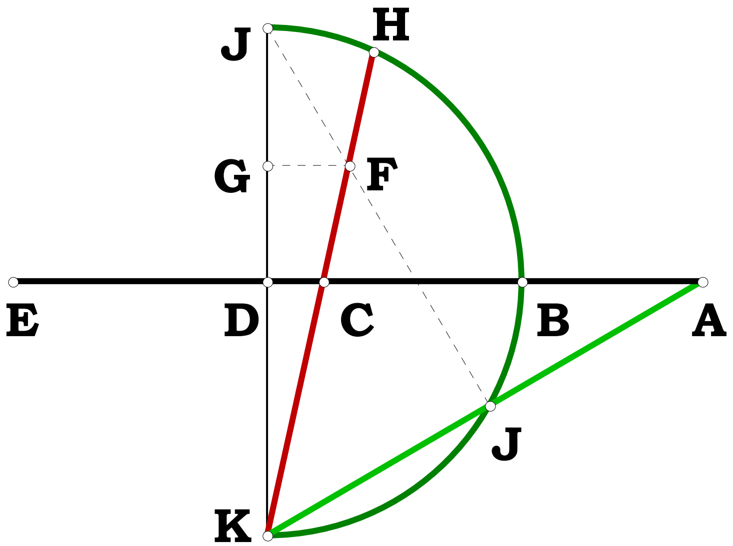
$$BC := BD - CD \quad CE := BE - BC$$

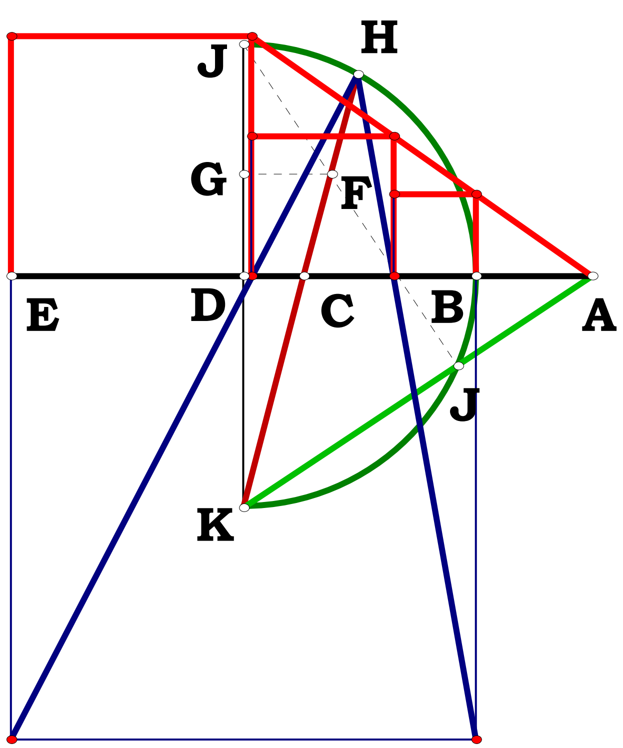
$$CJ := \sqrt{BC \cdot CE} \quad AJ := \sqrt{(AB + BC)^2 + CJ^2}$$

$$AJ - \sqrt{N} = 0 \qquad AJ - \sqrt{AB \cdot AE} = 0$$

$$AC := AB + BC \qquad 2 \cdot AB \cdot \frac{N}{(1 + N)} - AC = 0$$

012596 On Cubes





Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.

$N := 9 \qquad BE := 1$

$BD := \dfrac{BE}{2} \quad DK := BD \quad DJ := BD \quad JK := BE \quad DE := BD$

$BC := \dfrac{BD}{N} \quad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$

$HK := \dfrac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \dfrac{CH}{2}$

$FK := CK + CF \quad GK := \dfrac{DK \cdot FK}{CK} \quad FG := \dfrac{CD \cdot FK}{CK}$

$GJ := JK - GK \quad AD := \dfrac{GJ \cdot DK}{FG} \quad AE := AD + DE$

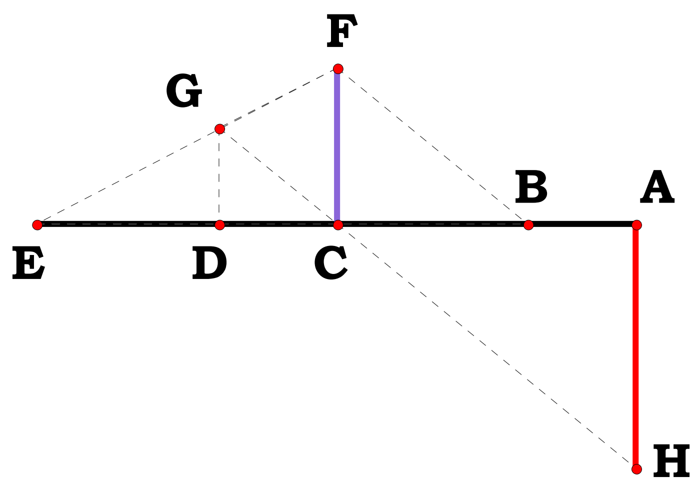
$AB := AE - BE$

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$\frac{8 \cdot N^3 - 12 \cdot N^2 + 6 \cdot N - 1}{8 \cdot N^3 - 12 \cdot N^2 + 6 \cdot N - 2} \qquad \frac{1}{8 \cdot N^3 - 12 \cdot N^2 + 6 \cdot N - 2}$$

012996 Linear division $\dfrac{N_1\cdot(N_2+2\cdot N_3)}{2\cdot(N_2+N_3)}$



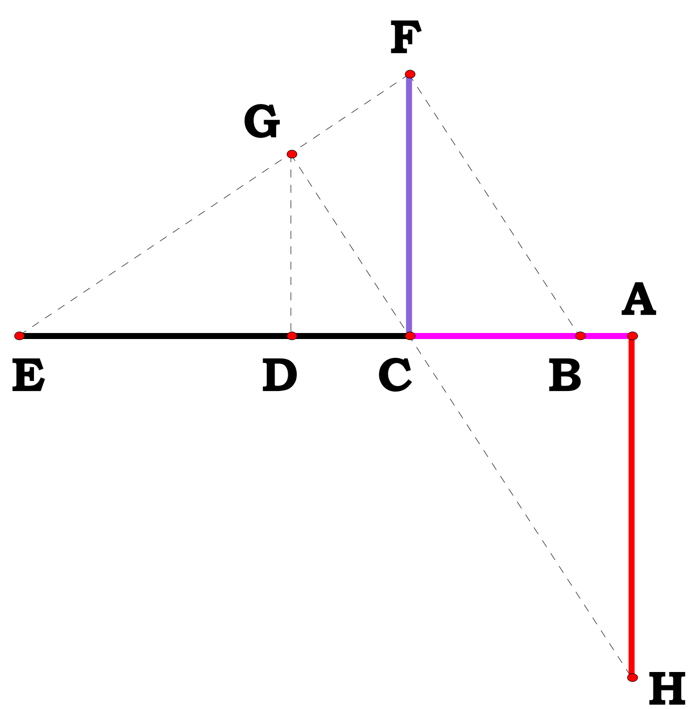
$N_1 := 2 \quad N_2 := 3 \quad N_3 := 9$

$AE := N_1 \quad AH := N_2 \quad AC := \dfrac{AE}{2} \quad CF := N_3 \quad BC := \dfrac{AC\cdot CF}{AH} \quad CE := AC$

$BE := CE + BC \quad CD := \dfrac{BC\cdot CE}{BE} \quad DE := CE - CD \quad AD := AC + CD \quad DG := \dfrac{AH\cdot CD}{AC} \quad BC - \dfrac{N_1\cdot N_3}{2\cdot N_2} = 0$

$DE - \dfrac{N_1\cdot N_2}{2\cdot(N_2+N_3)} = 0 \quad AD - \dfrac{N_1\cdot(N_2+2\cdot N_3)}{2\cdot(N_2+N_3)} = 0 \quad CD - \dfrac{N_1\cdot N_3}{2\cdot(N_2+N_3)} = 0 \quad DG - \dfrac{N_2\cdot N_3}{N_2+N_3} = 0$

Linear division $\dfrac{N_2\cdot(N_1-N_3)^2}{N_1\cdot N_2 - N_2\cdot N_3 + N_3\cdot N_4}$



$N_1 := 4 \quad N_2 := 4 \quad N_3 := 2 \quad N_4 := 3$

$AE := N_1 \quad AH := N_2 \quad AC := N_3 \quad CF := N_4 \quad BC := \dfrac{AC\cdot CF}{AH} \quad CE := AE - AC$

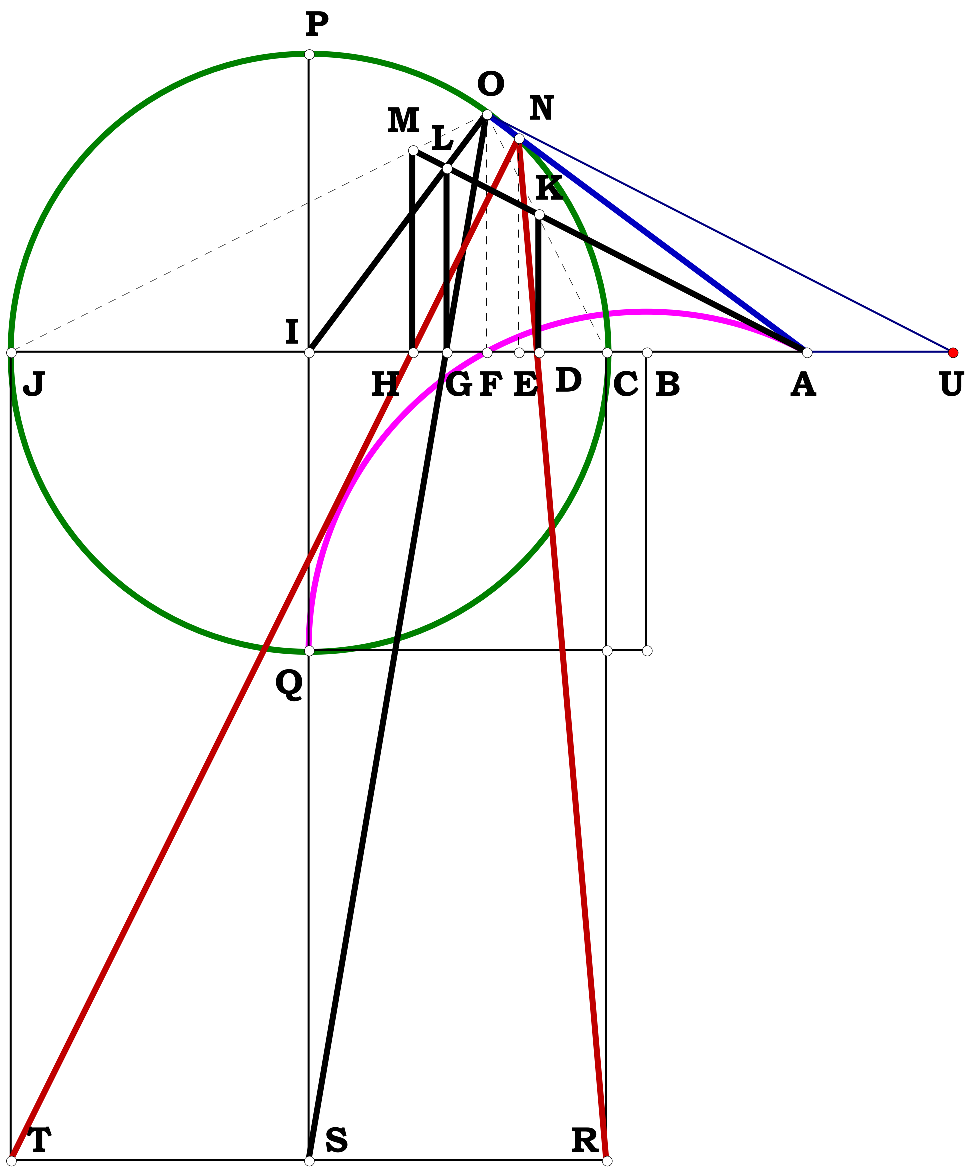
$BE := CE + BC \quad CD := \dfrac{BC\cdot CE}{BE} \quad DE := CE - CD \quad AD := AC + CD \quad DG := \dfrac{CF\cdot CE}{BE}$

$BC - \dfrac{N_3\cdot N_4}{N_2} = 0 \quad DE - \dfrac{N_2\cdot(N_1-N_3)^2}{N_1\cdot N_2 - N_2\cdot N_3 + N_3\cdot N_4} = 0 \quad AD - \dfrac{N_3\cdot(N_1\cdot N_2 + N_1\cdot N_4 - N_2\cdot N_3)}{N_1\cdot N_2 - N_2\cdot N_3 + N_3\cdot N_4} = 0$

$CD - \dfrac{N_3\cdot N_4\cdot(N_1-N_3)}{N_1\cdot N_2 - N_2\cdot N_3 + N_3\cdot N_4} = 0 \quad DG - \dfrac{N_2\cdot N_4\cdot(N_1-N_3)}{N_1\cdot N_2 - N_2\cdot N_3 + N_3\cdot N_4} = 0$

013196 On Gemini Roots

Hitting AO from any RT while maintaining Gemini Roots.



$N_1 := 5 \quad N_2 := 3 \quad BC := 1$

$BJ := BC \cdot N_1 \quad CJ := BJ - BC \quad CI := \dfrac{CJ}{2}$

$IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF$

$FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2 \quad HS := CR$

$FI := FJ - IJ \quad FG := \dfrac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \dfrac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \dfrac{HS \cdot GO}{OS} \quad FU := \dfrac{AG \cdot FO}{GL}$

$AH := \dfrac{FU \cdot AJ}{FU + FJ} \quad DK := \dfrac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \dfrac{AG \cdot DK}{GL} \quad AC := AF - CF \quad CD := AD - AC$

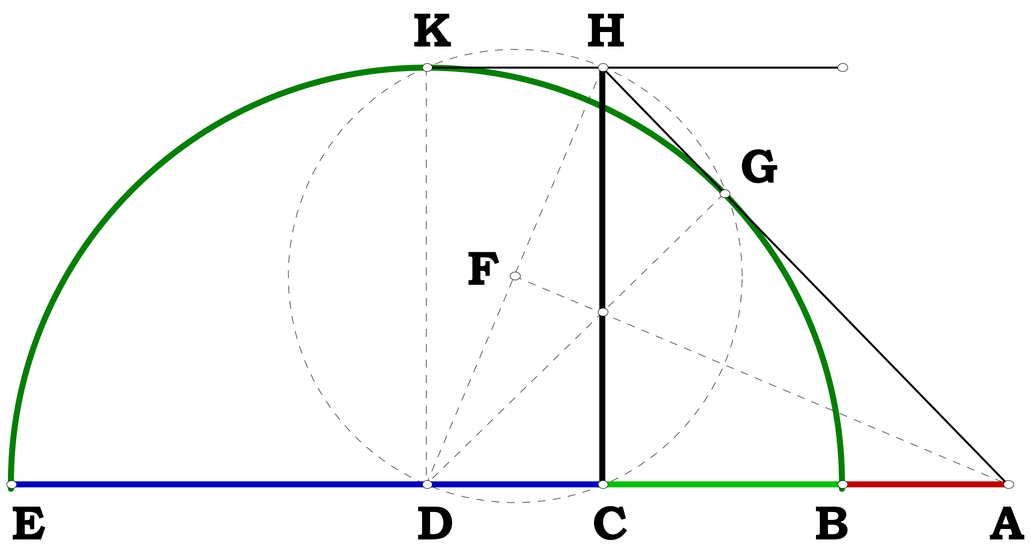
$CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \dfrac{CR \cdot DH}{CD + HJ} \quad CE := \dfrac{CD \cdot (CR + EN)}{CR}$

$AE := AC + CE \quad \dfrac{AF}{FO} - \dfrac{AE}{EN} = 0 \quad BD := BC + CD \quad BH := BC + CH$

$$\sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$$

020296 Find A Segment

Given BE and BC such that $\sqrt{(AB + BE) \cdot AB} = AB + BC$, find AB.



$N_1 := 2 \qquad N_2 := 5$

$BC := N_1 \quad BE := N_2 \quad BD := \frac{BE}{2} \quad CD := BD - BC$

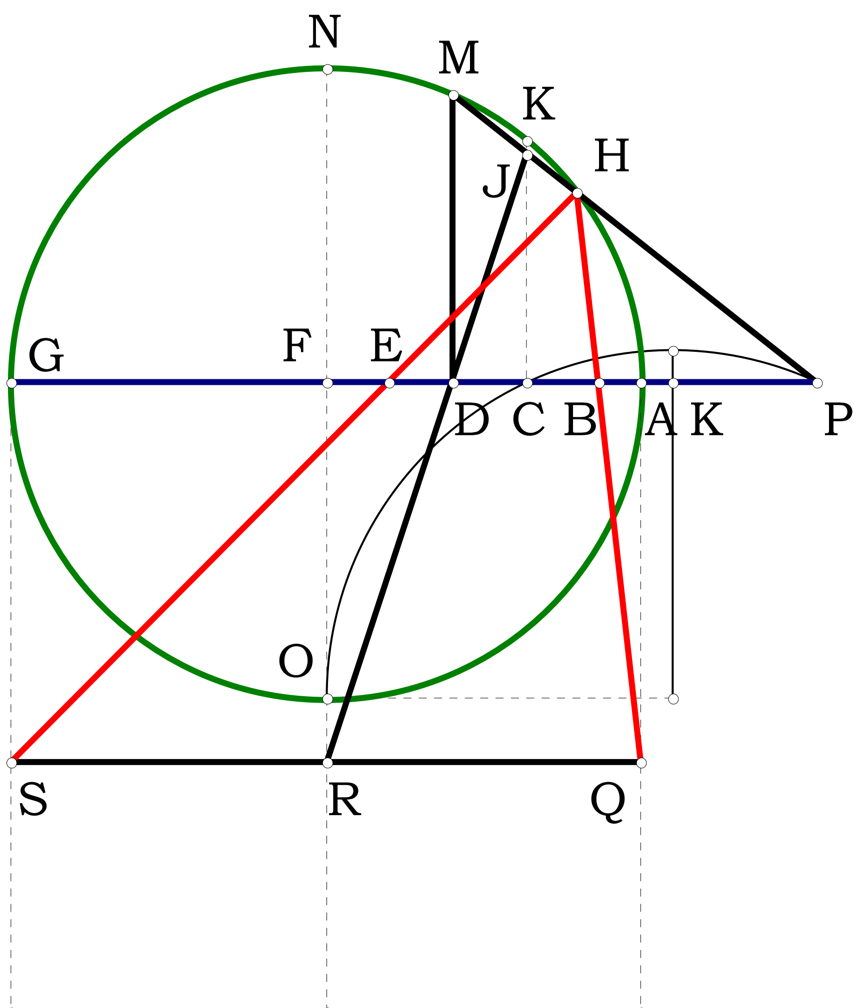
$CH := BD \quad DH := \sqrt{CD^2 + CH^2} \quad DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD} \quad AB := AD - BD$

$AB - \frac{{N_1}^2}{N_2 - 2 \cdot N_1} = 0$

$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$

021496.MCD Or, the 17 decimal place rustic solution.



Use iteration to find any root pair for BE.
Remember that when N is set to 2, we have cube roots.

$$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$$

$$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$$

$$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$$

$$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$$

$$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$$

$$\Delta := 40 \quad \delta := 0 .. \Delta$$

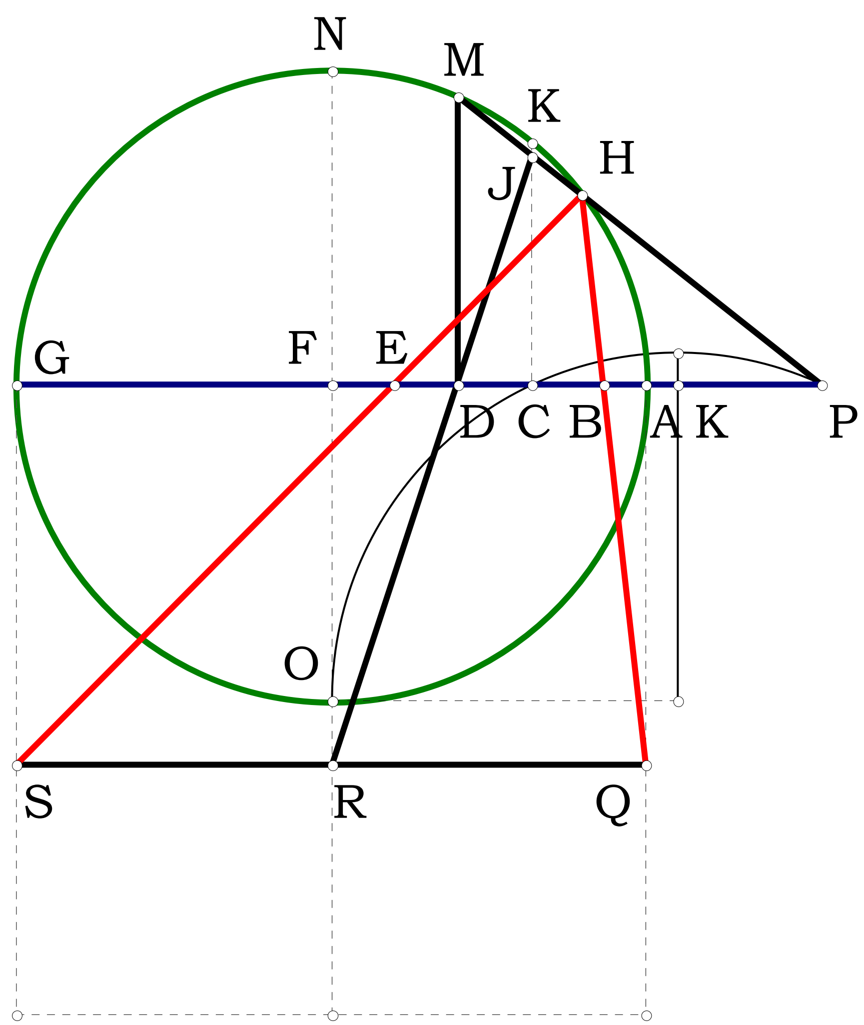
$$\begin{pmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{pmatrix} := \begin{bmatrix} EK \\ \dfrac{EG \cdot GN}{GN + EK} \\ AG - \dfrac{EG \cdot GN}{GN + EK} \\ GI + \dfrac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\dfrac{EG \cdot GN}{GN + EK}\right)\right] - AC \\ \sqrt{\left[\left[AG - \left(\dfrac{EG \cdot GN}{GN + EK}\right)\right] - AC\right] \cdot \left[GI + \left(\dfrac{EG \cdot GN}{GN + EK}\right)\right]} \end{bmatrix}$$

$$\begin{pmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{pmatrix} := \begin{pmatrix} \dfrac{FL_\delta \cdot AE}{AF_\delta} \\ \dfrac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{pmatrix}$$

$$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$$

$$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$$

$$BD := BC + CD \quad BH := BC + CD + DH$$



$$\frac{DH}{\sqrt{CD \cdot HI}} = 1$$

$$BE - \sqrt{BD \cdot BH} = 0$$

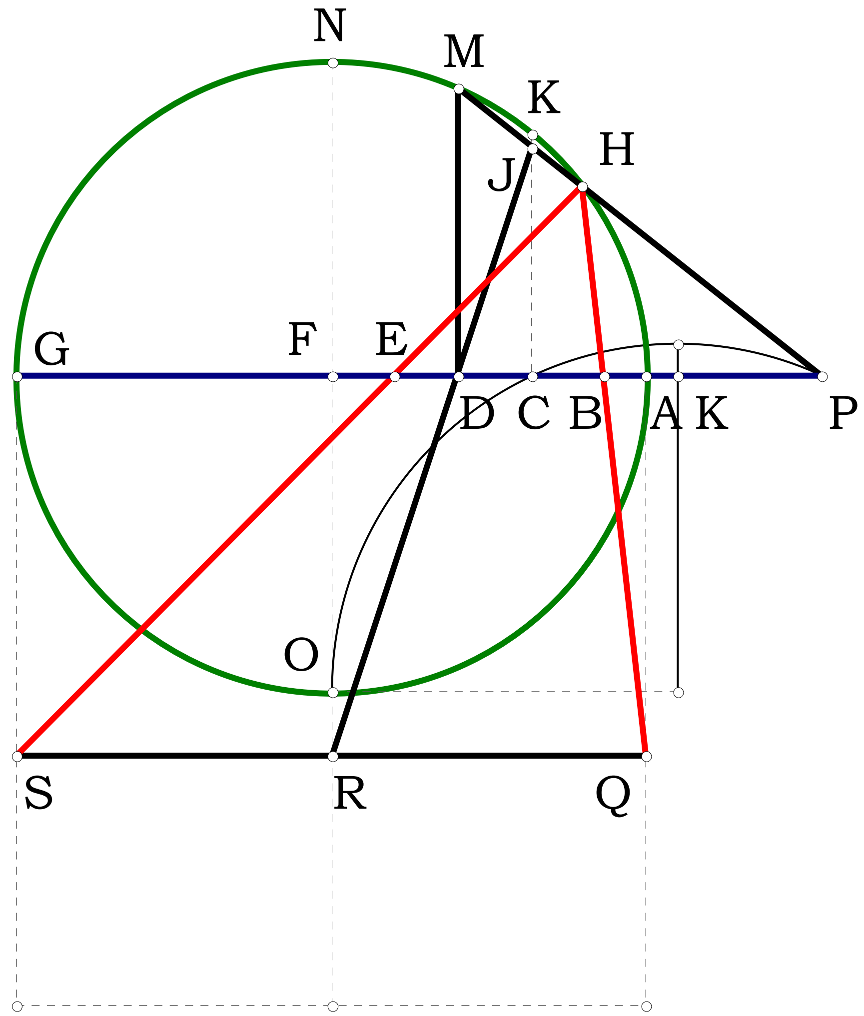
The next two equations are for the Delian Problem only.
Resolution set to max of the program.

$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0$$

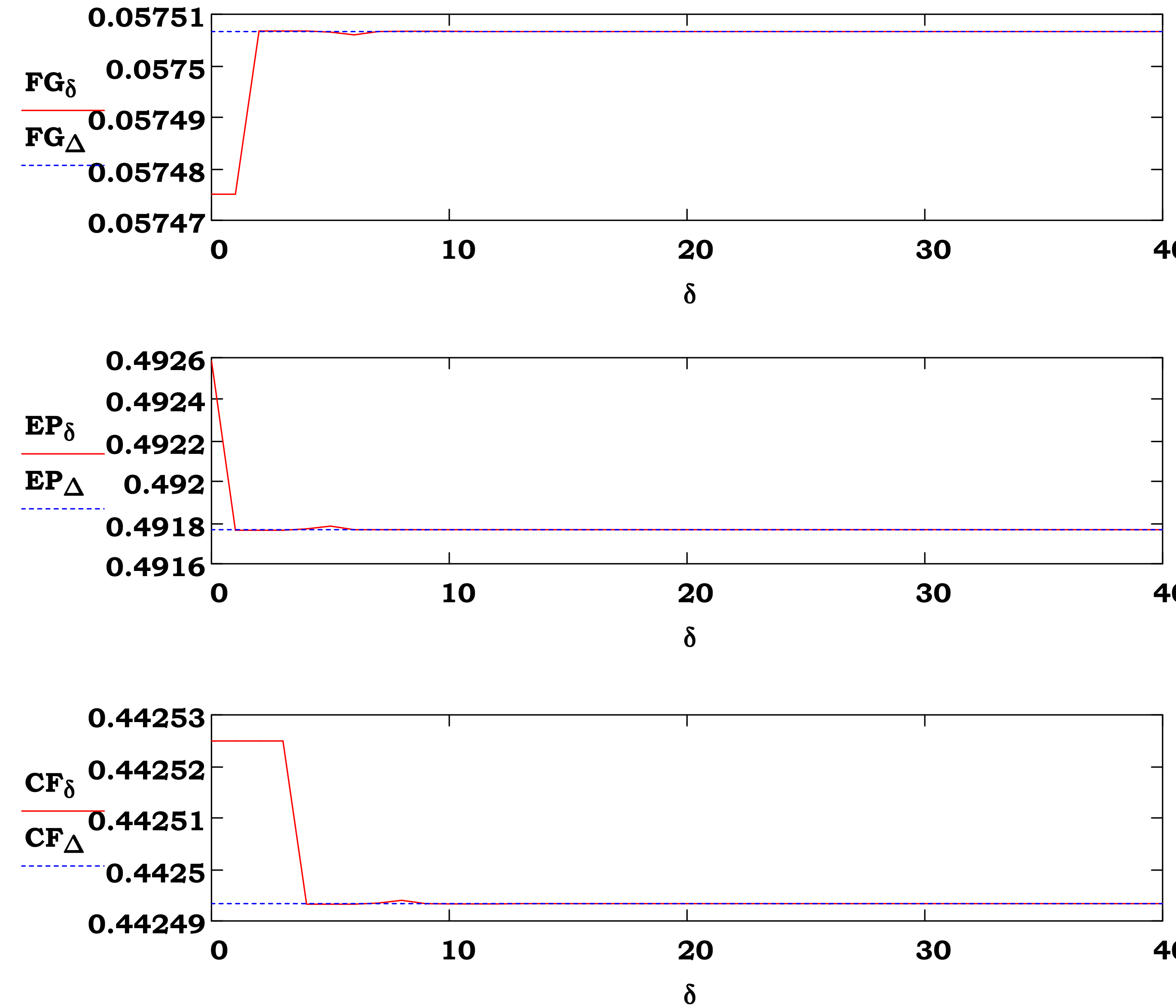
$$BD = 1.2599210498948732 \qquad 2^{\frac{1}{3}} = 1.2599210498948732$$

17 decimal places. Good to the limits of the program.

$$BH = 1.5874010519681994 \qquad 4^{\frac{1}{3}} = 1.5874010519681994$$



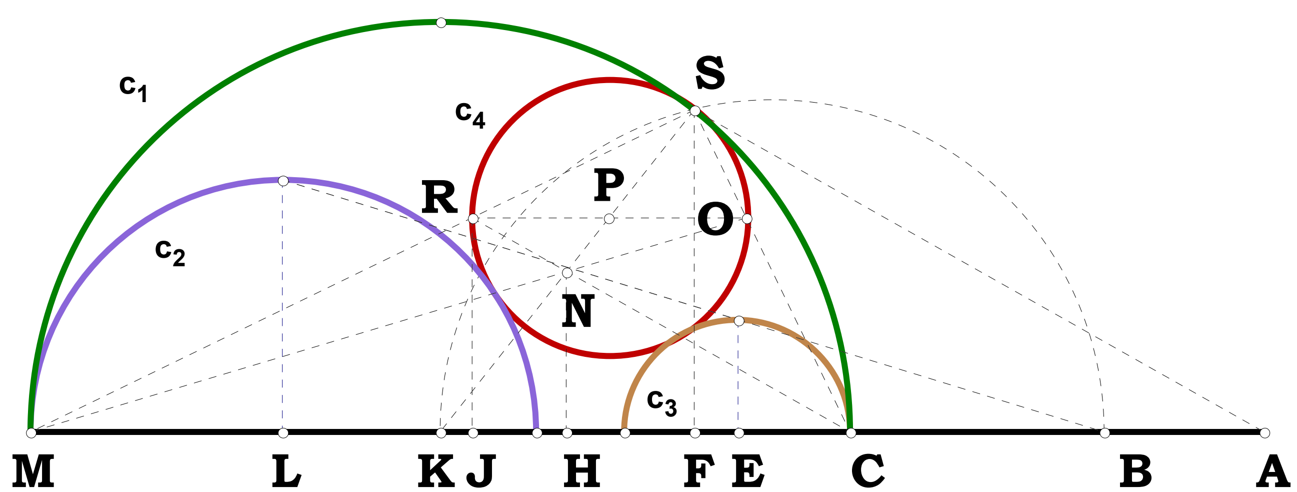
A graphics peek.



The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist. The solution is only good to material differences, on the atomic level, so to speak.

041496 Method for Unequals

Given c_1 , c_2 , c_3, find c_4. I had this sketched out in 95, but if I put it there I would have had a lot of document links to redo in "The Quest."



$N_1 := 7 \qquad N_2 := 3$

$CM := 1 \quad CK := \dfrac{CM}{2} \quad CE := \dfrac{CM}{N_1}$

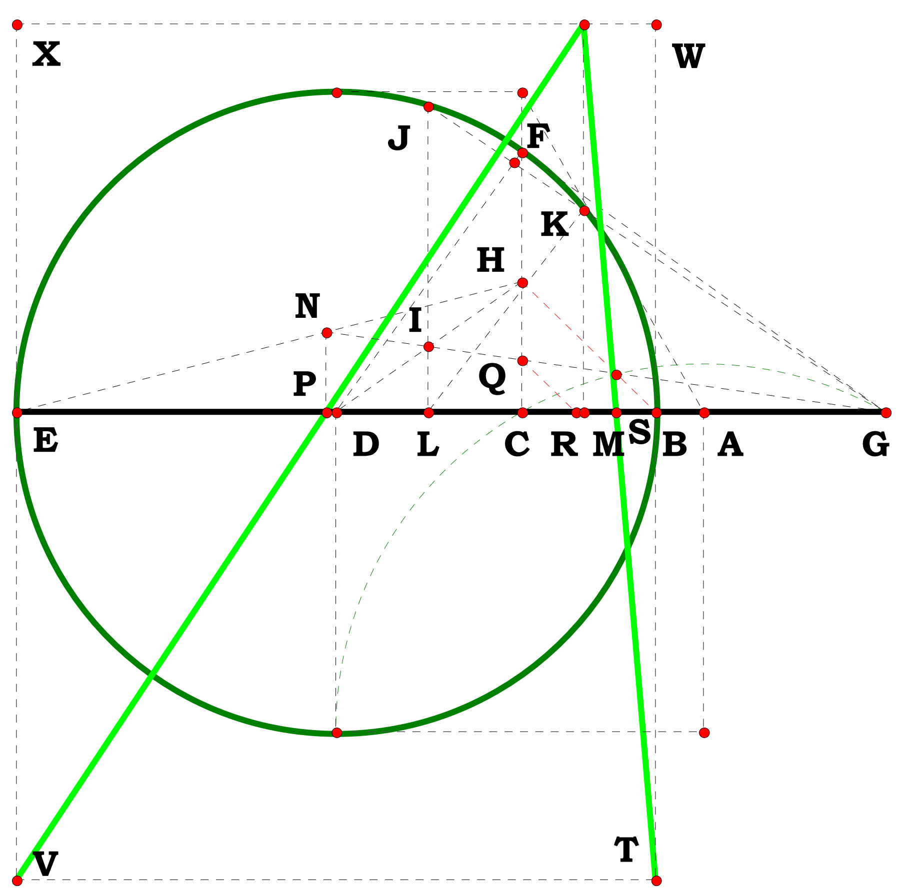
$LM := \dfrac{CM}{N_2} \quad EL := CM - (CE + LM)$

$BL := \dfrac{EL \cdot LM}{LM - CE} \quad BM := BL + LM \quad BC := BM - CM \quad BK := \dfrac{CM}{2} + BC \quad R_1 := LM \quad R_2 := CE \quad D := EL \quad KS := CK \quad EH := \dfrac{\left(R_2^{\,2} + D^2 - R_1^{\,2}\right)}{2 \cdot D}$

$FK := \dfrac{KS^2}{BK} \quad CF := CK - FK \quad FM := CM - CF \quad FS := \sqrt{CF \cdot FM} \quad HK := CK - (CE + EH) \quad CH := CK - HK \quad HN := \dfrac{FS \cdot HK}{FK} \quad AF := \dfrac{CH \cdot FS}{HN}$

$JR := \dfrac{FS \cdot CM}{AF + FM} \quad RO := \dfrac{CM \cdot (FS - JR)}{FS} \quad PS := \dfrac{RO}{2} \quad PS - \dfrac{\left(N_2 - 2\right) \cdot \left(N_1 - 2\right)}{2 \cdot \left(N_1 \cdot N_2 - 4\right)} = 0$

041596.MCD



On Gemini Roots

$N_1 := 3 \qquad N_2 := 4$

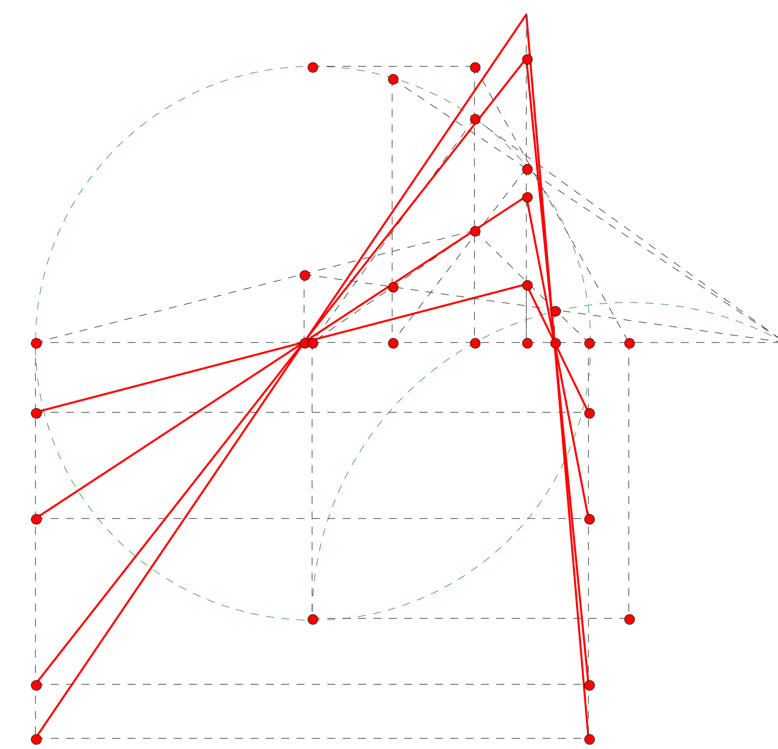
$AB := N_1 \quad BE := N_2 \qquad BD := \dfrac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE} \quad BC := AC - AB$

$CE := BE - BC \quad CF := \sqrt{BC \cdot CE} \quad CD := BD - BC$

$CG := \dfrac{CF^2}{CD} \quad BG := CG - BC \quad EG := BG + BE$

$CH := \dfrac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2} \quad DI := \dfrac{1}{2} \cdot DH$

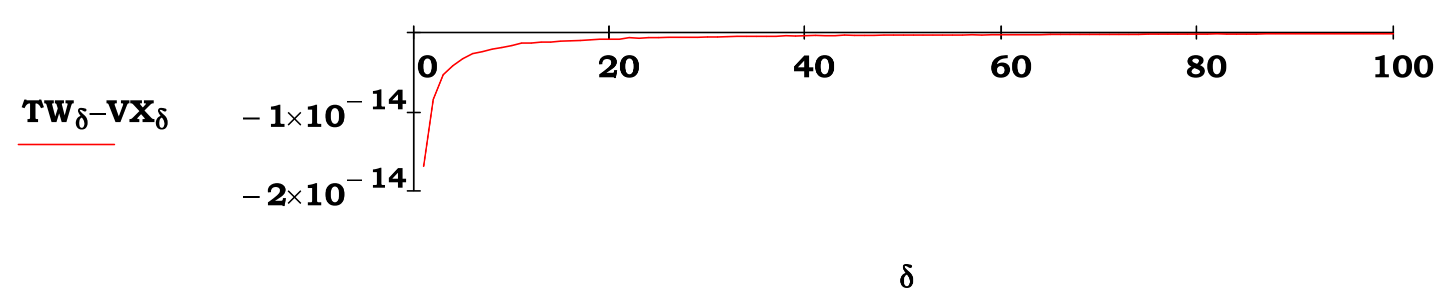


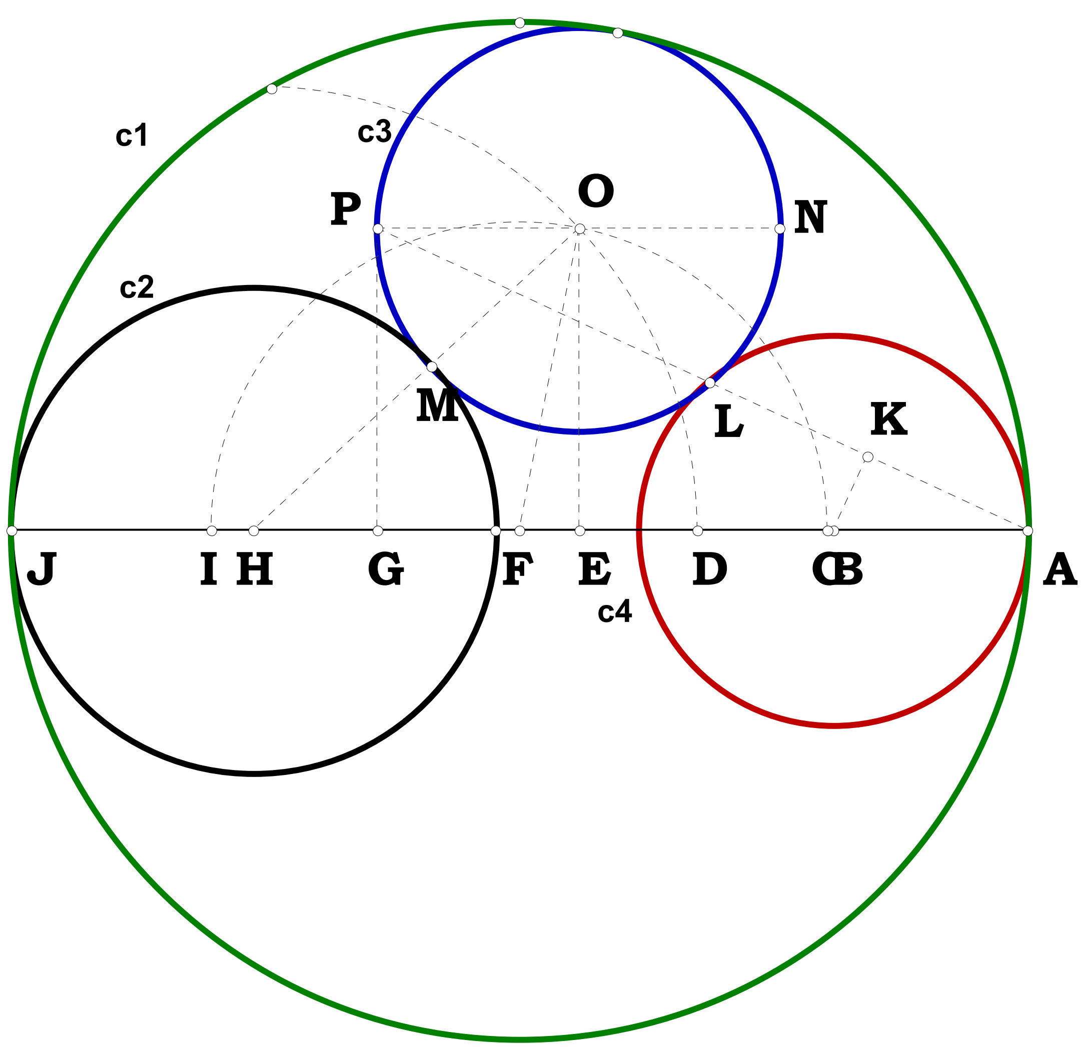
$DL := \dfrac{CD \cdot DI}{DH} \quad BL := BD - DL \quad EL := BE - BL \quad JL := \sqrt{BL \cdot EL} \quad GL := BL + BG \quad GJ := \sqrt{JL^2 + GL^2}$

$GK := \dfrac{BG \cdot EG}{GJ} \quad GM := \dfrac{GL \cdot GK}{GJ} \quad BM := GM - BG \quad EM := BE - BM \quad IL := \sqrt{DI^2 - DL^2} \quad CO := \dfrac{GL \cdot CH}{IL}$

$NP := \dfrac{CH \cdot EG}{(CO + CE)} \quad EP := \dfrac{CE \cdot NP}{CH} \quad CQ := \dfrac{IL \cdot CG}{GL} \quad CR := \dfrac{BC \cdot CQ}{CH} \quad GR := CG - CR \quad BS := \dfrac{CR \cdot BG}{GR}$

$\delta := 1 .. 100 \quad E_\delta := \dfrac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta \quad TW_\delta := \dfrac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \dfrac{EV_\delta \cdot EM}{EP}$





041696 Given Three Radii

Given c1, c2 and c3 find c4 such that AB is collinear with c1 and c2.

$N_1 := 4 \quad N_2 := 16$

$AJ := 1 \quad AF := \dfrac{AJ}{2} \quad HJ := \dfrac{AJ}{N_1} \quad NO := \dfrac{AJ}{N_2}$

$HM := HJ \quad MO := NO \quad HO := HM + MO$

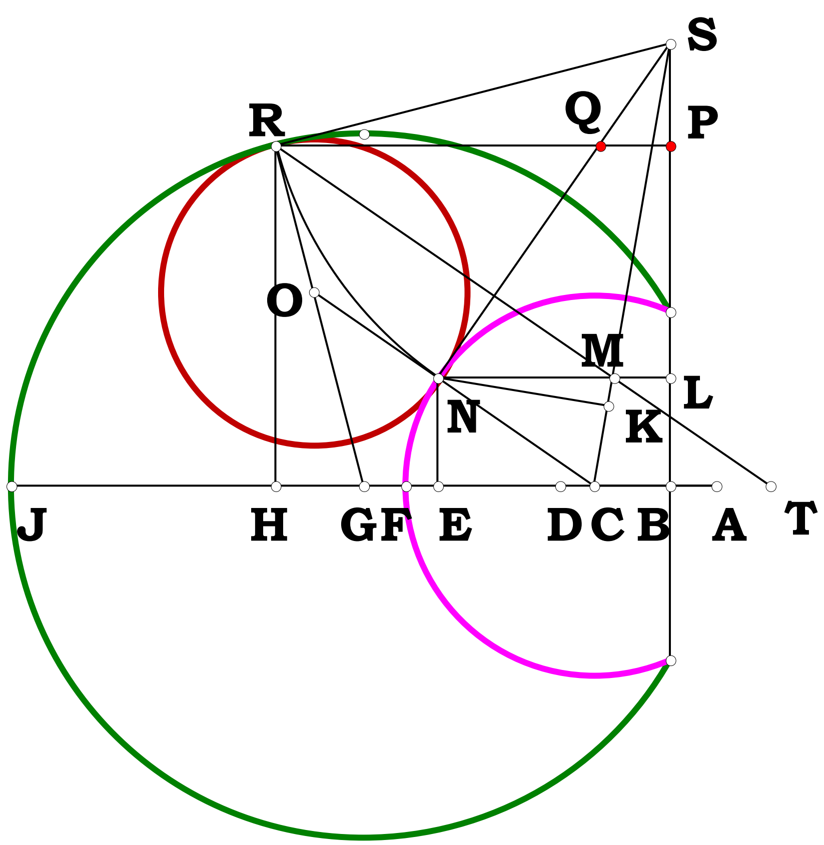
$FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$S_1 := FH \quad S_2 := FO \quad S_3 := HO \quad EH := \dfrac{{S_3}^2 + {S_1}^2 - {S_2}^2}{2 \cdot S_1} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO \quad AP := \sqrt{AG^2 + GP^2}$

$PL := \dfrac{AG \cdot (NO + OP)}{AP} \quad AL := AP - PL \quad AB := \dfrac{AP \cdot AL}{2 \cdot AG} \quad AB - \dfrac{N_2 \cdot N_1 - 2 \cdot N_2 - 2 \cdot N_1}{2 \cdot \left(N_2 \cdot N_1 - 2 \cdot N_2 - 4\right)} = 0$

041796 A Circle In A Crescent



$R_1 := 4.65667 \quad R_2 := 2.50804 \quad D_1 := 3.05114 \quad D_2 := 5.20739$

$AG := R_1 \quad AJ := 2 \cdot R_1 \quad CF := R_2 \quad CG := D_1 \quad BG := \dfrac{\left(R_1{}^2 + D_1{}^2 - R_2{}^2\right)}{2 \cdot D_1}$

$BC := CG - BG \quad FG := CG - CF \quad AF := AG - FG \quad AD := \dfrac{AF}{2} \quad AB := AG - BG$

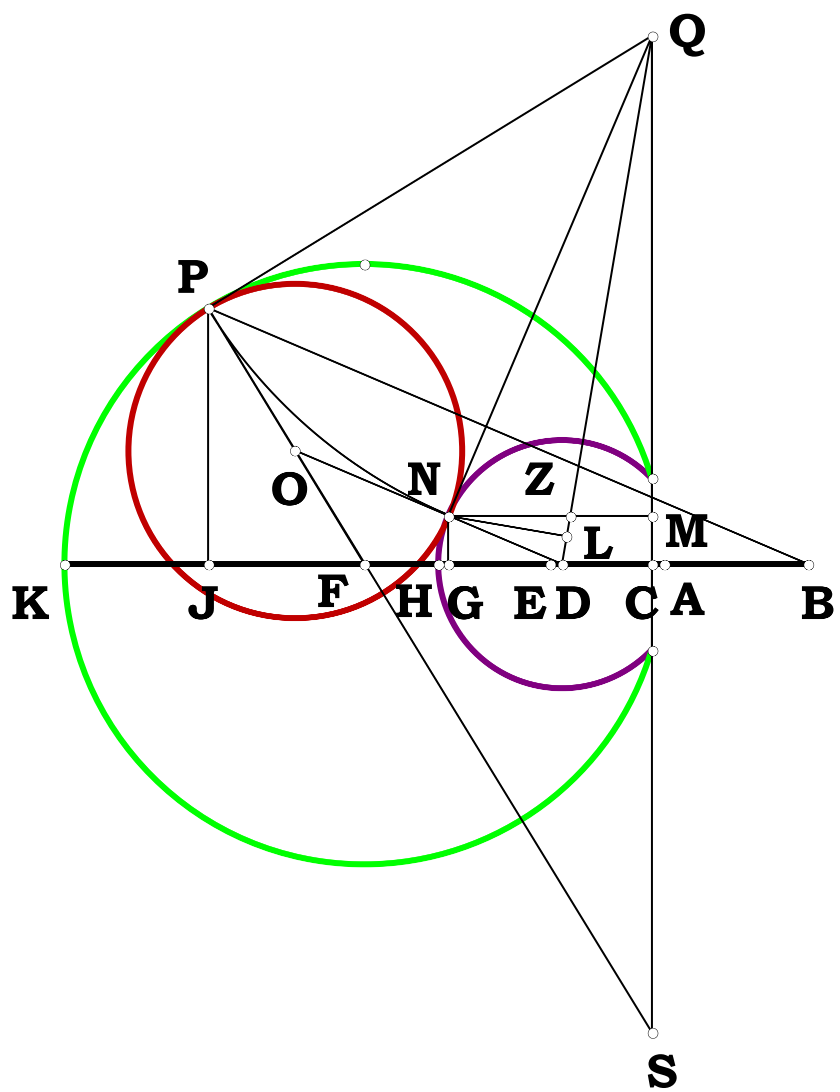
$BJ := AJ - AB \quad BH := D_2 \quad AH := BH + AB \quad GH := AH - AG \quad HR := \sqrt{AG^2 - GH^2}$

$AP := HR \quad PR := BH \quad PS := \dfrac{GH \cdot PR}{HR} \quad BS := AP + PS \quad RS := \sqrt{PR^2 + PS^2} \quad NS := RS$

$CN := CF \quad CS := \sqrt{NS^2 + CN^2} \quad CK := \dfrac{CN^2}{CS} \quad SK := CS - CK \quad KN := \sqrt{CN^2 - CK^2} \quad KM := \dfrac{BC \cdot KN}{BS}$

$SM := SK + KM \quad SL := \dfrac{BS \cdot SM}{CS} \quad BL := BS - SL \quad EN := BL \quad CE := \sqrt{CN^2 - EN^2}$

$HT := \dfrac{CE \cdot HR}{EN} \quad GT := HT - GH \quad GO := \dfrac{AG \cdot CG}{GT} \quad OR := AG - GO \quad OR = 2.005063$



041796b A Circle In A Crescent

$R_1 := 5.79438 \quad R_2 := 2.40272 \quad D_1 := 3.82231 \quad D_2 := 6.96436$

$AF := R_1 \quad AK := 2 \cdot R_1 \quad FP := AF \quad FK := AF$

$DH := R_2 \quad DF := D_1 \quad CD := \dfrac{DH^2 + DF^2 - AF^2}{2 \cdot DF}$

$FH := DF - DH \quad AH := AF - FH \quad AE := \dfrac{AH}{2} \quad EH := AE \quad AD := DF - AF$

$DE := AE - AD \quad DN := DH \quad AC := CD - AD \quad CK := AK - AC \quad CJ := D_2 \quad CF := CK - FK$

$CF := DF - CD \quad FJ := CJ - CF \quad JP := \sqrt{FP^2 - FJ^2} \quad FS := \dfrac{FP \cdot CF}{FJ} \quad PS := FS + FP$

$QS := \dfrac{FP \cdot PS}{JP} \quad PQ := \dfrac{FJ \cdot QS}{FP} \quad CS := \dfrac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad DQ := \sqrt{CD^2 + CQ^2}$

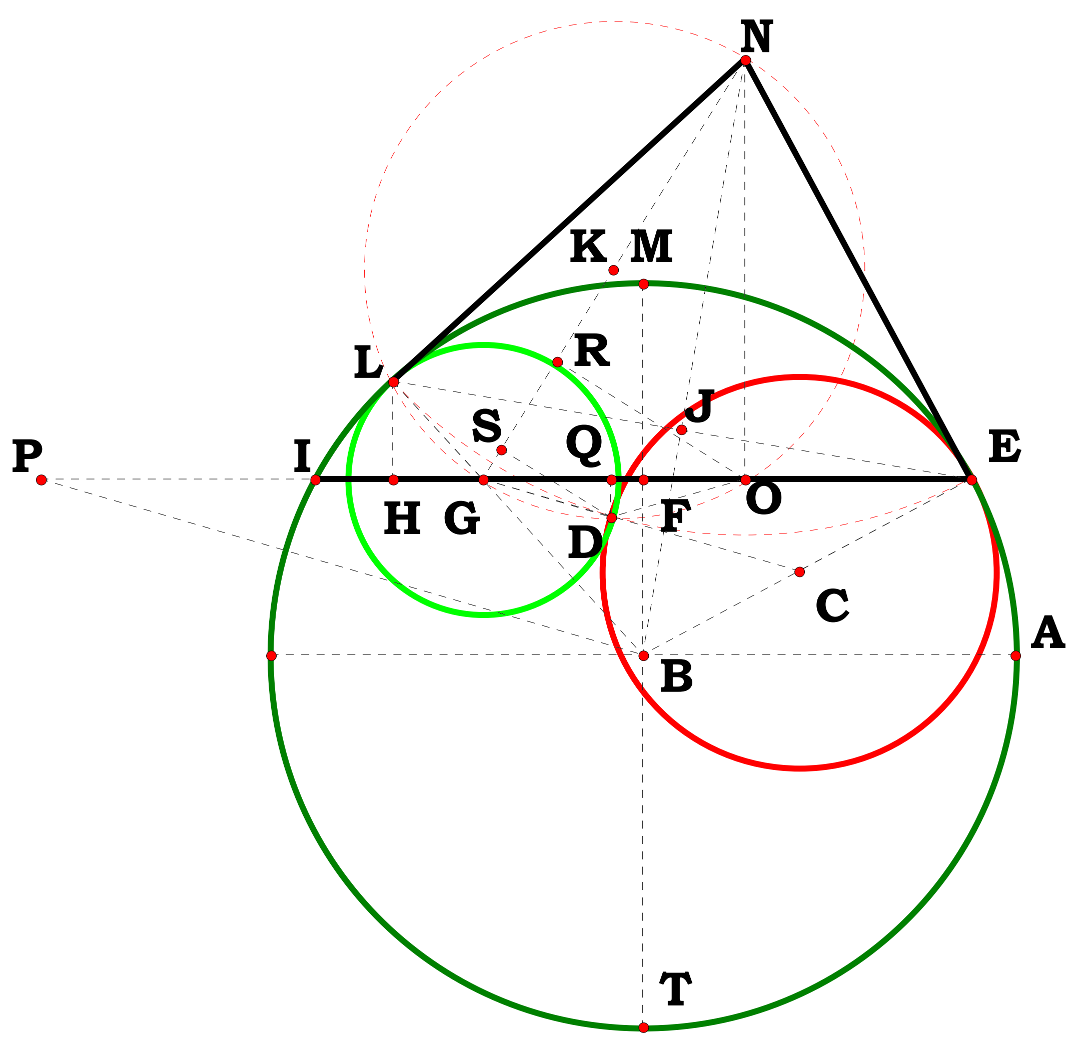
$DL := \dfrac{DH^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2} \quad LZ := \dfrac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ \quad MQ := \dfrac{CQ \cdot QZ}{DQ}$

$CM := CQ - MQ \quad GN := CM \quad DG := \sqrt{DN^2 - GN^2} \quad BJ := \dfrac{DG \cdot JP}{GN} \quad BF := BJ - FJ$

$FO := \dfrac{FP \cdot DF}{BF} \quad OP := FP - FO \quad OP = 2.915191$

042296a

Given BF as a ratio to BM and EG as a ratio to EI, what is CE?



$N_1 := 9.8425 \quad D_1 := .26381 \quad D_2 := .74461$

$MT := N_1 \quad AB := \dfrac{MT}{2} \quad BM := AB \quad BE := AB \quad BL := AB$

$MF := D_1 \cdot N_1 \quad BF := N_1 - MF - AB \quad EF := \sqrt{\sqrt{\left(BE^2 - BF^2\right)^2}}$

$EI := 2 \cdot EF \quad EG := EI \cdot D_2 \quad FG := EG - EF \quad BG := \sqrt{BF^2 + FG^2} \cdot \dfrac{FG}{\sqrt{FG^2}} \quad GL := BL - BG$

$DG := GL \quad GH := \dfrac{FG \cdot GL}{BG} \quad HL := \sqrt{GL^2 - GH^2} \quad EH := EG + GH \quad EL := \sqrt{EH^2 + HL^2}$

$JL := \dfrac{EL}{2} \quad BJ := \sqrt{BL^2 - JL^2} \quad LN := \dfrac{BL \cdot JL}{BJ} \quad GN := \sqrt{LN^2 + GL^2} \quad JN := \sqrt{LN^2 - JL^2}$

$EJ := JL \quad EN := \sqrt{JN^2 + EJ^2} \quad S_1 := EG \quad S_2 := E\, S_3 := GN \quad GO := \dfrac{S_3{}^2 + S_1{}^2 - S_2{}^2}{2 \cdot S_1}$

$NO := \sqrt{GN^2 - GO^2} \quad NR := \dfrac{NO^2}{GN} \quad GS := \dfrac{DG^2}{GN} \quad RS := GN - (NR + GS) \quad DT := RS$

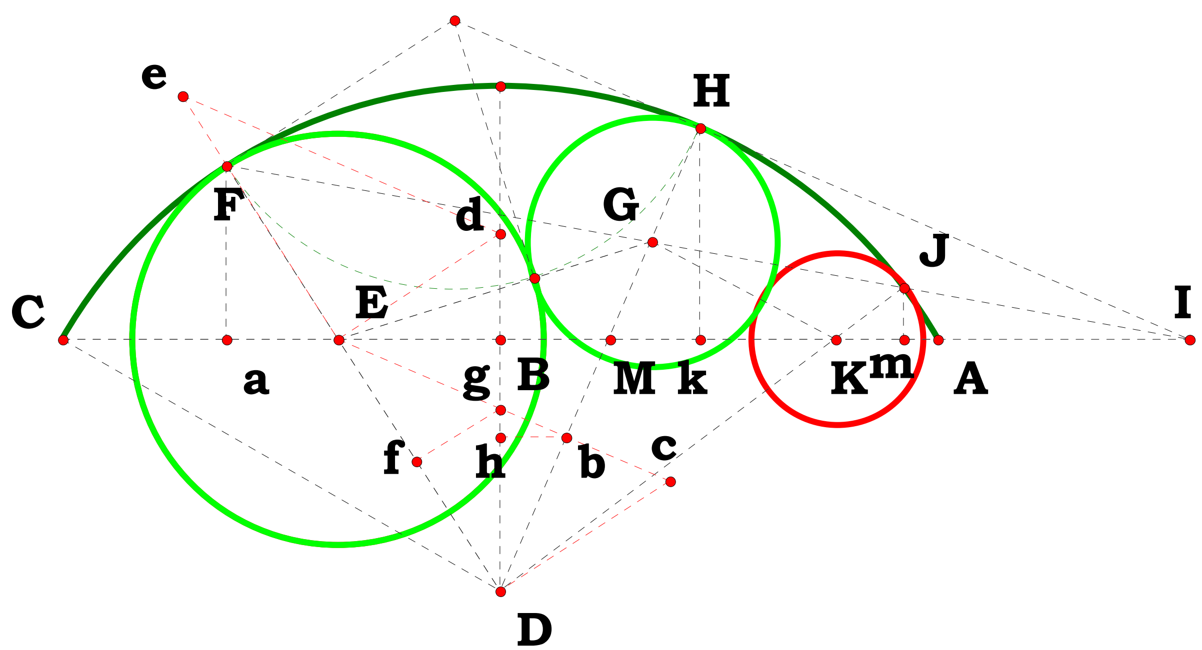
$DS := \sqrt{DG^2 - GS^2} \quad RT := DS \quad OR := \sqrt{NO^2 - NR^2} \quad OT := OR - RT \quad DO := \sqrt{DT^2 + OT^2}$

$S_1 := GO \quad S_2 := DG \quad S_3 := DO \quad OQ := \dfrac{S_3{}^2 + S_1{}^2 - S_2{}^2}{2 \cdot S_1} \quad GQ := GO - OQ$

$DQ := \sqrt{DO^2 - OQ^2} \quad FP := \dfrac{GQ \cdot BF}{DQ} \quad CE := \sqrt{\left(\dfrac{BE \cdot EG}{FP + EF}\right)^2} \quad CE = 2.583347$

042296.MCD

Place EF and GH and find JK.



$N_1 := 11.59521 \quad N_2 := 3.34617 \quad D_1 := .812152 \; D_2 := 3.19428$

$AC := N_1 \quad AB := \frac{AC}{2} \quad BC := AB \quad BD := N_2 \quad Aa := AC \cdot D_1 \quad Ba := Aa - AB$

$CD := \sqrt{BD^2 + BC^2} \quad EF := \frac{CD \cdot \left(\sqrt{CD^2 - Ba^2} - BD\right)}{\sqrt{CD^2 - Ba^2}} \quad DE := CD - EF$

$BE := \sqrt{DE^2 - BD^2} \quad AE := AB + BE \quad GH := \frac{AE - EF}{D_2} \quad EG := EF + GH$

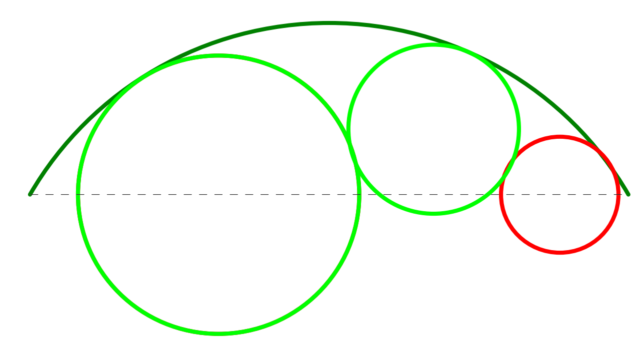
$DG := CD - GH \quad Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG} \quad Eb := \sqrt{DE^2 - Db^2} \quad Ec := \frac{DE^2}{Eb}$

$Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD} \quad Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$

$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db} \quad DM := \sqrt{BD^2 + BM^2}$

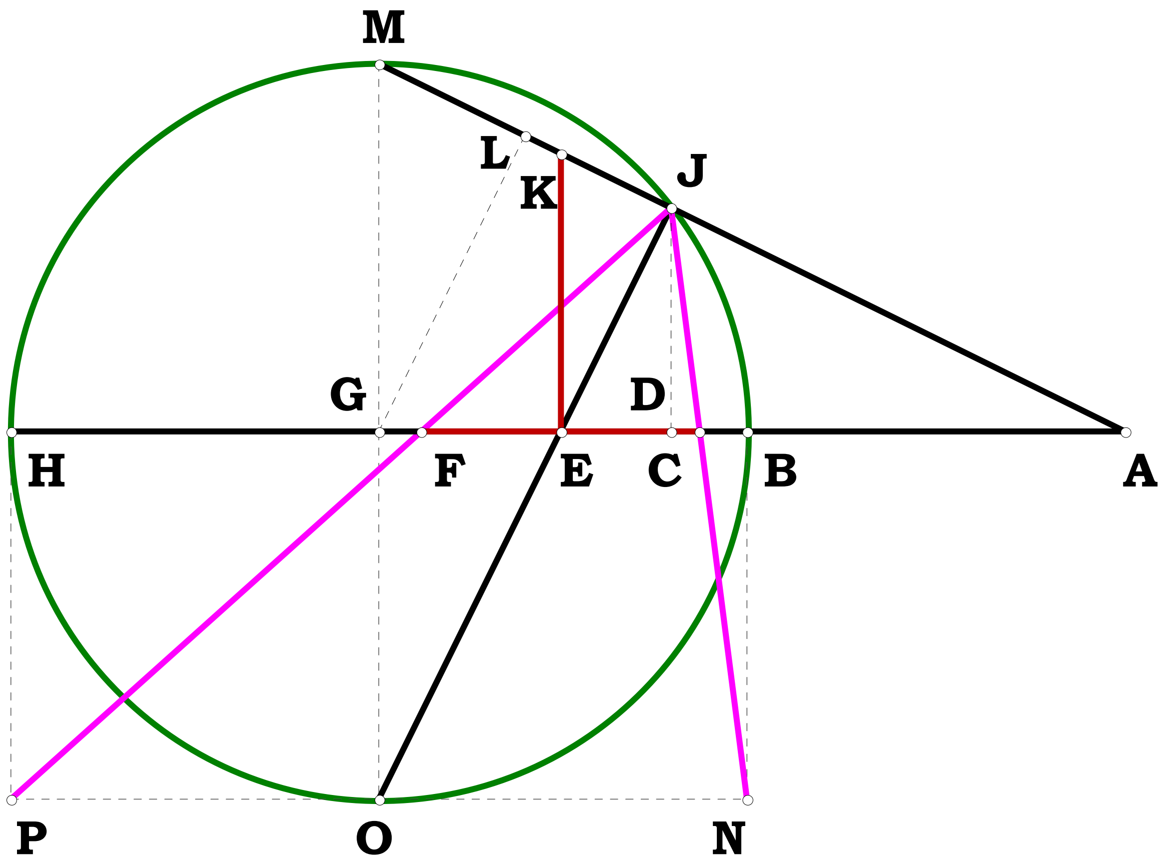
$Bk := \frac{BM \cdot CD}{DM} \quad HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM} \quad Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2} \quad Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$

$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI} \quad Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm} \quad JK = 1.126755$



042396a

Is CF always equal to EK?



$N_1 := 3$ $\quad AB := 1$

$AH := AB \cdot N_1 \quad BH := AH - AB \quad BG := \dfrac{BH}{2} \quad BN := BG \quad GO := BG \quad HP := BG$

$GM := BG \quad GH := BG \quad AG := AH - GH \quad AM := \sqrt{GM^2 + AG^2} \quad AL := \dfrac{AG^2}{AM}$

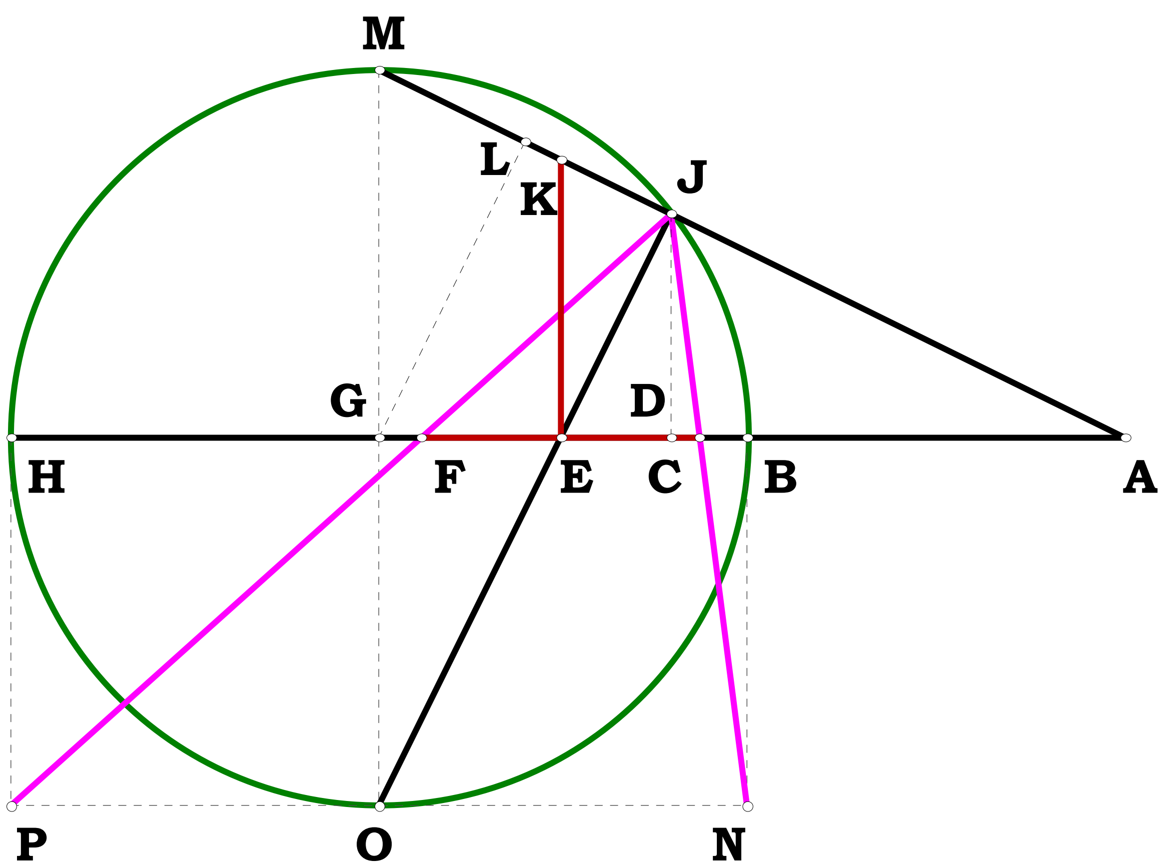
$LM := AM - AL \quad JL := LM \quad AJ := AM - (JL + LM) \quad AD := \dfrac{AG \cdot AJ}{AM}$

$BD := AD - AB \quad DH := BH - BD \quad DJ := \sqrt{BD \cdot DH} \quad BC := \dfrac{BD \cdot BN}{BN + DJ}$

$DF := \dfrac{DH \cdot DJ}{BN + DJ} \quad CD := BD - BC \quad CF := CD + DF \quad CE := \dfrac{CF}{2} \quad BE := BC + CE$

$AE := AB + BE \quad EK := \dfrac{GM \cdot AE}{AG} \quad EK - CF = 0 \quad EK = 0.75$

Definitions.



$$BH - (N_1 - 1) = 0 \qquad BG - \frac{N_1 - 1}{2} = 0 \qquad AG - \left(\frac{1}{2}\cdot N_1 + \frac{1}{2}\right) = 0$$

$$AM - \frac{1}{2}\cdot\sqrt{2\cdot {N_1}^2 + 2} = 0 \qquad AL - \frac{1}{2}\cdot\frac{(N_1 + 1)^2}{\sqrt{2\cdot {N_1}^2 + 2}} = 0 \qquad LM - \frac{1}{2}\cdot\frac{(N_1 - 1)^2}{\sqrt{2\cdot {N_1}^2 + 2}} = 0$$

$$AJ - \frac{2}{\sqrt{2\cdot {N_1}^2 + 2}}\cdot N_1 = 0 \qquad AD - (N_1 + 1)\cdot\frac{N_1}{({N_1}^2 + 1)} = 0 \qquad BD - \frac{(N_1 - 1)}{({N_1}^2 + 1)} = 0$$

$$DH - {N_1}^2\cdot\frac{(N_1 - 1)}{({N_1}^2 + 1)} = 0 \qquad DJ - \frac{(N_1 - 1)}{({N_1}^2 + 1)}\cdot N_1 = 0 \qquad BC - \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0$$

$$DF - 2\cdot {N_1}^3\cdot\frac{(N_1 - 1)}{\left[(N_1 + 1)^2\cdot({N_1}^2 + 1)\right]} = 0 \qquad CD - 2\cdot(N_1 - 1)\cdot\frac{N_1}{\left[({N_1}^2 + 1)\cdot(N_1 + 1)^2\right]} = 0$$

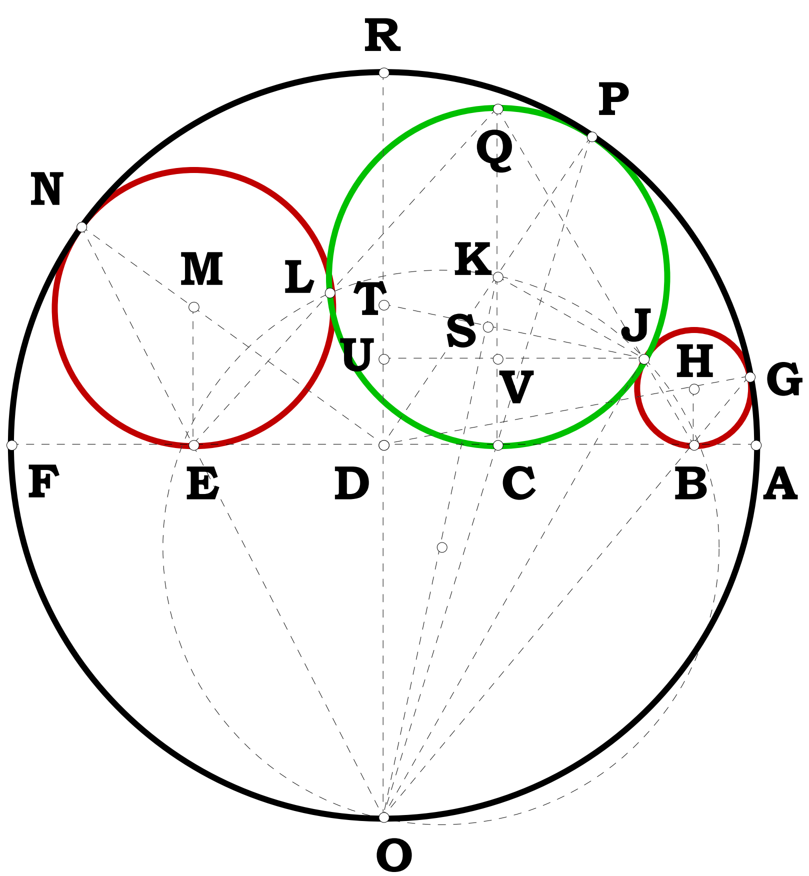
$$CF - 2\cdot N_1\cdot\frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad CE - N_1\cdot\frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad BE - \frac{(N_1 - 1)}{(N_1 + 1)} = 0$$

$$AE - 2\cdot\frac{N_1}{(N_1 + 1)} = 0 \qquad EK - 2\cdot(N_1 - 1)\cdot\frac{N_1}{(N_1 + 1)^2}$$

$$EK - CF = 0 \qquad EK = 0.75$$

Meditation : Do these equations satisfy the requirement that a definition must contain both form and matter?

Three Circles 042496



Given AC, find CK and BH.

$N_1 := 9.8425 \qquad D_1 := .36802$

$AF := N_1 \quad AD := \frac{AF}{2} \quad AC := AF \cdot D_1 \quad DO := AD \quad OR := AF$

$CD := AD - AC \quad CO := \sqrt{CD^2 + DO^2} \quad PO := \frac{DO \cdot OR}{CO} \quad CP := PO - CO \quad CK := \frac{DO \cdot CP}{PO}$

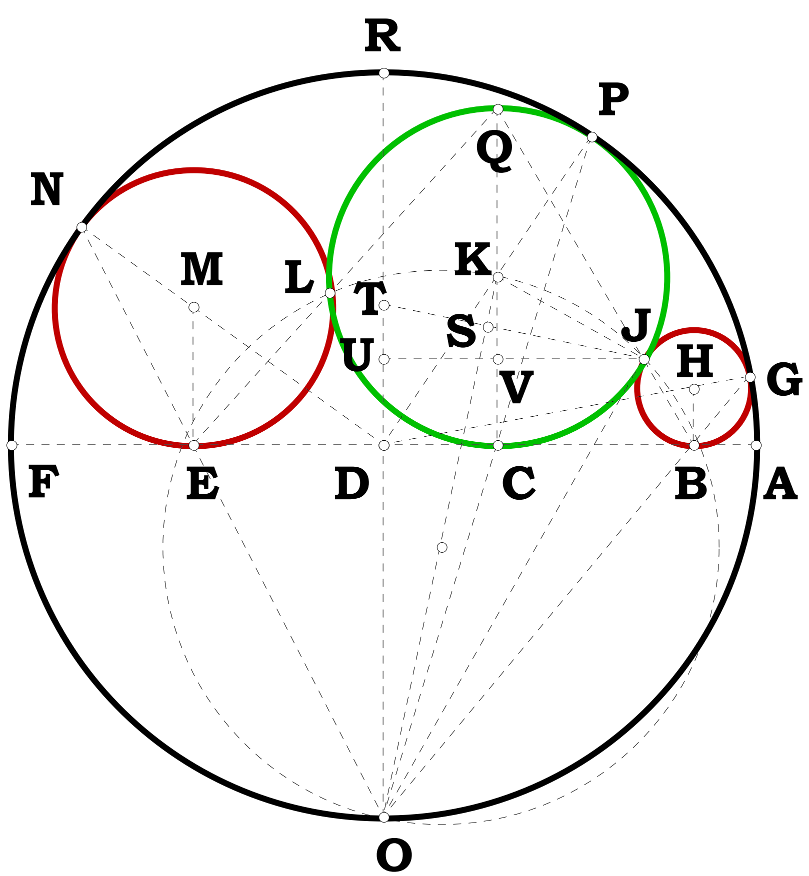
$JK := CK \quad KO := \sqrt{CD^2 + (DO + CK)^2} \quad JO := \sqrt{KO^2 - JK^2} \quad KS := \frac{JK^2}{KO} \quad SO := KO - KS$

$JS := \frac{JK \cdot SO}{JO} \quad ST := \frac{CD \cdot SO}{DO + CK} \quad JT := JS + ST \quad TO := \frac{KO \cdot ST}{CD} \quad TU := \frac{CD \cdot JT}{KO} \quad DU := TO - (DO + TU)$

$CV := DU \quad CQ := 2 \cdot CK \quad QV := CQ - CV \quad BH := \frac{CK \cdot CV}{QV} \quad BH = 0.812843$

Some Algebraic Names, or Definitions.

$$BH - N_1 \cdot \frac{D_1 \cdot (D_1 - 1) \cdot \left(2 \cdot D_1 - 2 \cdot D_1^{\,2} - 2 \cdot \sqrt{2} \cdot D_1 + \sqrt{2} - 2\right)}{2 \cdot D_1 + 6 \cdot D_1^{\,2} - 16 \cdot D_1^{\,3} + 8 \cdot D_1^{\,4} - 2 \cdot \sqrt{2} \cdot D_1 + \sqrt{2} + 2} = 0$$



$$AD-\frac{N_1}{2}=0 \quad AC-N_1\cdot D_1=0 \quad CD-\frac{N_1\cdot(1-2\cdot D_1)}{2}=0 \quad CO-\frac{N_1\cdot\sqrt{(2\cdot D_1{}^2-2\cdot D_1+1)}}{\sqrt{2}}=0$$

$$PO-\frac{N_1\cdot\sqrt{2}}{2\cdot\sqrt{2\cdot D_1{}^2-2\cdot D_1+1}}=0 \quad CP-N_1\cdot\frac{\sqrt{2}\cdot D_1\cdot(1-D_1)}{\sqrt{2\cdot D_1{}^2-2\cdot D_1+1}}=0 \quad CK-N_1\cdot\left(D_1-D_1{}^2\right)=0$$

$$KO-N_1\cdot\frac{\sqrt{(2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1)}}{\sqrt{2}}=0 \quad KS-N_1\cdot\frac{\sqrt{2}\cdot D_1{}^2\cdot(D_1-1)^2}{\sqrt{2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1}}=0$$

$$SO-N_1\cdot\frac{\sqrt{2}}{2\cdot\sqrt{2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1}}=0 \quad JS-N_1\cdot\frac{D_1\cdot(1-D_1)}{\sqrt{2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1}}=0$$

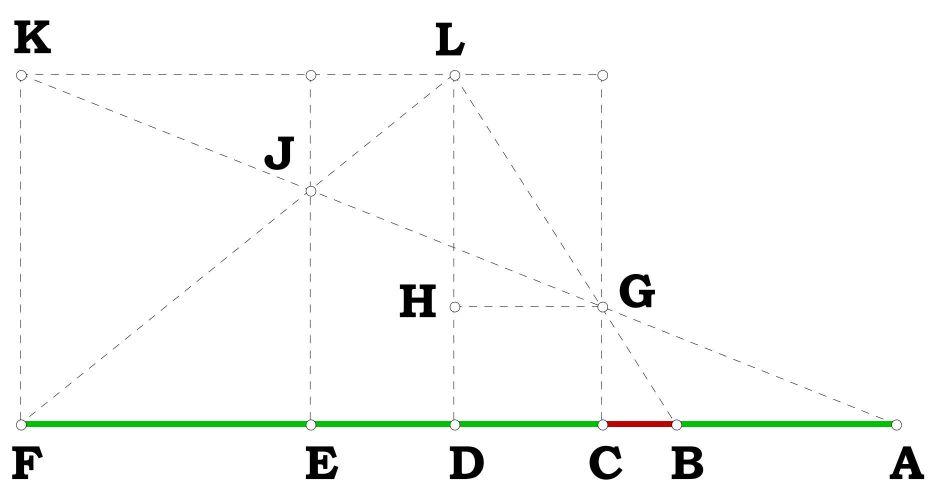
$$ST-N_1\cdot\frac{\sqrt{2}\cdot(1-2\cdot D_1)}{2\cdot\left(2\cdot D_1-2\cdot D_1{}^2+1\right)\cdot\sqrt{2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1}}=0 \quad JO-N_1\cdot\frac{1}{\sqrt{2}}=0$$

$$JT-N_1\cdot\frac{D_1+D_1{}^2-4\cdot D_1{}^3+2\cdot D_1{}^4-\sqrt{2}\cdot D_1+\frac{\sqrt{2}}{2}}{\left(2\cdot D_1-2\cdot D_1{}^2+1\right)\cdot\sqrt{2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1}}=0 \quad TO-N_1\cdot\frac{1}{2\cdot D_1-2\cdot D_1{}^2+1}=0$$

$$TU-N_1\cdot\left[\frac{1}{2\cdot D_1-2\cdot D_1{}^2+1}-\frac{\left(3\cdot\sqrt{2}-2\right)\cdot D_1{}^2-2\cdot\sqrt{2}\cdot D_1{}^3+\left(2-\sqrt{2}\right)\cdot D_1+1}{2\cdot\left(2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1\right)}\right]=0 \quad DU-N_1\cdot\left[\frac{\left(3\cdot\sqrt{2}-2\right)\cdot D_1{}^2-2\cdot\sqrt{2}\cdot D_1{}^3+\left(2-\sqrt{2}\right)\cdot D_1+1}{2\cdot\left(2\cdot D_1{}^4-4\cdot D_1{}^3+2\cdot D_1{}^2+1\right)}-\frac{1}{2}\right]=0$$

$$CQ-N_1\cdot 2\cdot\left(D_1-D_1{}^2\right)=0 \quad QV-N_1\cdot\frac{D_1\cdot(D_1-1)\cdot\left(2\cdot D_1+6\cdot D_1{}^2-16\cdot D_1{}^3+8\cdot D_1{}^4-2\cdot\sqrt{2}\cdot D_1+\sqrt{2}+2\right)}{2\cdot\left(4\cdot D_1{}^3-2\cdot D_1{}^4-2\cdot D_1{}^2-1\right)}=0$$

One Over N + One 042596



Construct 1/(N+1)

In this construction, N_2 has to be something, but it can be anything and will not change the equation.

$N_1 := 2.817 \quad N_2 := 3 \quad AC := 1$

$AF := AC \cdot N_1 \quad CF := AF - AC \quad CE := \frac{CF}{2} \quad AE := AC + CE \quad FK := N_2 \quad EJ := \frac{FK \cdot AE}{AF}$

$DL := FEF := CE \quad DF := \frac{EF \cdot DL}{EJ} \quad CG := \frac{FK \cdot AC}{AF} \quad CD := CF - DF \quad DH := CG$

$HL := DL - DH \quad BC := \frac{CD \cdot CG}{HL} \quad CF := N_1 - 1 \quad CE := \frac{1}{2} \cdot (N_1 - 1) \quad AE := \frac{1}{2} \cdot (1 + N_1)$

$FK := N_2 \quad EJ := \frac{1}{2} \cdot N_2 \cdot \frac{(1 + N_1)}{N_1} \quad DF := \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 \quad CG := \frac{N_2}{N_1} \quad CD := \frac{(N_1 - 1)}{(1 + N_1)}$

$HL := N_2 \cdot \frac{(N_1 - 1)}{N_1} \quad \frac{1}{N_1 + 1} - BC = 0$

04_26_96.MCD

Three Base Theorem.

$N_1 := .64966 \quad N_2 := 7.51417 \quad N_3 := 1.92131$

$BC := N_1 \quad CI := N_2 \quad CG := \dfrac{CI}{2} \quad BI := BC + CI$

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.

$BE := \sqrt{BC \cdot BI} \quad BM := N_3$

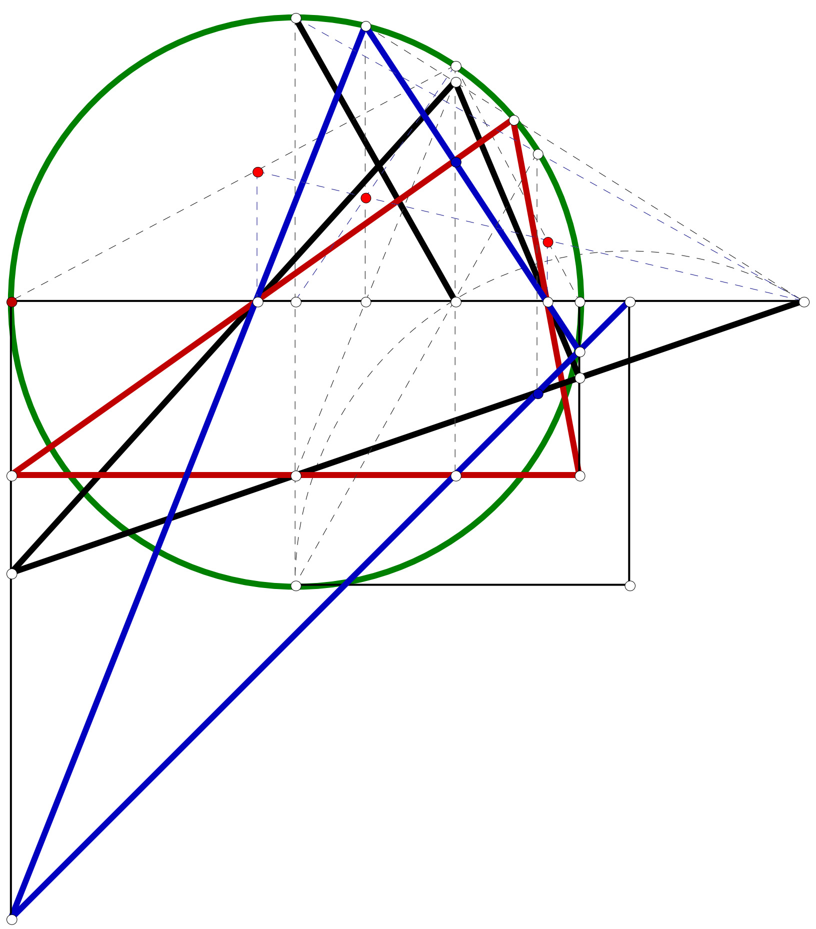
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

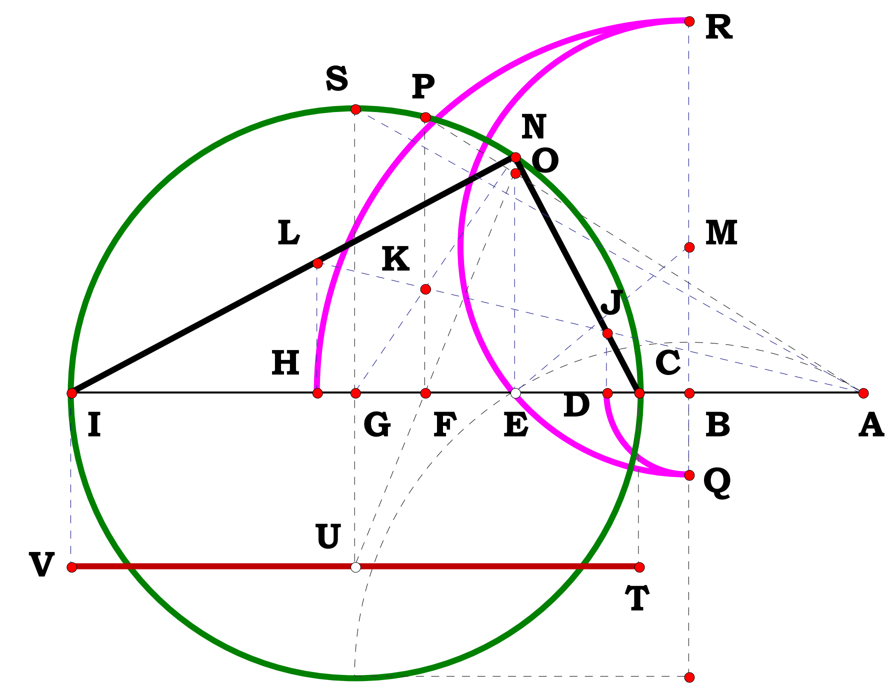
$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

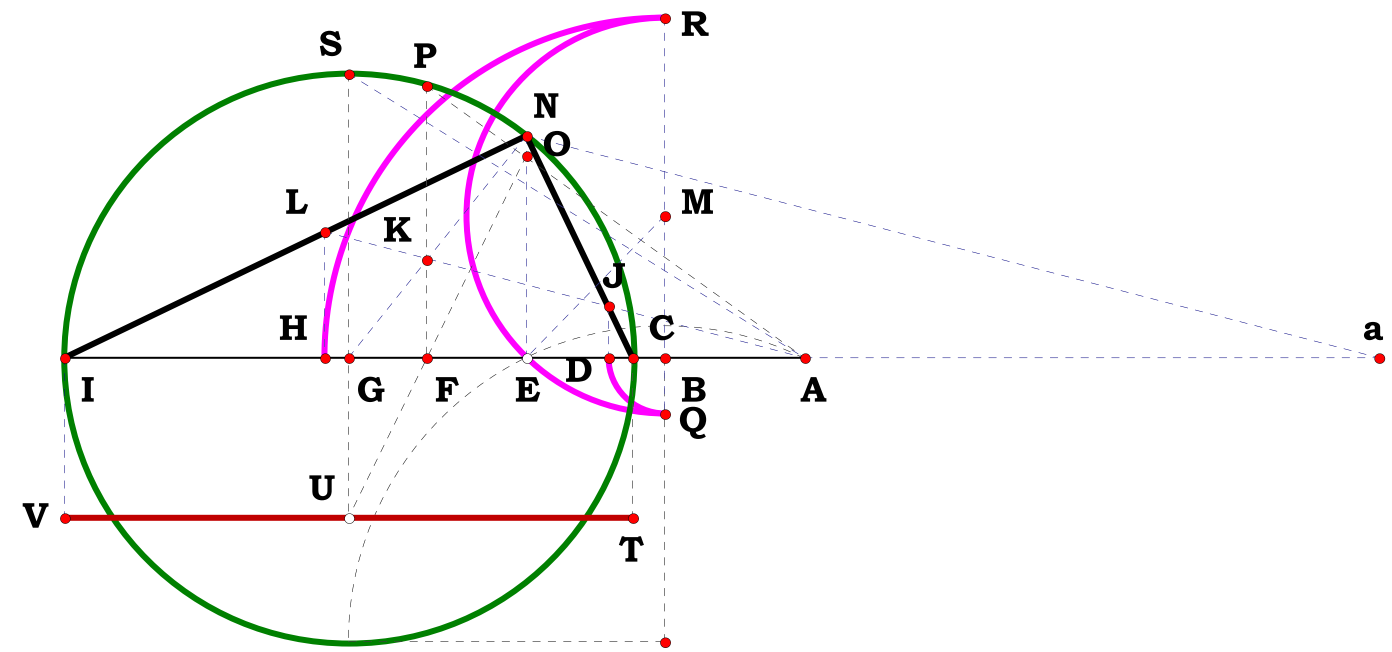
$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \dfrac{EN^2}{EG} \quad HI := EI - EH$

$HL := \dfrac{EN \cdot HI}{EI} \quad AG := AE + EG$







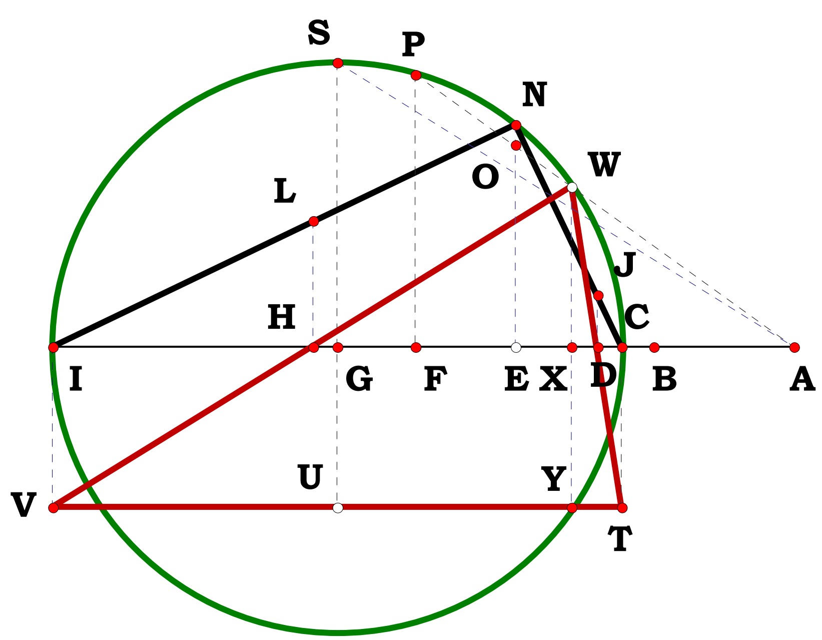
$$AH := AE + EH \qquad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \qquad CF := CG - FG$$

$$FI := CG + FG \qquad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \qquad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \qquad GU := \frac{EO \cdot FG}{EF}$$



$$AC := AE - CE \qquad AI := AC + CI \qquad AP := \sqrt{AF^2 + FP^2}$$

$$AW := \frac{AC \cdot AI}{AP} \qquad AX := \frac{AF \cdot AW}{AP} \qquad CX := AX - AC \qquad XI := CI - CX$$

$$WX := \sqrt{CX \cdot XI} \qquad XG := CG - CX \qquad YU := XG \qquad UV := CG$$

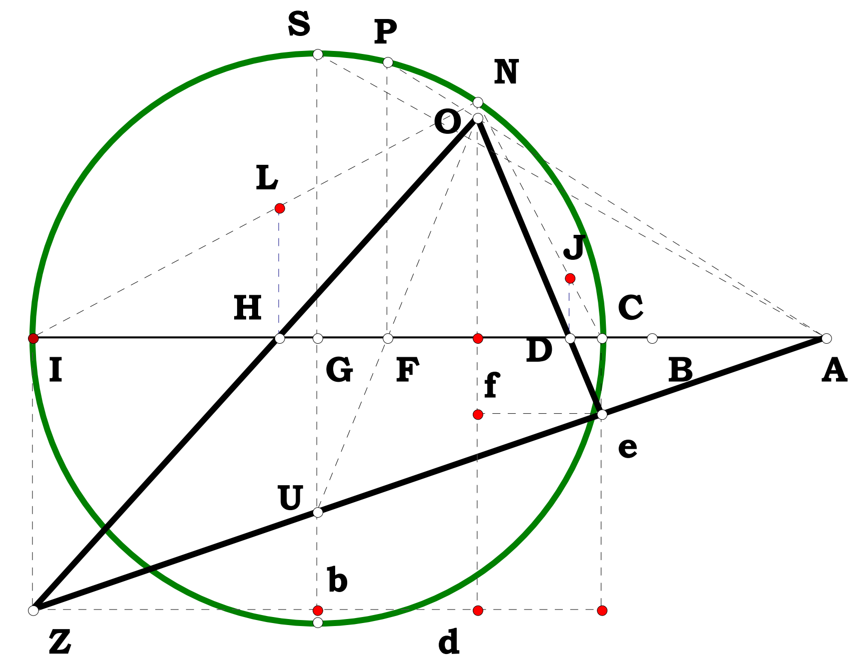
$$YV := YU + UV \qquad XH := \frac{YV \cdot WX}{WX + GU} \qquad CH := AH - AC \qquad \frac{CH}{XH + CX} = 1$$

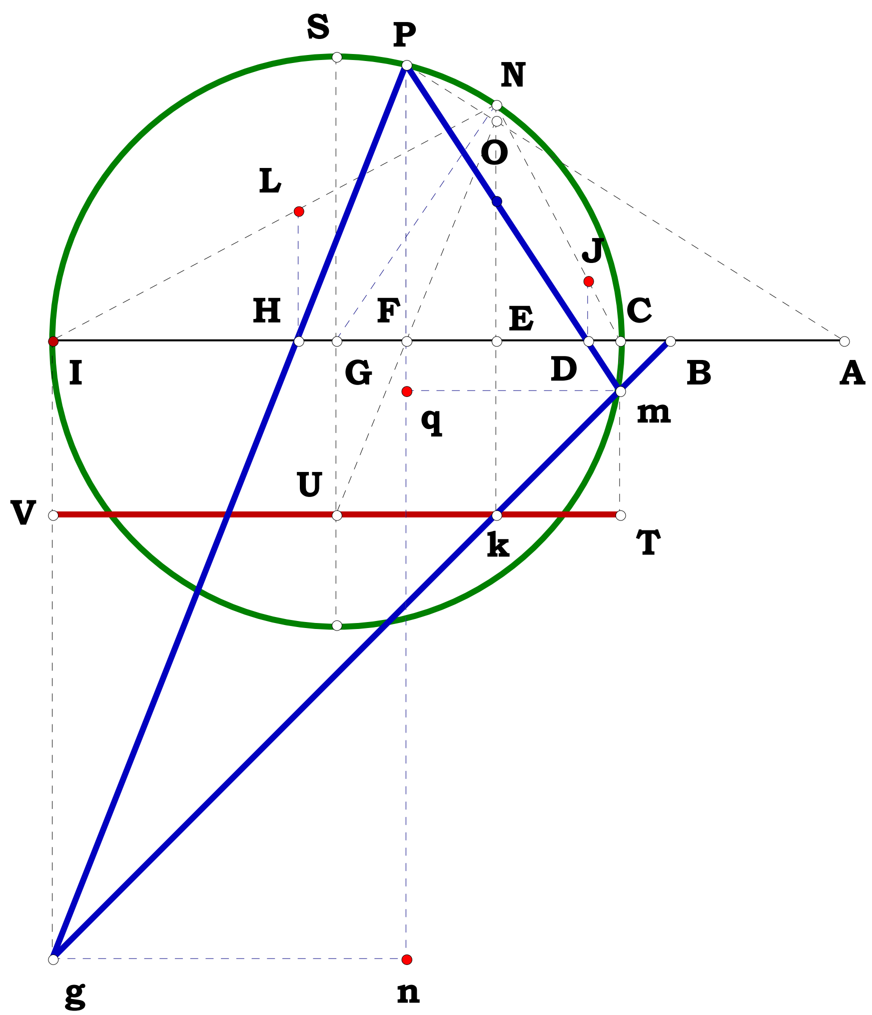
$$CD := BD - BC \qquad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$

$$\mathbf{IZ} := \frac{\mathbf{GU}\cdot\mathbf{AI}}{\mathbf{AG}} \qquad \mathbf{Ed} := \mathbf{IZ} \qquad \frac{\dfrac{\mathbf{EI}\cdot\mathbf{EO}}{\mathbf{EO+Ed}}}{\mathbf{EH}} = 1$$

$$\mathbf{Ce} := \frac{\mathbf{GU}\cdot\mathbf{AC}}{\mathbf{AG}} \qquad \mathbf{Ef} := \mathbf{Ce} \qquad \frac{\mathbf{CD}}{\dfrac{\mathbf{CE}\cdot\mathbf{Ce}}{\mathbf{EO+Ef}}} = 1$$





$$\mathbf{Ek} := \mathbf{GU} \qquad \mathbf{Ig} := \frac{\mathbf{Ek}\cdot\mathbf{BI}}{\mathbf{BE}} \qquad \mathbf{Cm} := \frac{\mathbf{Ek}\cdot\mathbf{BC}}{\mathbf{BE}}$$

$$\mathbf{Fn} := \mathbf{I}{\mid}\mathbf{gn} := \mathbf{FI} \qquad \mathbf{FH} := \frac{\mathbf{gn}\cdot\mathbf{FP}}{\mathbf{FP}+\mathbf{Fn}}$$

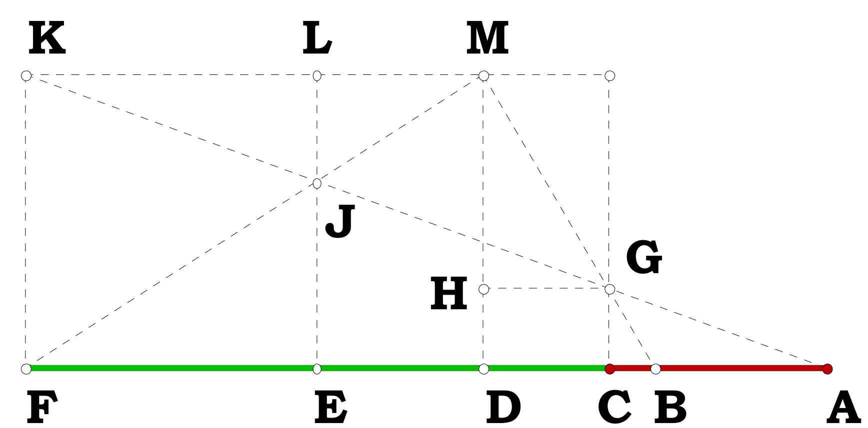
$$\frac{\mathbf{FH}}{\mathbf{AH}-\mathbf{AF}} = 1 \qquad \mathbf{DF} := \frac{\mathbf{CF}\cdot\mathbf{FP}}{\mathbf{FP}+\mathbf{Cm}}$$

$$\frac{\mathbf{CD}}{\mathbf{CF}-\mathbf{DF}} = 1$$

A Root Figure 042796

CD + BC is the square root of $\sqrt{BC \cdot BF}$. What is BC?

$N_1 := 7.72583 \quad N_2 := 1.65429 \qquad N_3 := 3.88233$



$CF := N_1 \quad CE := \dfrac{CF}{2} \quad CD := N_2 \quad FK := N_3$

$DM := FK \quad EL := FK \qquad DF := CF - CD$

$EF := \dfrac{CF}{2} \quad EJ := \dfrac{DM \cdot EF}{DF} \quad JL := EL - EJ$

$KL := EF \quad AF := \dfrac{KL \cdot FK}{JL} \quad AC := AF - CF \quad CG := \dfrac{FK \cdot AC}{AF}$

$DH := CG \quad HM := DM - DH \quad BC := \dfrac{CD \cdot DH}{HM}$

$BF := BC + CF \quad BD := BC + CD \qquad \sqrt{BC \cdot BF} - BD = 0$

Once again, FK makes the equation possible, but disappears in the result.

$$\left[\frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2}\right]^2 - \frac{{N_2}^2}{N_1 - 2 \cdot N_2} \cdot \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0$$

Definitions.

$$CF - N_1 = 0 \qquad DF - (N_1 - N_2) = 0 \qquad EF - \frac{N_1}{2} = 0 \qquad EJ - \frac{N_1 \cdot N_3}{2 \cdot (N_1 - N_2)} = 0 \qquad JL - \frac{N_3 \cdot (N_1 - 2 \cdot N_2)}{2 \cdot (N_1 - N_2)} = 0 \qquad AF - \frac{N_1 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0 \qquad AC - \frac{N_1 \cdot N_2}{N_1 - 2 \cdot N_2} = 0$$

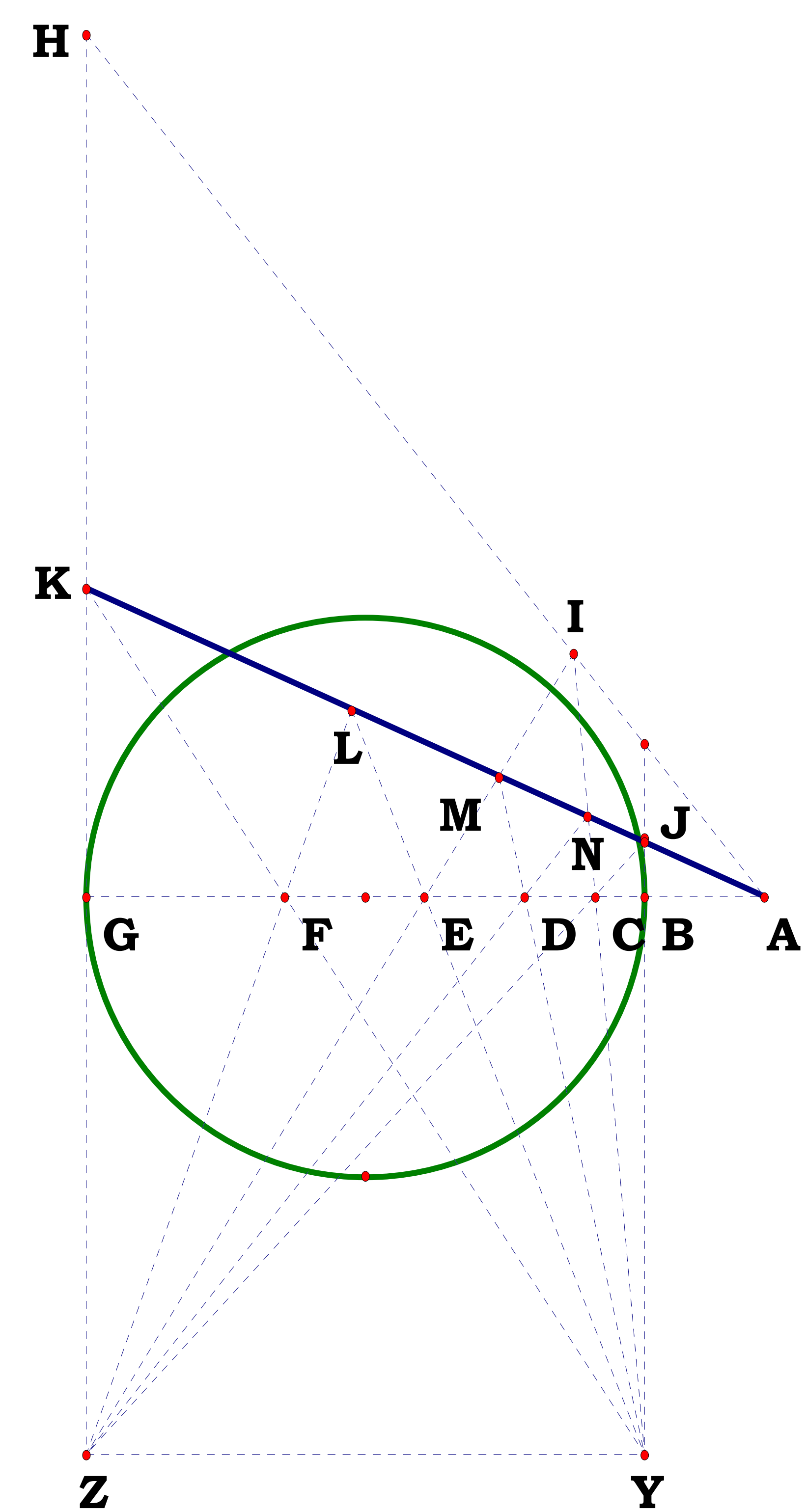
$$CG - \frac{N_2 \cdot N_3}{N_1 - N_2} = 0 \qquad HM - \frac{N_3 \cdot (N_1 - 2 \cdot N_2)}{N_1 - N_2} = 0 \qquad BC - \frac{{N_2}^2}{N_1 - 2 \cdot N_2} = 0 \qquad BF - \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0 \qquad BD - \frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0$$

042896.MCD

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use a 5^{th} root series for an example.



$N_1 := 2 \qquad N_2 := 6$

$AB := N_1 \quad AG := N_2 \quad AE := \left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$$GH := \frac{BY \cdot EG}{BE}$$

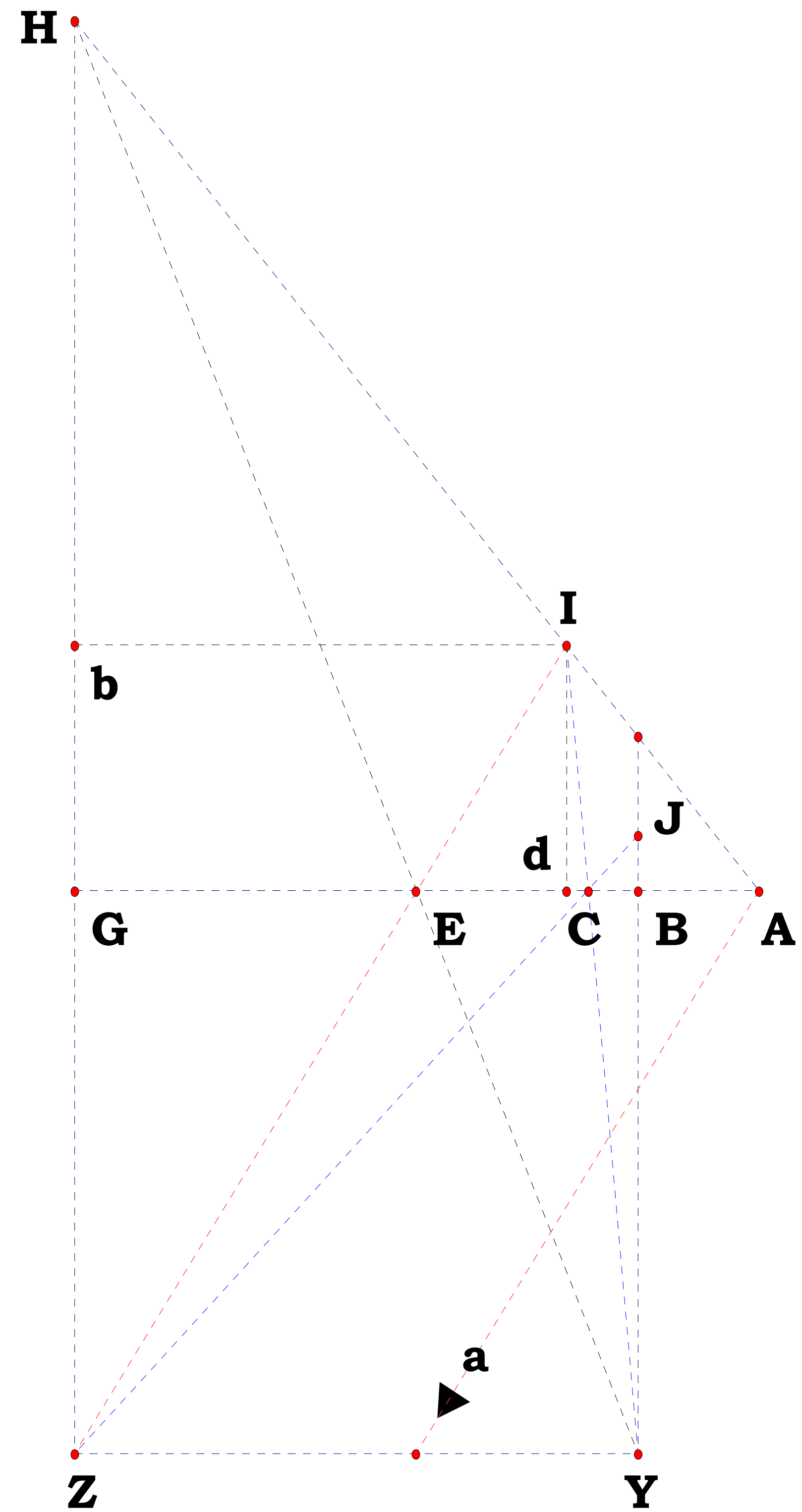
$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

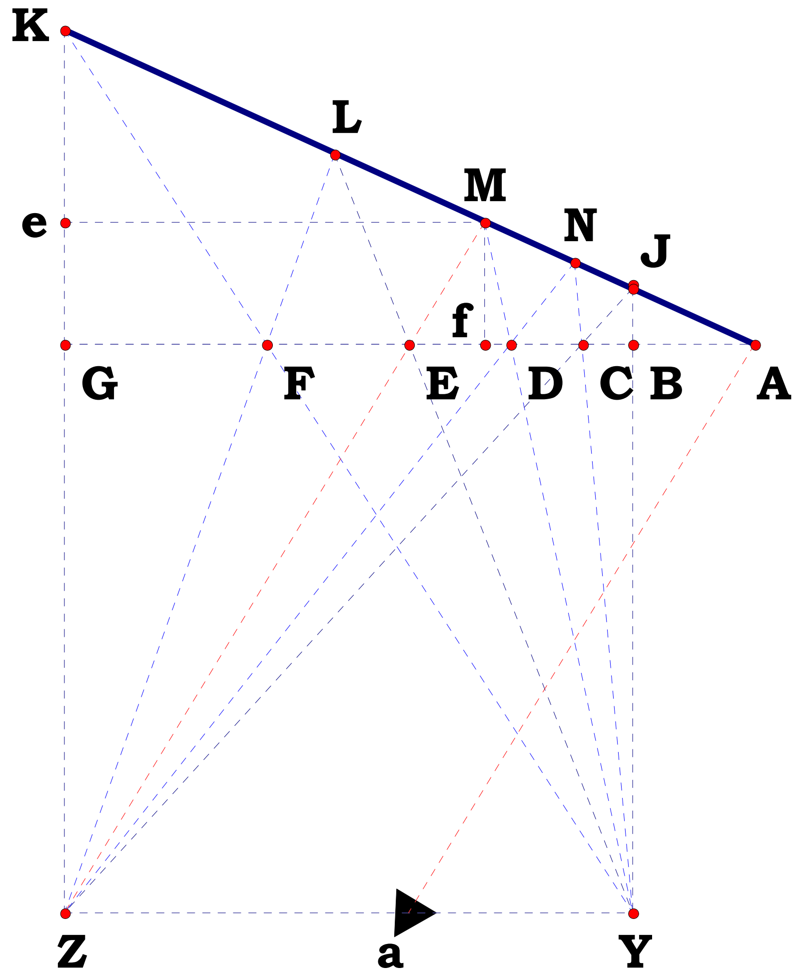
$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$





$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

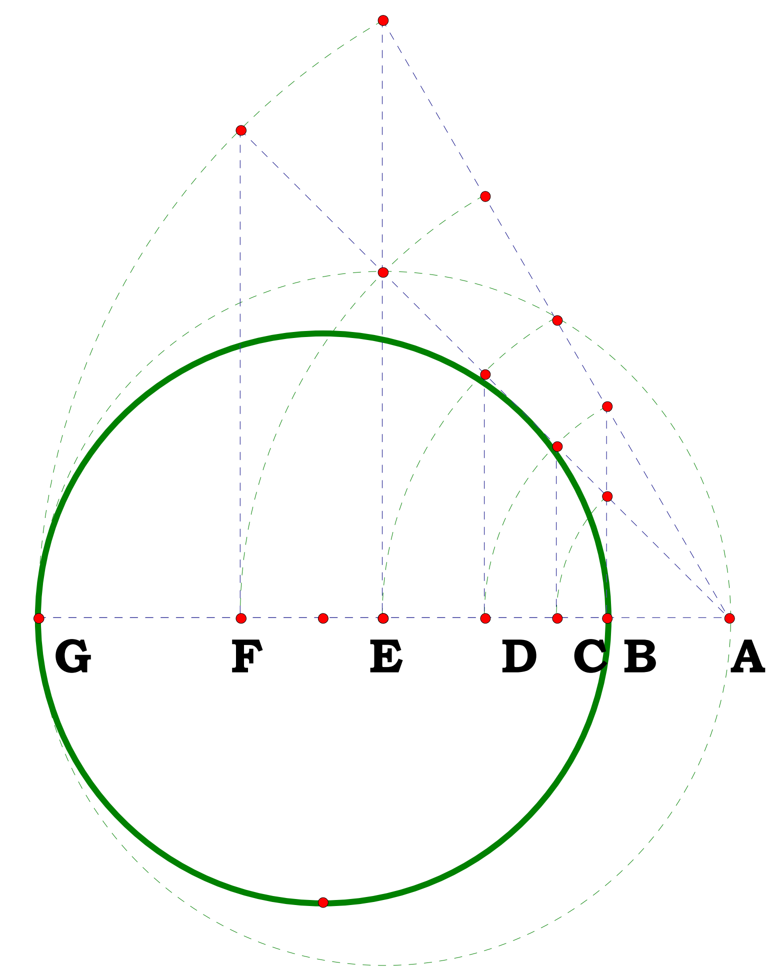
$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$

$$\frac{\left(AB^{5} \cdot AG^{0}\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^{4} \cdot AG^{1}\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^{3} \cdot AG^{2}\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^{2} \cdot AG^{3}\right)^{\frac{1}{5}}}{AE} = 1$$

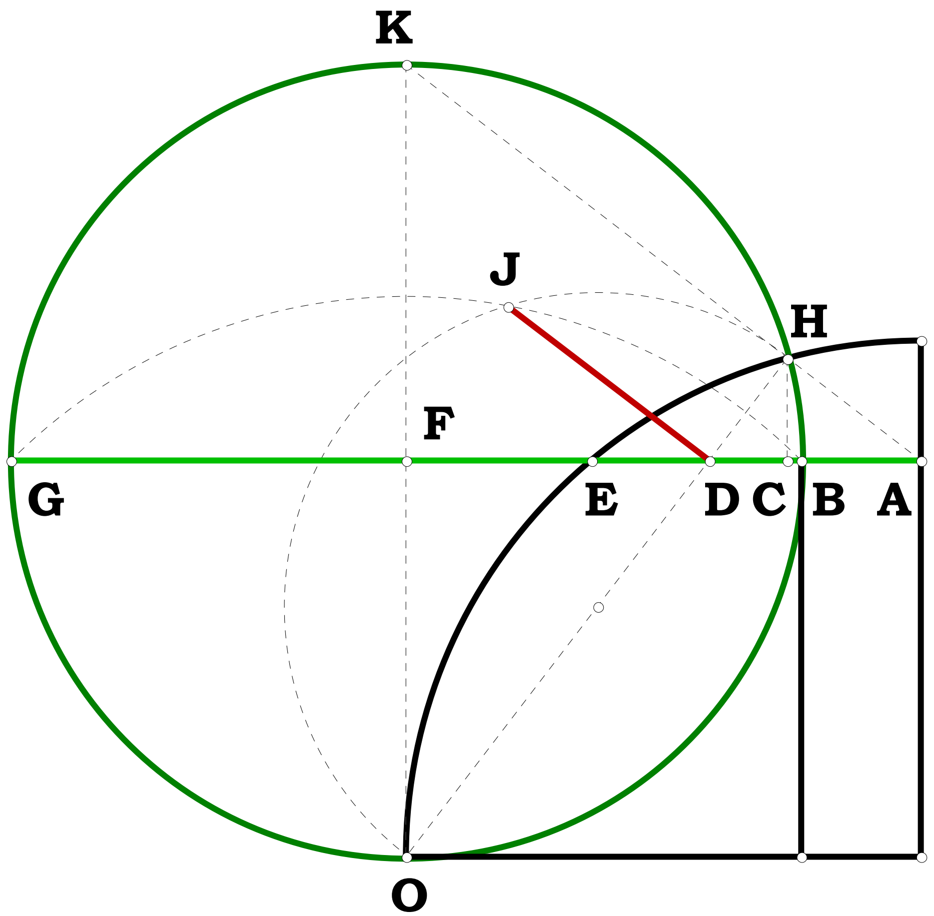
$$\frac{\left(AB^{1} \cdot AG^{4}\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^{0} \cdot AG^{5}\right)^{\frac{1}{5}}}{AG} = 1$$



If any of a prime root series can be given exactly, every root of the series can be determined exactly.

What is DJ? 042996

DJ is the Geometric name, what is its Algebraic name?



$$N := 5 \quad AB := 1 \quad AG := AB \cdot N \quad BG := AG - AB$$

$$BF := \frac{BG}{2} \quad FK := FO := BF \quad AF := BF + AB$$

$$DF := \frac{FK \cdot FO}{AF} \quad AK := \sqrt{AF^2 + FK^2} \quad KO := BG$$

$$HO := \frac{AF \cdot KO}{AK} \quad DO := \frac{AK \cdot FO}{AF} \quad DH := HO - DO$$

$$DJ := \sqrt{DH \cdot DO}$$

$$AG := N \quad BG := N - 1 \quad BF := \frac{N-1}{2} \quad AF := \frac{1}{2} \cdot N + \frac{1}{2}$$

$$DF := \frac{1}{2} \cdot \frac{(N-1)^2}{(N+1)} \quad AK := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$$

$$HO := (N+1) \cdot \frac{(N-1)}{\sqrt{2 \cdot N^2 + 2}} \quad DO := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2} \cdot \frac{(N-1)}{(N+1)}$$

$$DH := 2 \cdot N \cdot \frac{(N-1)}{\left[\sqrt{2 \cdot N^2 + 2} \cdot (N+1)\right]}$$

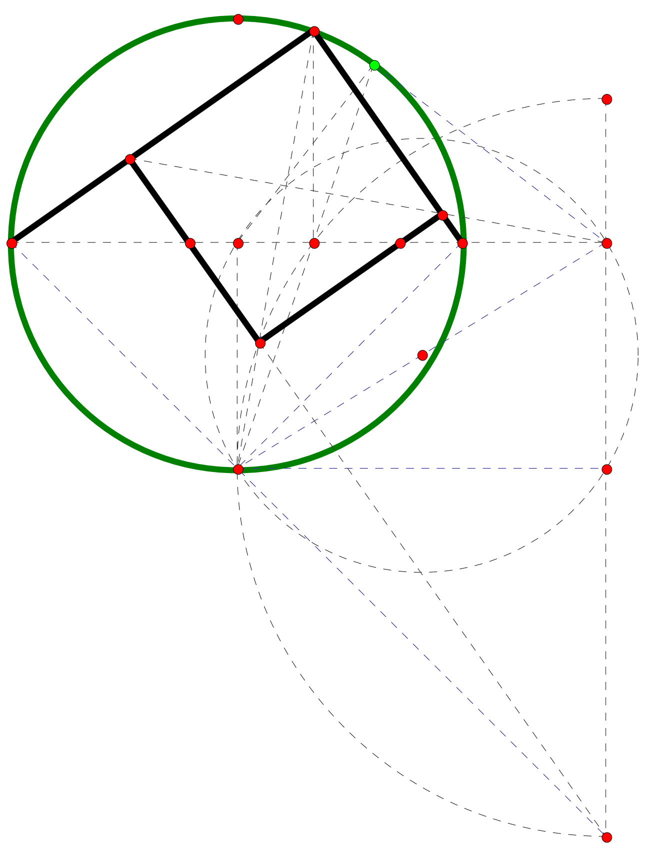
$$DJ := \sqrt{N \cdot \frac{(N-1)}{(N+1)}}$$

043096.MCD

Geometric Exponential Series of the form

$$\frac{\sum_{\delta} N^{\frac{Root-\delta}{Root}}}{N^{\frac{Root-1}{Root}}}, \quad \sum_{\delta} N^{\frac{Root-\delta}{Root}} \quad \text{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 010896 and 011696 for the sides of the right triangle.



$N = 4 \quad Root = 4 \qquad M = 1 \qquad BG := AB := M$

$AG := AB + BG \qquad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \qquad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

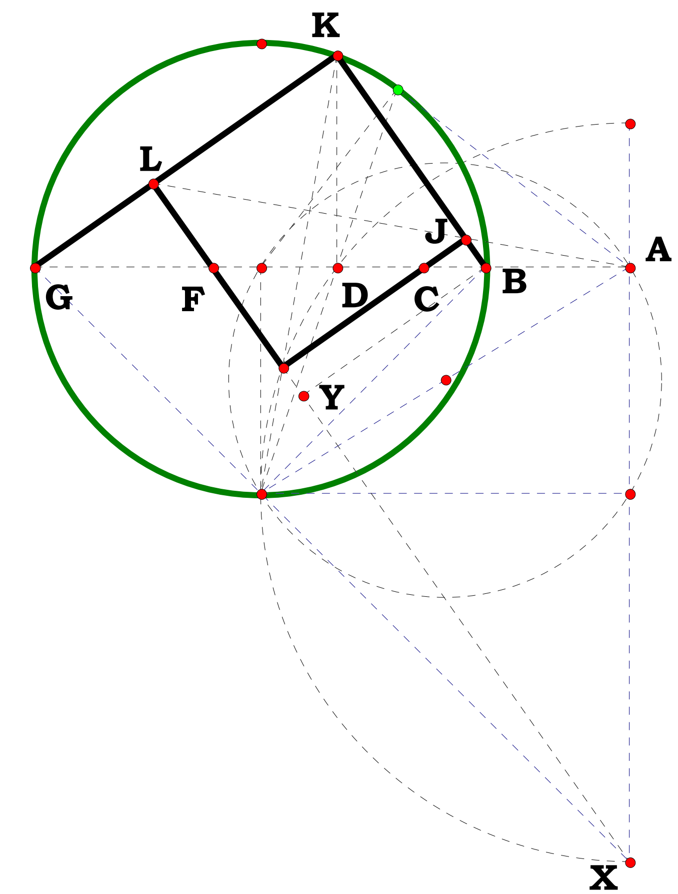
$BC := AC - AB \qquad FG := AG - AF \qquad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \qquad BD := \frac{FY \cdot BG}{FX} \qquad AD := BD + AB$

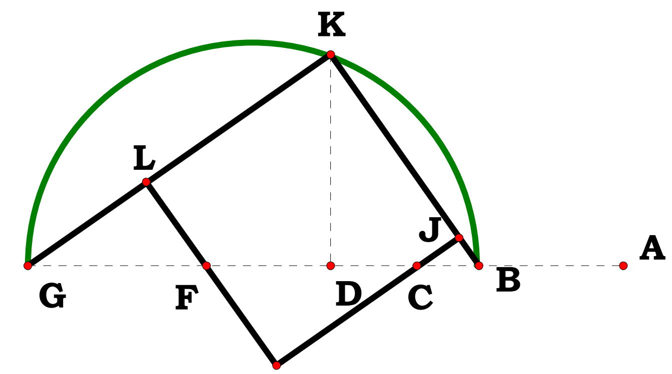
$DG := AG - AD \qquad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \qquad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \qquad GL := \frac{GK \cdot FG}{BG}$



Plug in BG here as N. AB as M. Plug in root series also.



$N \equiv 4 \quad Root \equiv 4 \quad \delta := 1 .. Root \quad M \equiv 1$

$GL = 1.376805 \quad BJ = 0.275361 \quad \frac{GL}{BJ} = 5 \quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root-1}{Root}}} = 2.415024 \quad \frac{GK}{GL} = 2.415024 \quad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}} = 8.075118 \quad \frac{BK}{BJ} = 8.075118$$

$$BM := \frac{BD \cdot BC}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$$

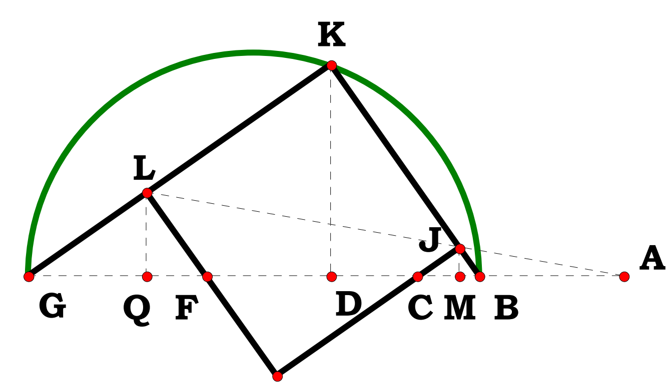
$\frac{AG}{FQ} = 9.768976$

$\frac{AG}{BM} = 32.66454$

On the left is the first and last of the series, on the right is the entire series.

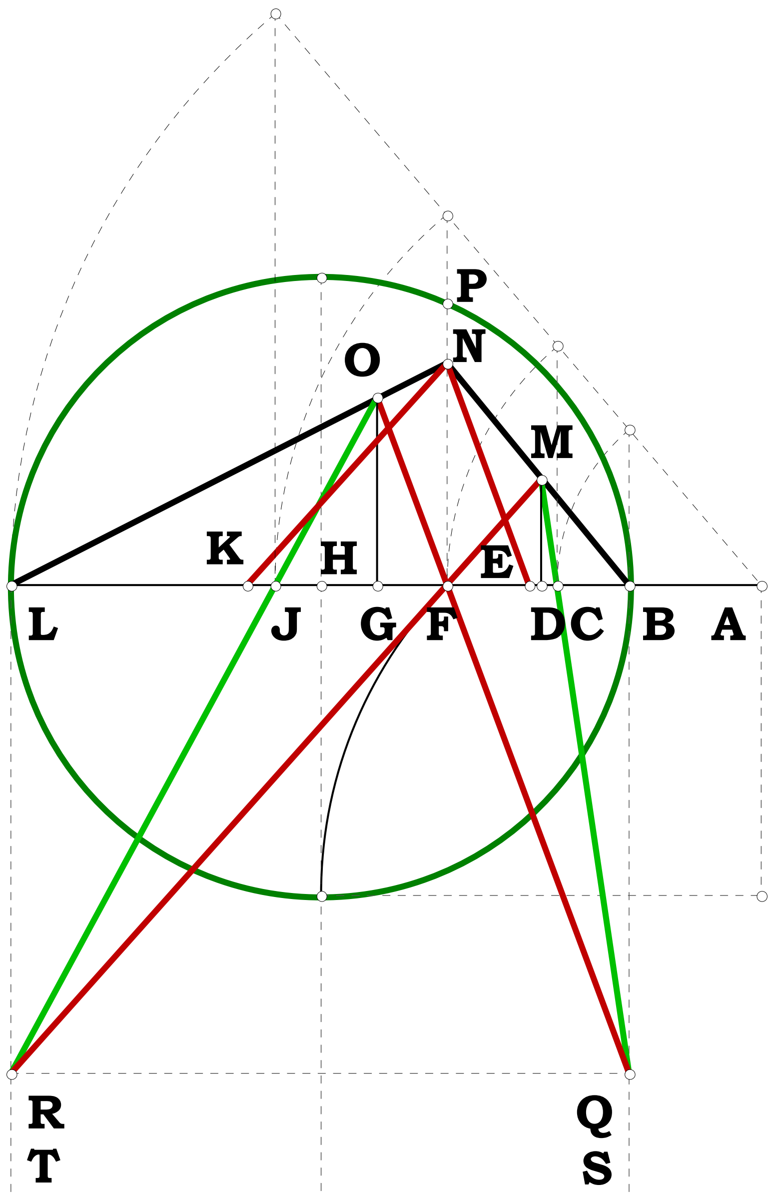
$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}} =$$

9.768976
14.608026
21.844094
32.66454



122096 Alternate Method Quad Roots

If FN:FP as BQ:BS then quad roots series can be divided off in the figure.



$N_1 := 2 \quad N_2 := .2$

$AB := 1 \quad AL := AB \cdot N_1$

$BL := AL - AB \quad BS := BL \quad LT := BL$

$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$

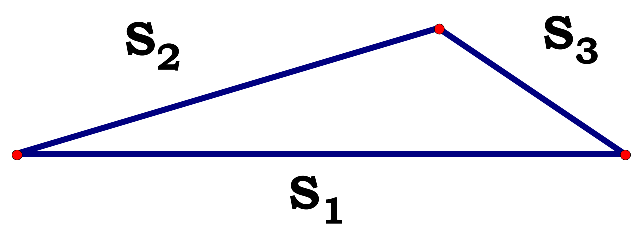
$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL$

$$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$$

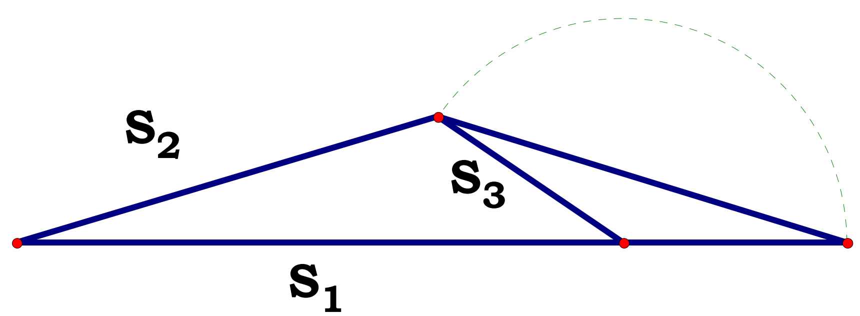
04_03_97.MCD

Not changing the height of a given triangle, or the length of the subtended side, what happens to it's area if we halve the angle of one side?



$$n := 1..3$$

$$S_1 := \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad S_2 := \begin{pmatrix} b \\ c \\ a \end{pmatrix} \quad S_3 := \begin{pmatrix} c \\ a \\ b \end{pmatrix}$$



$$Is\_This\_a\_Triangle := \left(S_{1_1} + S_{2_1} > S_{3_1}\right)\cdot\left(S_{1_1} + S_{3_1} > S_{2_1}\right)\cdot\left(S_{2_1} + S_{3_1} > S_{1_1}\right)$$

As was learned in school, the area of a triagle is given by $\frac{1}{2}\cdot B\cdot H.$

From 04_02_97.MCD I show that, for a given side, the height is given by;

$$H_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}}\cdot\sqrt{-S_{1_n} + S_{2_n} + S_{3_n}}\cdot\sqrt{S_{1_n} - S_{2_n} + S_{3_n}}\cdot\sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2\cdot S_{1_n}}$$

And since $B := S_1$

Area is defined as

$$A_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}}\cdot\sqrt{-S_{1_n} + S_{2_n} + S_{3_n}}\cdot\sqrt{S_{1_n} - S_{2_n} + S_{3_n}}\cdot\sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{4}$$

$$\frac{1\cdot B_n\cdot H_n}{2} - A_n =$$

0
0
0

$A_n =$

2.904738
2.904738
2.904738

$H_n =$

2.904738
1.936492
1.452369

What is the definition of acute, solely in terms of the sides of a triangle? Basically from this it can be argued that Euclid's definition of acute or obtuse was out of order.

$$\mathbf{Acute_n} := \mathbf{if}\left[\sqrt{\left(\mathbf{S_1}_{\mathbf{n}}\right)^2+\left(\mathbf{S_2}_{\mathbf{n}}\right)^2} > \mathbf{S_3}_{\mathbf{n}}, \mathbf{1}, \mathbf{0}\right] \qquad \mathbf{Acute2_n} := \mathbf{if}\left[\sqrt{\left(\mathbf{S_1}_{\mathbf{n}}\right)^2+\left(\mathbf{S_3}_{\mathbf{n}}\right)^2} > \mathbf{S_2}_{\mathbf{n}}, \mathbf{1}, \mathbf{0}\right]$$

$\mathbf{Acute_n} =$

0
1
1

$\mathbf{Acute2_n} =$

1
0
1

$\mathbf{H_n} =$

2.904738
1.936492
1.452369

$\mathbf{S_1}_{\mathbf{n}} =$

2
3
4

$\mathbf{S_2}_{\mathbf{n}} =$

3
4
2

$\mathbf{S_3}_{\mathbf{n}} =$

4
2
3

if we halve $\angle\, \mathbf{S_1S_2}$

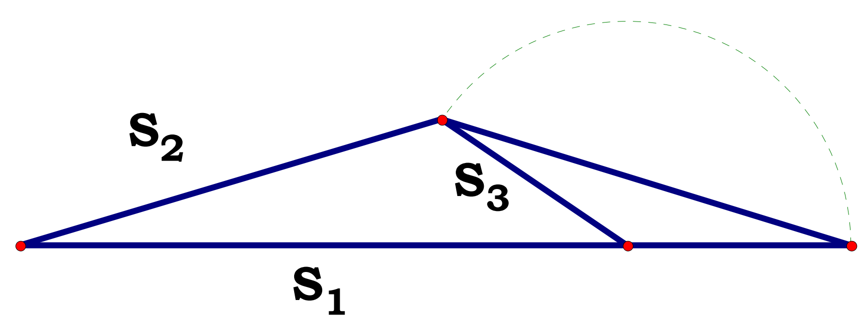
$$\frac{\left(\mathbf{S_1}_{\mathbf{n}}+\mathbf{S_2}_{\mathbf{n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n} =$$

4.357106
3.872983
1.452369

if we halve $\angle \mathbf{S_1S_3}$

$$\frac{\left(\mathbf{S_1}_{\mathbf{n}}+\mathbf{S_3}_{\mathbf{n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n} =$$

5.809475
1.936492
2.178553



Is_This_a_Triangle = 1

Since the greater angle is subtended by the greater side, halving the lesser angle increases the area of the triangle by the greater amount.

$$\left(\mathbf{S_2}_{\mathbf{n}} > \mathbf{S_3}_{\mathbf{n}}\right) - \left[\frac{\left(\mathbf{B_n}+\mathbf{S_2}_{\mathbf{n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n} > \frac{\left(\mathbf{B_n}+\mathbf{S_3}_{\mathbf{n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n}\right] =$$

0
0
0

Given two sides of a triangle, the height and if the angle contained by the two sides is acute or not, find the remaining side. What would happen if you were given just the equation and had no idea what the equation represented? You could not possibly solve it so quickly.

$$H_n = \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2 \cdot S_{1_n}}$$

Given S_1, S_2 and $\sqrt{S_1{}^2 + S_2{}^2} > S_3$, find S_3.

$Acute_n =$

0
1
1

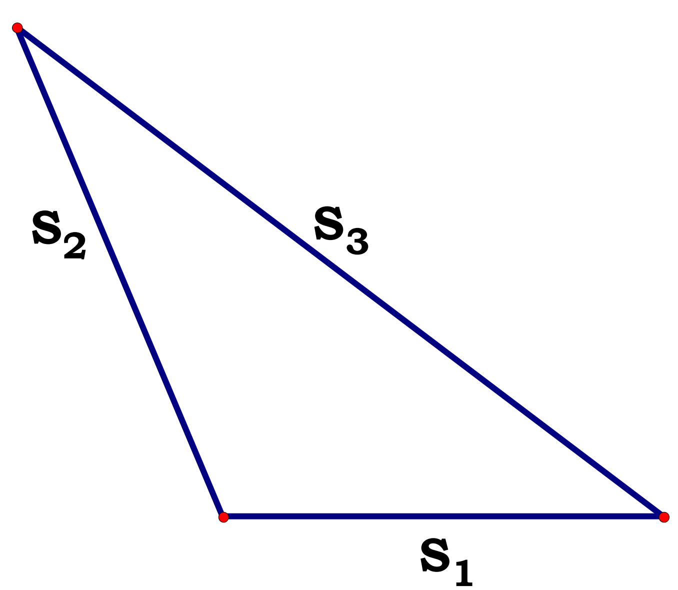
$Is\_This\_a\_Triangle = 1$

$$S_{4_n} := \sqrt{\left(S_{2_n}\right)^2 - \left(H_n\right)^2}$$

$$S_{x_n} := \text{if}\left(Acute_n, S_{1_n} - S_{4_n}, S_{1_n} + S_{4_n}\right)$$

$a \equiv 2 \quad b \equiv 3 \quad c \equiv 4$ ← Plug your values in here.

$$S_{3_n} := \sqrt{\left(H_n\right)^2 + \left(S_{x_n}\right)^2}$$



$S_{1_n} =$

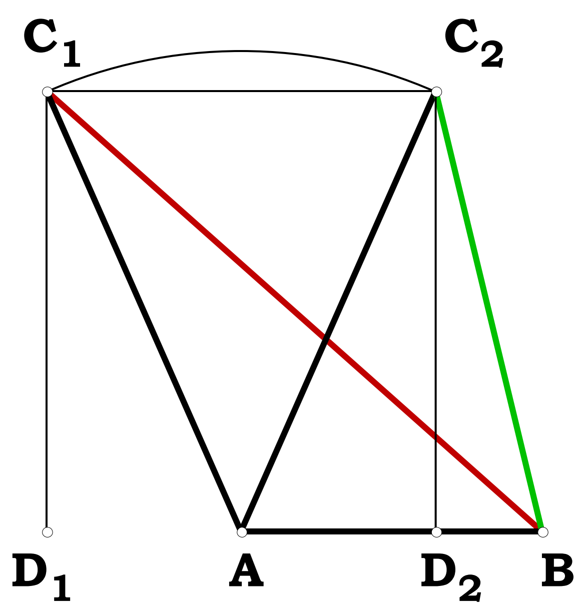
2
3
4

$S_{2_n} =$

3
4
2

$S_{3_n} =$

4
2
3



040497 Triangles

Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.

$$AB := 5 \qquad AC := 4 \qquad CD := 3$$

$$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$$

$$BC_1 := \sqrt{CD^2 + BD_1{}^2} \qquad BC_2 := \sqrt{CD^2 + BD_2{}^2}$$

$$BC_1 = 8.213252 \qquad BC_2 = 3.813461$$

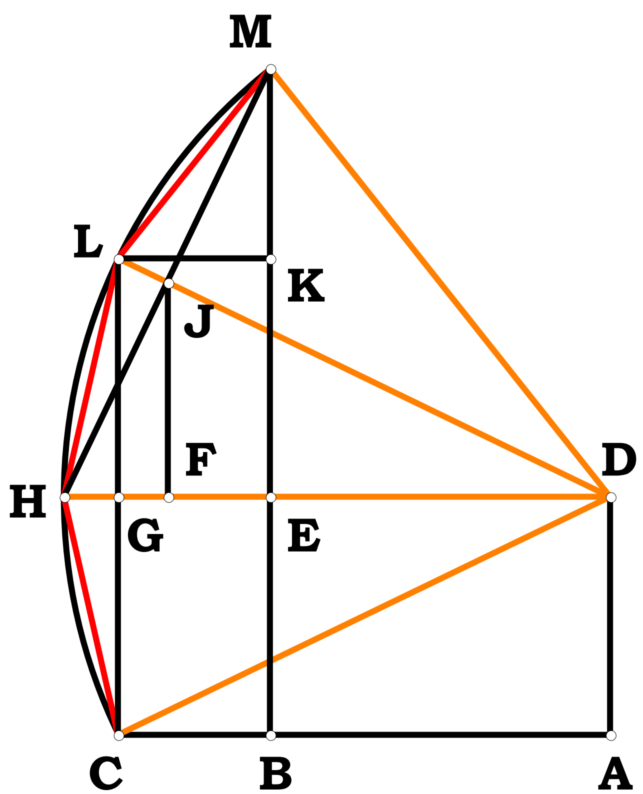
$$S_1 := AB \quad S_2 := AC \quad S_3 := BC_1$$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

$$S_1 := AB \quad S_2 := AC \quad S_3 := BC_2$$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

042897



$N_1 := 3.14854 \qquad N_2 := 6.50875 \qquad AD := N_1 \qquad AC := N_2$

$CD := \sqrt{AD^2 + AC^2} \qquad DH := CD$

$CG := AD \qquad DG := AC \qquad GH := DH - DG \qquad CH := \sqrt{GH^2 + CG^2}$

$HJ := CG \qquad DJ := DG$

$FH := \dfrac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH} \qquad EF := FH \qquad DE := DH - (EF + FH)$

$AB := DE \qquad EG := DG - DE \qquad LM := CH \qquad LK := EG$

$KM := \sqrt{LM^2 - LK^2} \qquad BE := AD \qquad BK := 2 \cdot BE \qquad BM := BK + KM$

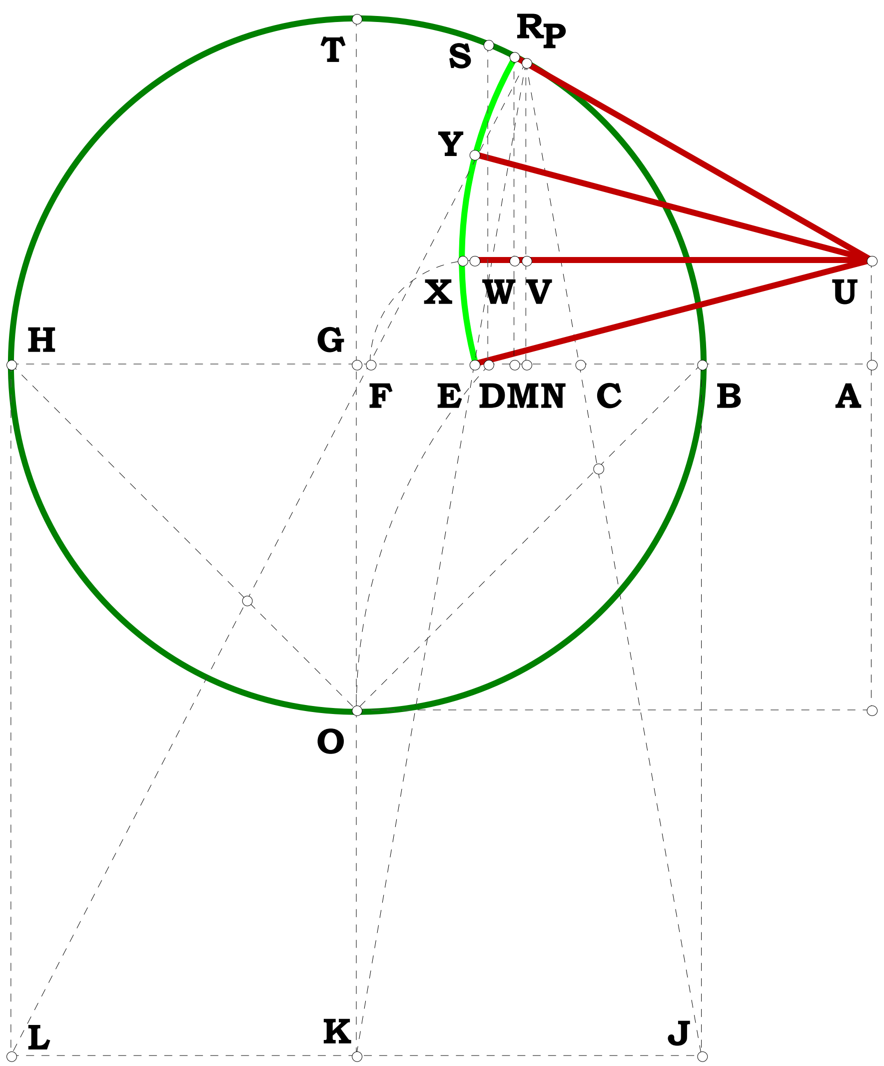
Some definitions:

$\sqrt{N_1^2 + N_2^2} - CD = 0 \qquad \sqrt{N_1^2 + N_2^2} - N_2 - GH = 0 \qquad \sqrt{\left[2 \cdot \left(N_1^2 + N_2^2 - N_2 \cdot \sqrt{N_1^2 + N_2^2}\right)\right]} - CH = 0 \qquad \dfrac{N_1^2}{\sqrt{N_1^2 + N_2^2}} - FH = 0 \qquad \dfrac{\left(N_2^2 - N_1^2\right)}{\sqrt{N_1^2 + N_2^2}} - DE = 0$

$$2 \cdot N_1 + \frac{\sqrt{2 \cdot N_2^3 \cdot \sqrt{N_1^2 + N_2^2} + N_1^4 + 5 \cdot N_1^2 \cdot N_2^2 - 2 \cdot N_2 \cdot \left(N_1^2 + N_2^2\right)^{\frac{3}{2}} - 2 \cdot N_1^2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}}}{\sqrt{N_1^2 + N_2^2}} - BM = 0$$

Trisection and the Cube Roots 042997

If trisection can be placed at RUE, then PV is proportional to RW.



$N := 5 \quad AB := 1 \quad AH := AB \cdot N$

$BH := AH - AB \quad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \quad AF := \left(AB \cdot A\right)^{\frac{1}{3}} \quad CF := AF - AC \quad CE := \frac{CF}{2}$

$AE := AC + CE$

$AU := CE \quad NV := AU \quad MW := AU$

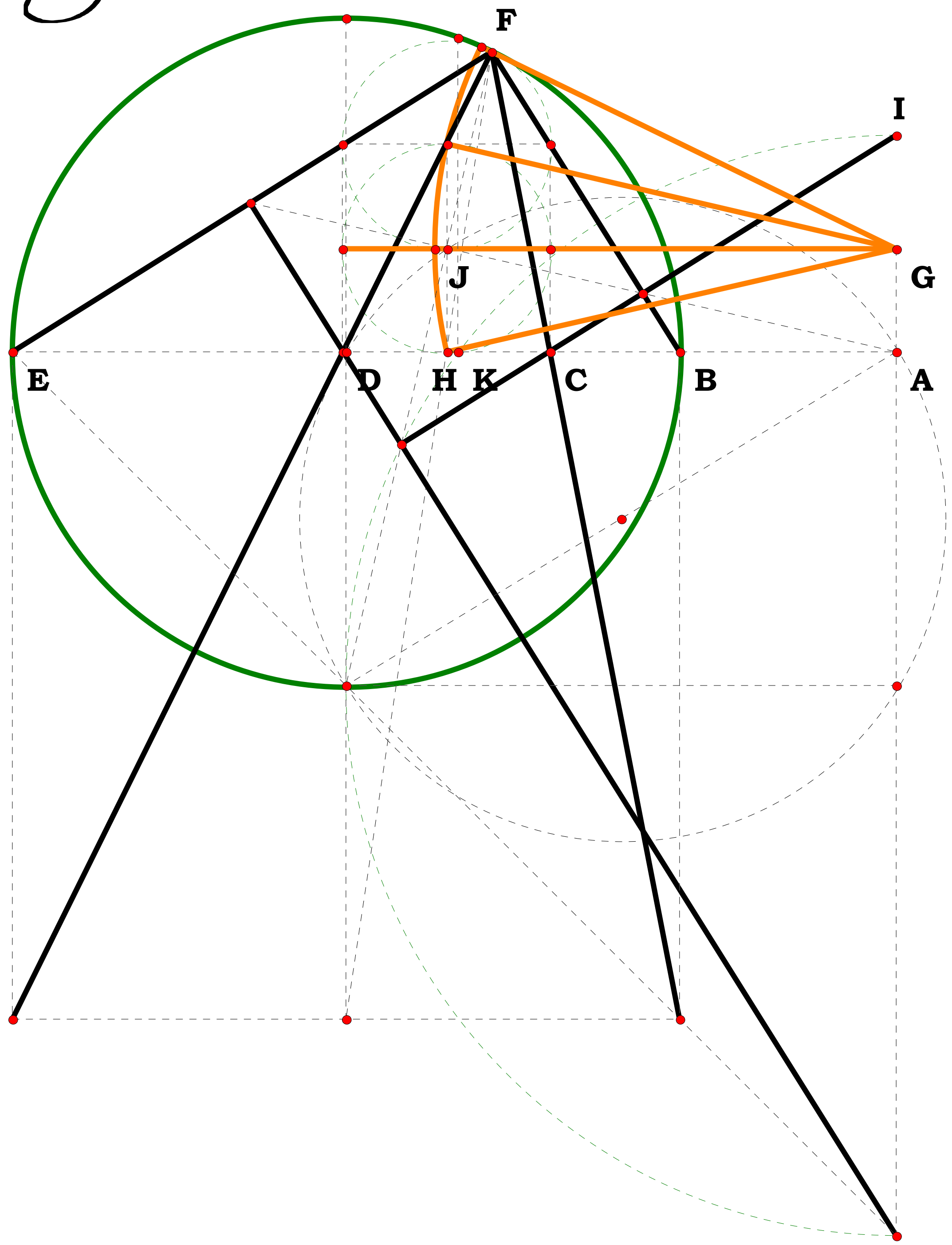
(For the next two equations see 042897.)

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} \qquad MR := 2 \cdot AU + AU \cdot \sqrt{\frac{5 \cdot \left(\frac{AE}{AU}\right)^2}{\left(\frac{AE}{AU}\right)^2 + 1} - \frac{4 \cdot \frac{AE}{AU}}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} + \frac{1}{\left(\frac{AE}{AU}\right)^2 + 1}}$$

$RW := MR - MW \quad BC := AC - AB \quad FH := AH - AF$

$$CN := \frac{BC \cdot CF}{BC + FH} \quad NP := \frac{BJ \cdot CN}{BC} \quad PV := NP - NV \quad AN := AC + CN \quad UV := AN \quad UW := AM \quad \frac{RW \cdot UV}{UW} - PV = 0$$

From a Single Point.



AB = 3.24039 cm
AC = 5.17367 cm
AD = 8.26039 cm
AE = 13.18872 cm
AK = 6.53732 cm

$$\frac{AK}{AB} = 2.01745$$

$$\frac{AE}{AB} = 4.07011$$

$$\frac{AC}{AB} = 1.59662$$

$$\frac{AD}{AB} = 2.54920$$

$$\left(\frac{AE}{AB}\right)^{\frac{1}{3}} - \frac{AC}{AB} = 0.00000$$

$$\left(\frac{AE}{AB}\right)^{\frac{2}{3}} - \frac{AD}{AB} = 0.00000$$

$$\left(\frac{AE}{AB}\right)^{\frac{1}{2}} - \frac{AK}{AB} = 0.00000$$

$m\angle FGH = 38.82047°$
$m\angle JGH = 12.94016°$

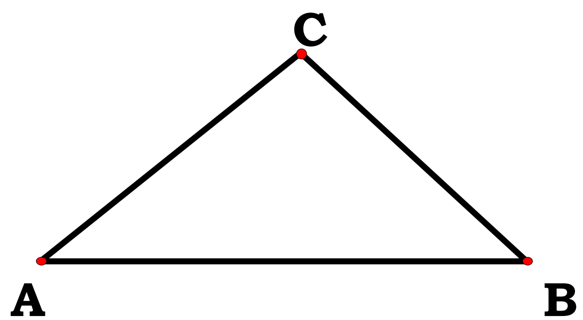
$$\frac{m\angle FGH}{m\angle JGH} = 3.00000$$

Start with H. Construct the two tangent circles. With those project to G, from the tangent at F and the enter line of the first circle for the tangent string; which will also construct the unit line.
Project from I perpendicular to BF to etc. etc.

An Indeterminate Problem Reduced To An Equation 072903

Page 5 of *A Treatise on Algebraic Geometry* by Rev. Dionysius Lardner, 1831

Given the base AB, and the sum of the sides (AC and BC) of a triangle, to find the vertex (C).



Let AB = *a*, AC = *y*, and CB = *x*, and the the excess of the sum of the sides above the base be *d*.

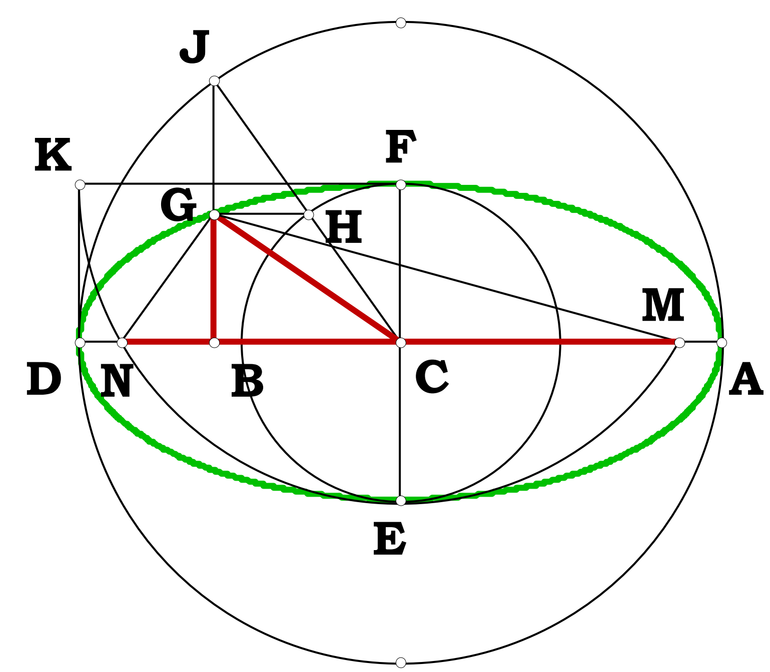
$\therefore\ y + x = a + b.$

Any values of *y* and *x*, which fulfill the conditions of this equation, represent the sides of the triangle, whose vertex solves the problem.

The ability to render a solution to a problem is first tendered by determining if the problem is indeed a problem first in grammar. The problem as stated makes an elementary grammar error, that of that of number itself. The sum of two sides yields a plurality, while the "the vertex" is decidely singular. Even so, anyone with any wit at all does not tender an answer to a problem by giving synonyms. y and x are synonyms for a and b. Any triangle can be seen as three potential ellipses, one for each side, while the remaining two as the sum of foci. Therefor, there was no indeterminate problem save 3, two grammatical, and the third a pretense of an answer. A vertex is not a difference, therefore it cannot possibly solve any problem.

09/11/97 The Ellipse

Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.



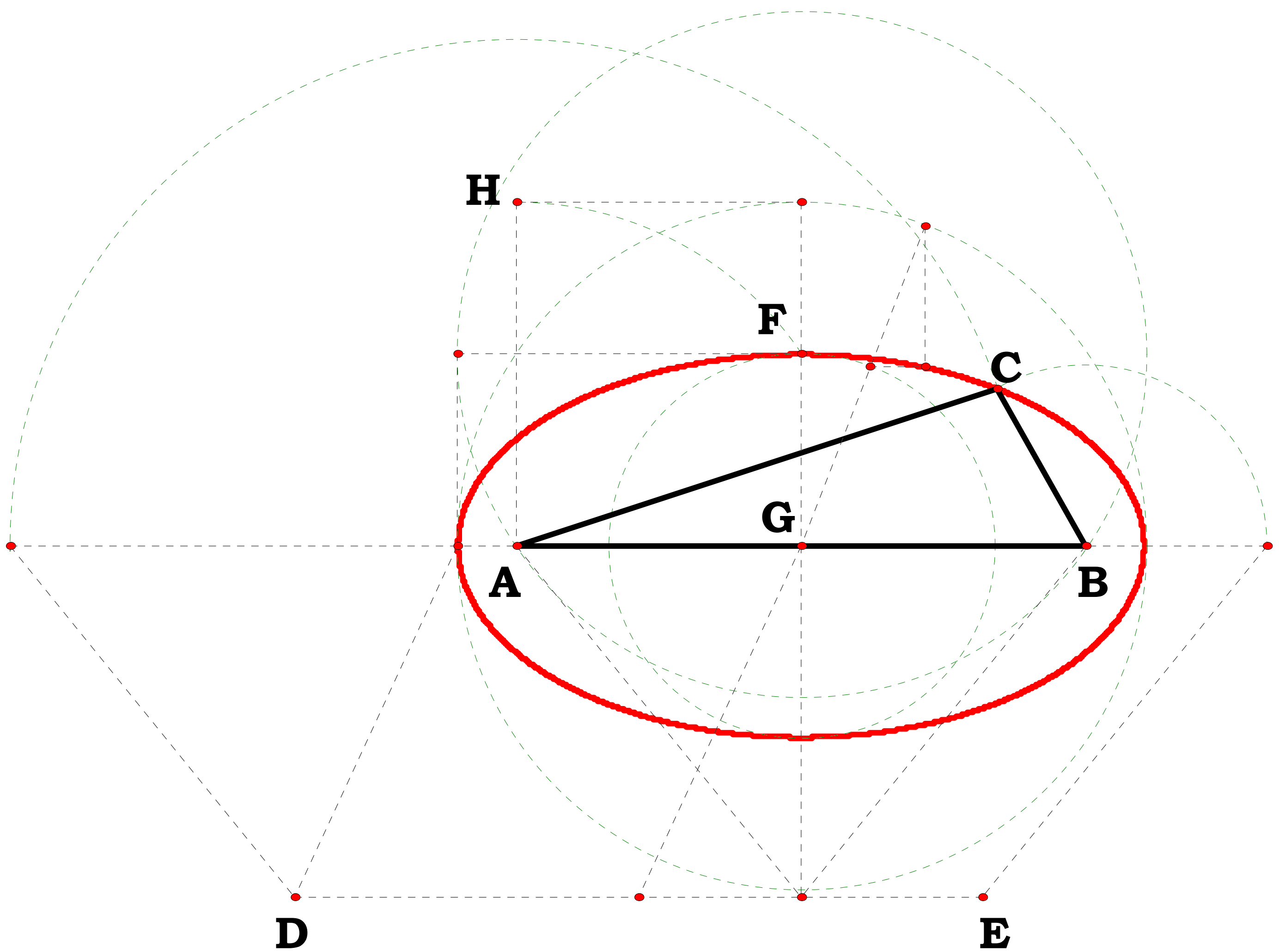
$N_1 := 2.028 \qquad N_2 := 1.035$

$AD := 3.333 \quad EF := \frac{AD}{N_1} \quad AB := \frac{AD}{N_2} \quad BD := AD - AB \quad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \quad BC := AC - AB \quad CH := \frac{EF}{2} \quad CJ := AC \quad BG := \frac{BJ \cdot CH}{CJ} \quad CG := \sqrt{BG^2 + BC^2}$

$MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2} \qquad \frac{AD \cdot \sqrt{4 \cdot N_2 - 4 + N_2^{\,2} \cdot N_1^{\,2} - 4 \cdot N_2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,2}}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{AD \cdot \sqrt{N_2 - 1}}{(N_2 \cdot N_1)} - BG = 0 \qquad \frac{AD \cdot \sqrt{(N_1^{\,2} - 1)}}{N_1} - MN = 0$

Given triangle ABC, and AB as base, describe the Ellipse



$S_1 := 8.14917 \qquad S_2 := 7.23745 \qquad S_3 := 2.58277$

$AB := S_1$

$AC := S_2$

$BC := S_3$

$DE := AC + BC$

$AH := \dfrac{DE}{2} \qquad AG := \dfrac{AB}{2} \qquad FG := \sqrt{AH^2 - AG^2}$

$$FG - \frac{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2} = 0$$

The ratio of the ellipse is thus;

$$\frac{AH}{FG} - \frac{(S_2 + S_3)}{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}} = 0$$

From any point on DE, one can find everything and not once think about x and y.

020298

$\Delta := 4 \quad \delta := 0 .. \Delta - 1$

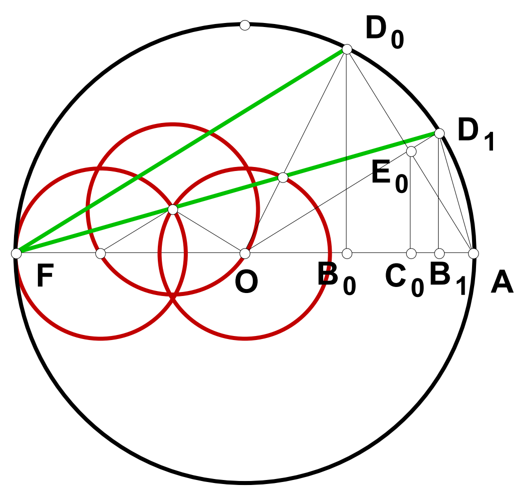
$N := .656 \quad AF := 2.3754$

$AO := \frac{AF}{2} \quad AB_0 := N \quad BD_0 := \sqrt{AB_0 \cdot (AF - AB_0)}$

$CE_0 := \frac{BD_0}{2} \quad AC_0 := \frac{AB_0}{2}$

$OC_0 := AO - AC_0$

$OE_0 := \sqrt{(CE_0)^2 + (OC_0)^2}$



$$\begin{pmatrix} OB_{\delta+1} \\ AB_{\delta+1} \\ BD_{\delta+1} \\ CE_{\delta+1} \\ AC_{\delta+1} \\ OC_{\delta+1} \\ OE_{\delta+1} \end{pmatrix} := \begin{bmatrix} OC_\delta \cdot \frac{AO}{OE_\delta} \\ AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{OE_\delta} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{(2 \cdot OE_\delta)} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta + OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot \sqrt{AO \cdot \frac{(AF \cdot OE_\delta + 4 \cdot OC_\delta \cdot AO - OC_\delta \cdot AF)}{OE_\delta}} \end{bmatrix}$$

$\sqrt{(AB_\delta)^2 + (BD_\delta)^2}$

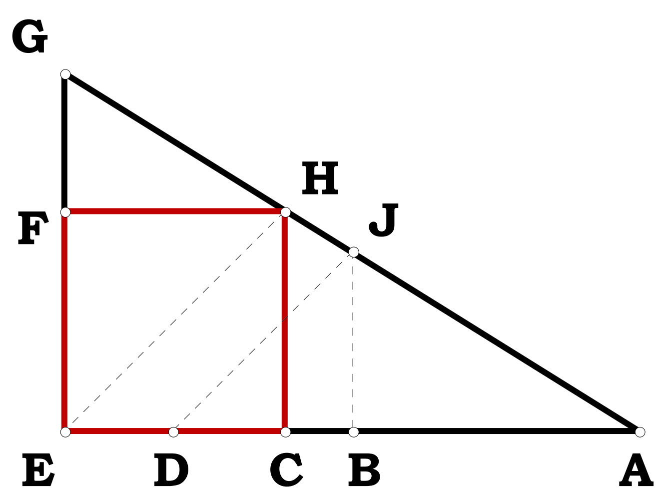
$AD_\delta := \sqrt{(AB_\delta)^2 + (BD_\delta)^2} \qquad AD = \begin{pmatrix} 1.248304 \\ 0.648825 \\ 0.327541 \\ 0.164163 \end{pmatrix}$

Length of cord by progressive bisections.

I have no idea why I did this figure, it was so long ago.

A Square In A Triangle 021098

What is the Algebraic Name for the square as given in a right triangle?
What is the Algebraic name for the ratio AE/AC?



$N_1 := 2.98958 \qquad N_2 := 1.86690$

$AE := N_1 \quad EG := N_2$

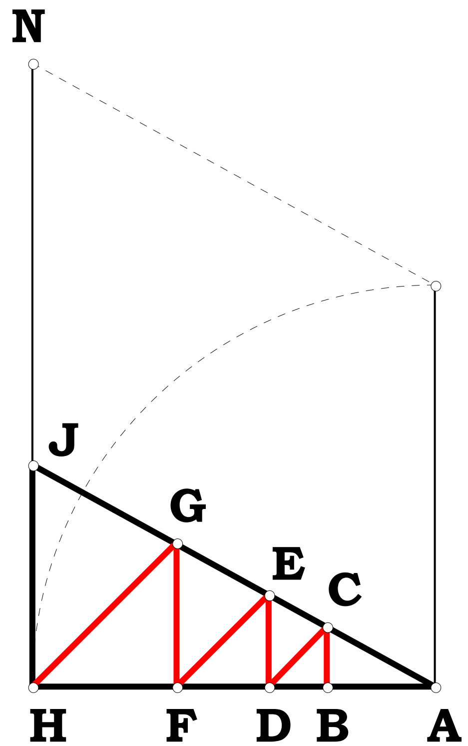
$AB := \dfrac{AE}{2} \quad BJ := \dfrac{EG}{2} \quad BD := BJ$

$AD := AB + BD \quad CE := BD \cdot \dfrac{AE}{AD}$

$AC := AE - CE \quad FG := EG - CE$

$$AB - \frac{N_1}{2} = 0 \qquad BJ - \frac{N_2}{2} = 0 \qquad AD - \left(\frac{N_1}{2} + \frac{N_2}{2}\right) = 0 \qquad CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0 \qquad AC - \frac{{N_1}^2}{N_1 + N_2} = 0$$

$$FG - \frac{{N_2}^2}{N_1 + N_2} = 0 \qquad \frac{AE}{AC} - \left(\frac{N_1 + N_2}{N_1}\right) = 0 \qquad \frac{EG}{FG} - \frac{N_1 + N_2}{N_2} = 0$$



Alternate Method Root Series 022598

Given a length and a unit, raise that length to any whole power.

Given for the third power.

$N_1 := 4 \qquad N_2 := 3$

$AH := N_1 \quad HN := AH \cdot N_2$

$HJ := HN - AH \quad FH := \dfrac{AH \cdot HJ}{AH + HJ} \quad AF := AH - FH$

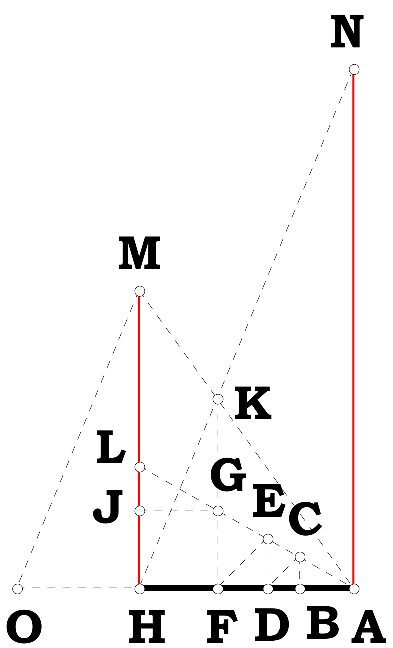
$FG := FH \quad DF := \dfrac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \dfrac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$\dfrac{AH}{AF} - {N_2}^1 = 0 \qquad \dfrac{AH}{AD} - {N_2}^2 = 0 \qquad \dfrac{AH}{AB} - {N_2}^3 = 0$

${N_2}^3 = 27$

Sum DIvided by One Powered 022598B



$N_1 := 5 \quad N_2 := 6 \quad AH := 1 \quad HM := AH \cdot N_1 \quad AN := AH \cdot N_2$

$HO := \frac{AH \cdot HM}{AN} \quad AO := AH + HO \quad AF := \frac{AH^2}{AO}$

$FH := AH - AF \quad FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0$$

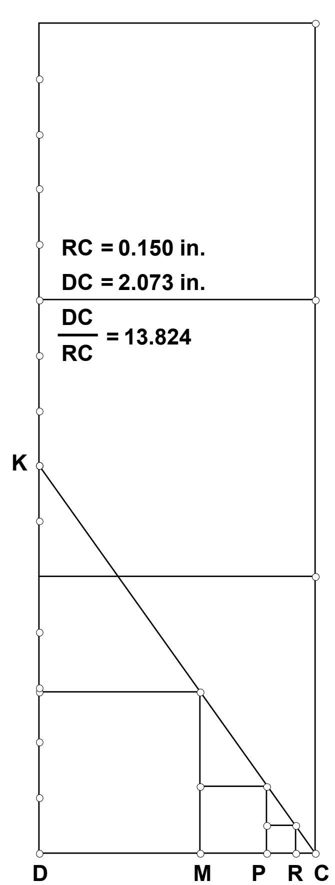
Doing the Math

These plates sat, never actually being wrote up, in their directory, but included in the Delian Quest. Because they are so elementary, I assumed that doing math with a geometric figure was known. I was a bit conflicted about this, however, working 12 hours a day for years on end tends to dull the senses. Then, I got to thinking about them again in 2007. I even did a couple of searches on the internet to see if anyone had actually developed doing the math with a simple geometric figure and could not find anything. Then I found scraps in old books found on the Internet Archive where certain operations were fragmented and really undeveloped. Then I started to realize and understand that BAM was not developed as a language. If it had been, there would be no talk of non-Euclidean Geometry, there would only be embarrassment of its memory.

One can see that it is directly derived from plate A on this date. BAM (Basic Analog Mathematics) has its roots in exponential series.

4
3+4/5
3+3/5
3+2/5
3+1/5
3
2+4/5
2+3/5
2+2/5
2+1/5
2
1+4/5
1+3/5
1+2/5
1+1/5
Show
Hide

M^N



$4^3 = 64.000$

$\left(3+\frac{4}{5}\right)^3 = 54.872$

$\left(3+\frac{3}{5}\right)^3 = 46.656$

$\left(3+\frac{2}{5}\right)^3 = 39.304$

$\left(3+\frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2+\frac{4}{5}\right)^3 = 21.952$

$\left(2+\frac{3}{5}\right)^3 = 17.576$

$\left(2+\frac{2}{5}\right)^3 = 13.824$

$\left(2+\frac{1}{5}\right)^3 = 10.648$

$\left(1+\frac{4}{5}\right)^3 = 5.832$

$\left(1+\frac{3}{5}\right)^3 = 4.096$

$\left(1+\frac{2}{5}\right)^3 = 2.744$

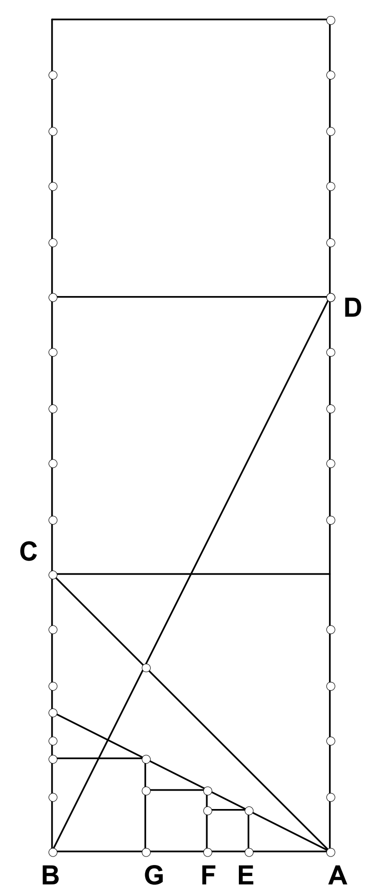
$\left(1+\frac{1}{5}\right)^3 = 1.728$

BC: 2+4/5, 2+3/5, 2+2/5, 2+1/5, 2, 1+4/5, 1+3/5, 1+2/5, 1+1/5, 4, 3+4/5, 3+3/5, 3+2/5, 3+1/5, 3

$\frac{AB}{AG} = 1.50000$

$\frac{AB}{AF} = 2.25000$

$\frac{AB}{AE} = 3.37500$



Given line AB, divide it by the equation

$$\frac{BC+AD^{W}}{AD} = 5.06250$$

AD: 2+4/5, 2+3/5, 2+2/5, 2+1/5, 2, 1+4/5, 1+3/5, 1+2/5, 1+1/5, 4, 3+4/5, 3+3/5, 3+2/5, 3+1/5, 3

where W is a whole number.

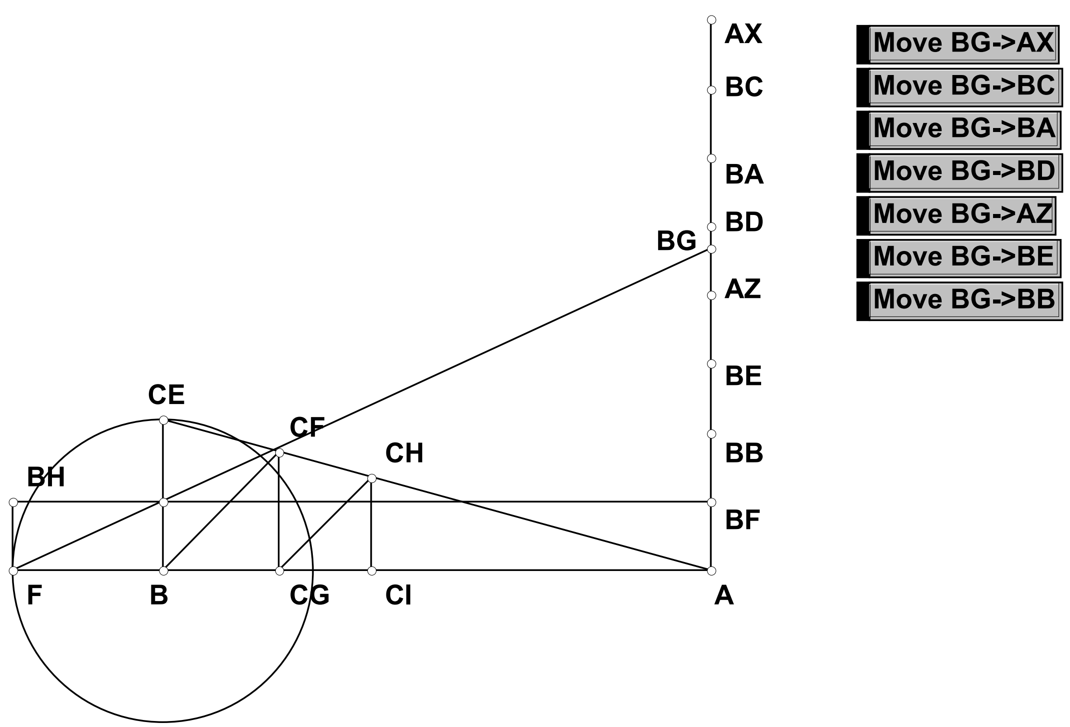
$\frac{BC+AD^{1}}{AD} = 1.50000$

$\frac{BC+AD^{2}}{AD} = 2.25000$

$\frac{BC+AD^{3}}{AD} = 3.37500$

022698

Now ain't this just typically human. Some of the more defined plates that led to Basic Analog Mathematics promptly get wrote up in 0816 2015. I am on the ball! It appears I never even bothered to do a pdf file of these. They are, however, not fundamentally distinct from some previous write ups except, these are series format. So, I think I will forgo the write ups again!



$\frac{AF}{FB} = 4.672$

$AB = 3.348$ in.

$FB = 0.912$ in.

$AF = 4.260$ in.

$\frac{AF}{AB} = 1.272$

$ACG = 2.632$ in.

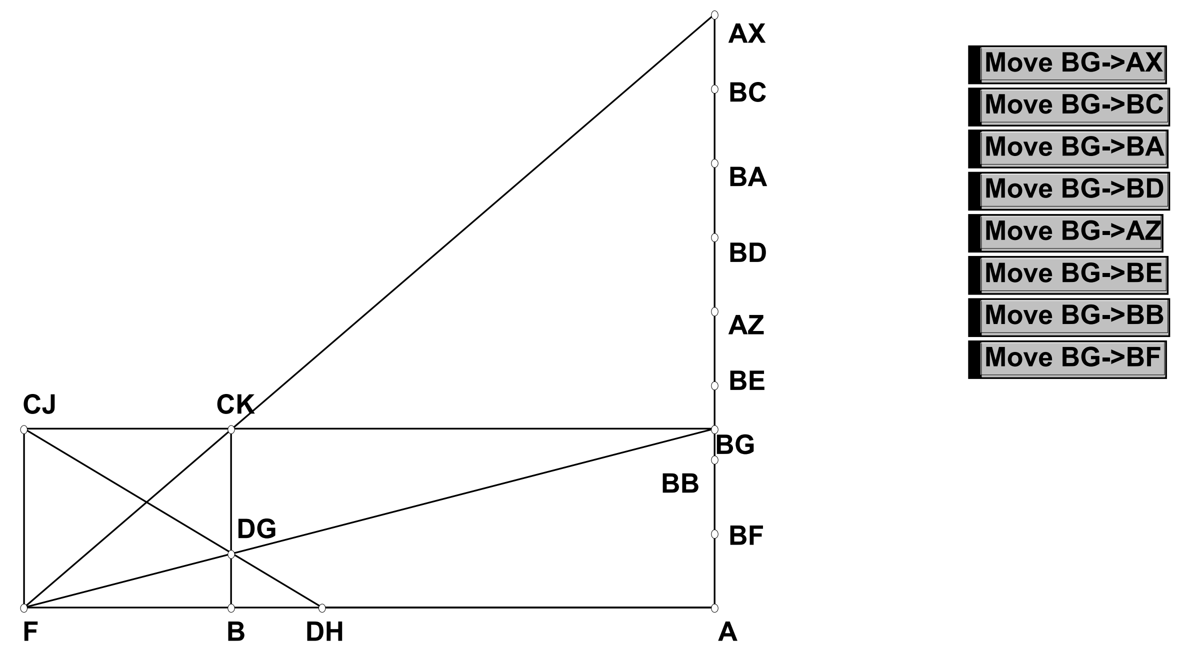
$ACI = 2.068$ in.

$\frac{AF}{ACI} = 2.060$

$\frac{AF}{ACG} = 1.619$

$\frac{AF^3}{AB} = 2.060$

$\frac{AF^2}{AB} = 1.619$



$AF = 4.271$ in.

$AB = 2.985$ in.

$\frac{AF}{AB} = 1.431$

$\frac{AF}{DHF} = 2.321$

$\frac{DHF}{AF} = 0.431$

$DHF = 1.840$ in.

$BF = 1.286$ in.

$\frac{AAX}{BGA} = 3.321$

$\frac{AAX}{BGA} - 1 = 2.321$

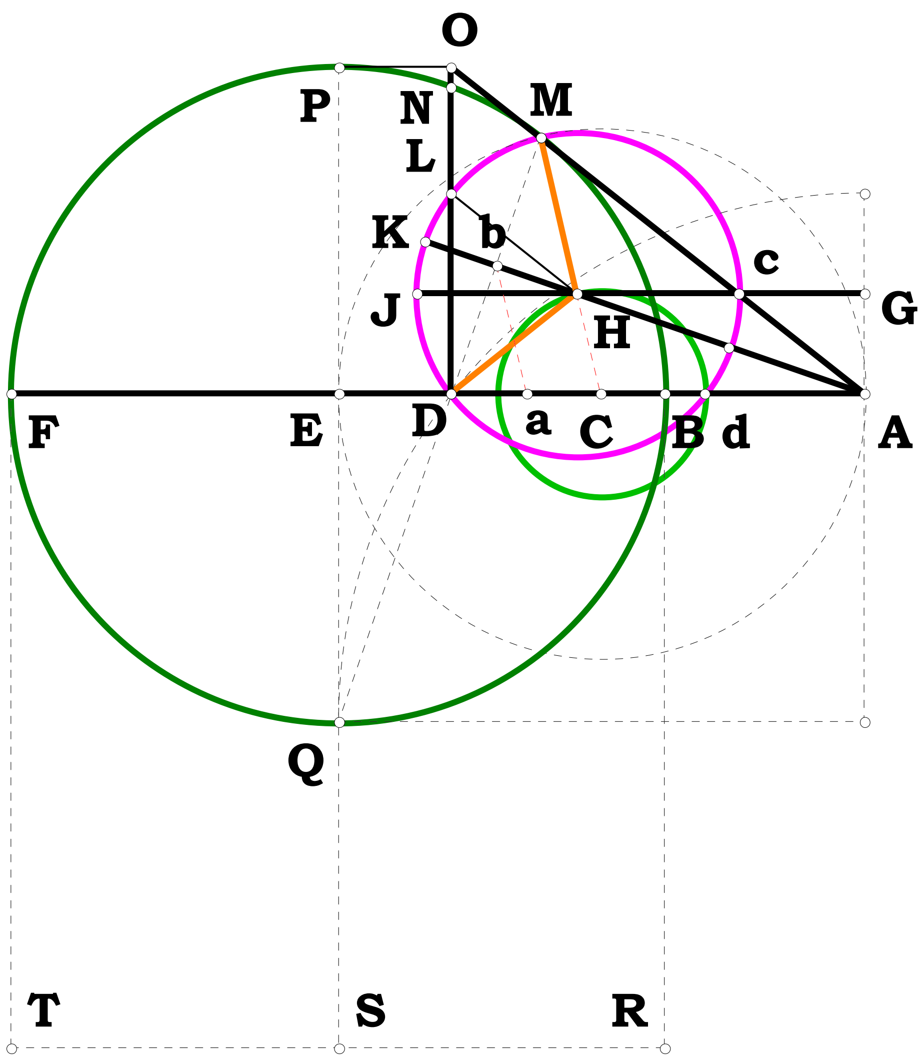
$BGA = 1.105$ in.

$AAX = 3.670$ in.

$$\left(\frac{1}{\frac{AAX}{BGA} - 1}\right) \cdot BF = 0.554 \text{ in.}$$

A Square Root Figure And Triseciton 042398

In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?



$N_1 := 2.64583 \qquad N_2 := 11.29771$

$AB := N_1 \qquad AF := N_2 \qquad BF := AF - AB$

$AD := (AB \cdot AF)^{\frac{1}{2}} \qquad BE := \dfrac{BF}{2} \qquad BD := AD - AB \qquad DE := BE - BD$

$EQ := BE \qquad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \qquad PQ := BF \qquad QM := \dfrac{EQ \cdot PQ}{DQ}$

$DM := QM - DQ \qquad AE := AB + BE \qquad AC := \dfrac{AE}{2} \qquad Db := \dfrac{DM}{2}$

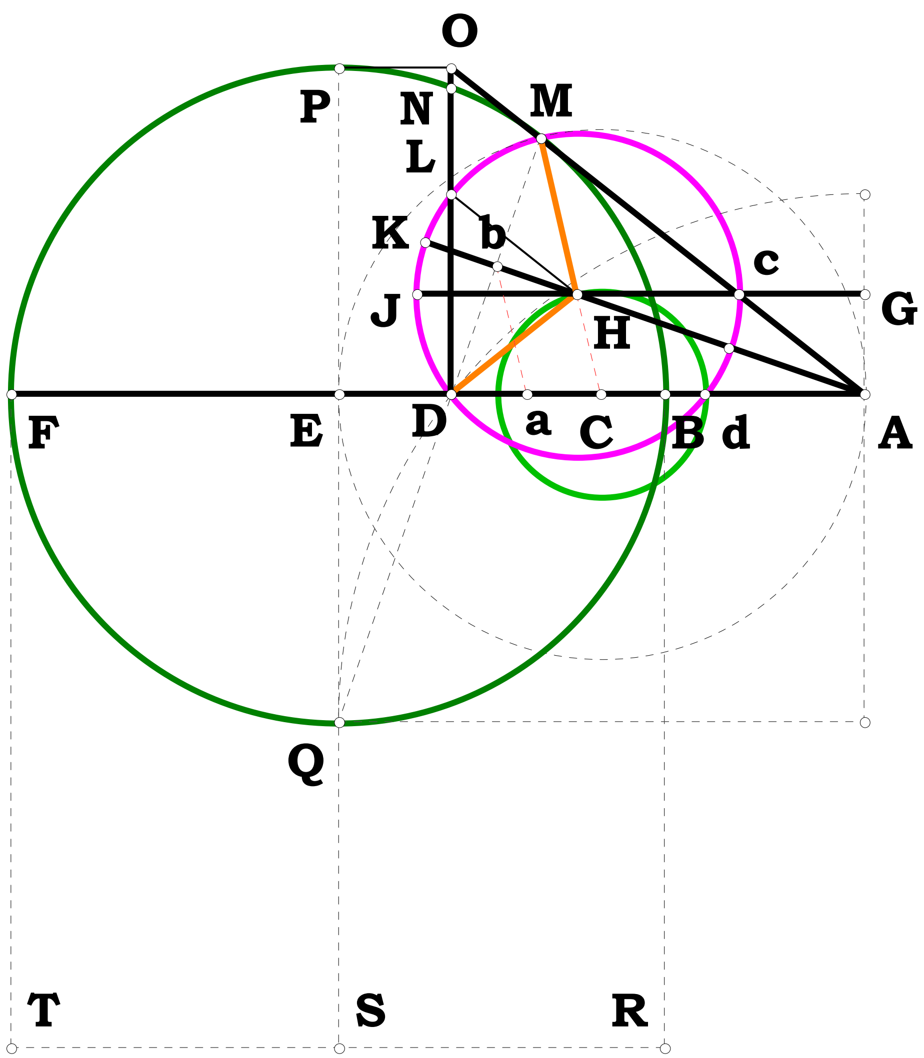
$CM := AC \qquad ab := \dfrac{CM \cdot Db}{DM} \qquad CD := AD - AC$

$Ca := \dfrac{CD}{2} \qquad Aa := AC + Ca \qquad CH := \dfrac{ab \cdot AC}{Aa}$

$AM := AD \qquad Ac := \dfrac{AM \cdot CH}{CM} \qquad HM := CM - CH \qquad HM - Ac = 0$

$$Ac - \frac{\sqrt{N_1 \cdot N_2} \cdot (N_1 + N_2)}{N_1 + N_2 + 4 \cdot \sqrt{N_1 \cdot N_2}} = 0$$

$$HM - \frac{\sqrt{N_1 \cdot N_2} \cdot (N_1 + N_2)}{N_1 + N_2 + 4 \cdot \sqrt{N_1 \cdot N_2}} = 0$$



Definitions:

$$AB - N_1 = 0 \qquad AF - N_2 = 0 \qquad BF - (N_2 - N_1) = 0 \qquad AD - (N_1 \cdot N_2)^{\frac{1}{2}} = 0$$

$$BE - \frac{N_2 - N_1}{2} = 0 \qquad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0 \qquad DE - \frac{N_1 + N_2 - 2\cdot\sqrt{N_1 \cdot N_2}}{2} = 0$$

$$DQ - \frac{\sqrt{\left[(N_1 + N_2)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1 \cdot N_2}\right)\right]}}{\sqrt{2}} = 0 \qquad QM - \frac{\sqrt{2}\cdot(N_1 - N_2)^2}{2\cdot\sqrt{(N_1 + N_2)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1 \cdot N_2}\right)}} = 0$$

$$DM - \frac{N_1\cdot\sqrt{2\cdot N_1\cdot N_2} + N_2\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{\sqrt{(N_1 + N_2)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1 \cdot N_2}\right)}} = 0 \qquad AE - \frac{N_1 + N_2}{2} = 0 \qquad AC - \frac{N_1 + N_2}{4} = 0$$

$$Db - \frac{N_1\cdot\sqrt{2\cdot N_1\cdot N_2} + N_2\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{2\cdot\sqrt{(N_1 + N_2)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1 \cdot N_2}\right)}} = 0 \qquad ab - \frac{N_1 + N_2}{8} = 0$$

$$CD - \frac{4\cdot\sqrt{N_1 \cdot N_2} - N_2 - N_1}{4} = 0 \qquad Ca - \frac{4\cdot\sqrt{N_1 \cdot N_2} - N_2 - N_1}{8} = 0 \qquad Aa - \frac{N_1 + N_2 + 4\cdot\sqrt{N_1 \cdot N_2}}{8} = 0$$

$$CH - \frac{(N_1 + N_2)^2}{4\cdot\left(N_1 + N_2 + 4\cdot\sqrt{N_1 \cdot N_2}\right)} = 0$$

$$R := \frac{\sqrt{N_2}\cdot\left(1 + N_2\right)}{1 + N_2 + 4\cdot\sqrt{N_2}}$$

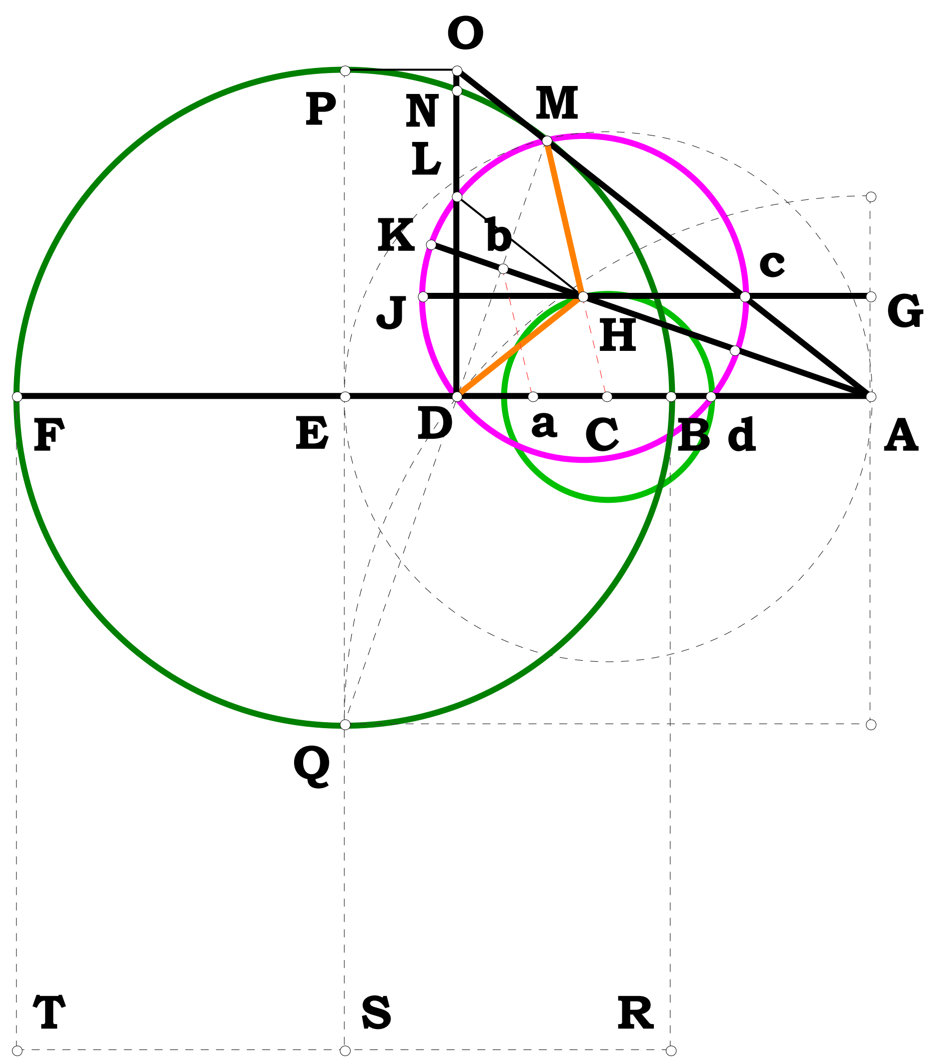
$$\frac{N_2 - N_1}{2} - BE = 0$$

$$\left[\frac{R}{3} + \left(\frac{R}{3} + \frac{2\cdot R^2}{3} + \frac{R^3}{27} + \sqrt{\frac{50\cdot R^2}{27} - \frac{44\cdot R^3}{27} - \frac{R^4}{9} - \frac{4\cdot R}{9} + \frac{1}{27}}\right)^{\frac{1}{3}} + \frac{\frac{R^2}{9} + \frac{4\cdot R}{3} - \frac{1}{3}}{\left(\frac{R}{3} + \frac{2\cdot R^2}{3} + \frac{R^3}{27} + \sqrt{\frac{50\cdot R^2}{27} - \frac{44\cdot R^3}{27} - \frac{R^4}{9} - \frac{4\cdot R}{9} + \frac{1}{27}}\right)^{\frac{1}{3}}}\right]^2 = 11.29771$$

$$N_2 - \left[\frac{R}{3} + \left(\frac{R}{3} + \frac{2\cdot R^2}{3} + \frac{R^3}{27} + \sqrt{\frac{50\cdot R^2}{27} - \frac{44\cdot R^3}{27} - \frac{R^4}{9} - \frac{4\cdot R}{9} + \frac{1}{27}}\right)^{\frac{1}{3}} + \frac{\frac{R^2}{9} + \frac{4\cdot R}{3} - \frac{1}{3}}{\left(\frac{R}{3} + \frac{2\cdot R^2}{3} + \frac{R^3}{27} + \sqrt{\frac{50\cdot R^2}{27} - \frac{44\cdot R^3}{27} - \frac{R^4}{9} - \frac{4\cdot R}{9} + \frac{1}{27}}\right)^{\frac{1}{3}}}\right]^2 = 0$$

A Square Root Figure And Triseciton 042398

In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?



$$N_1 := 2.64583 \qquad N_2 := 8.65188$$

$$AB := N_1 \qquad BF := N_2 \qquad AF := N_1 + N_2$$

$$AD := (AB \cdot AF)^{\frac{1}{2}} \qquad BE := \frac{BF}{2} \qquad BD := AD - AB \qquad DE := BE - BD$$

$$EQ := BE \qquad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \qquad PQ := BF \qquad QM := \frac{EQ \cdot PQ}{DQ}$$

$$DM := QM - DQ \qquad AE := AB + BE \qquad AC := \frac{AE}{2} \qquad Db := \frac{DM}{2}$$

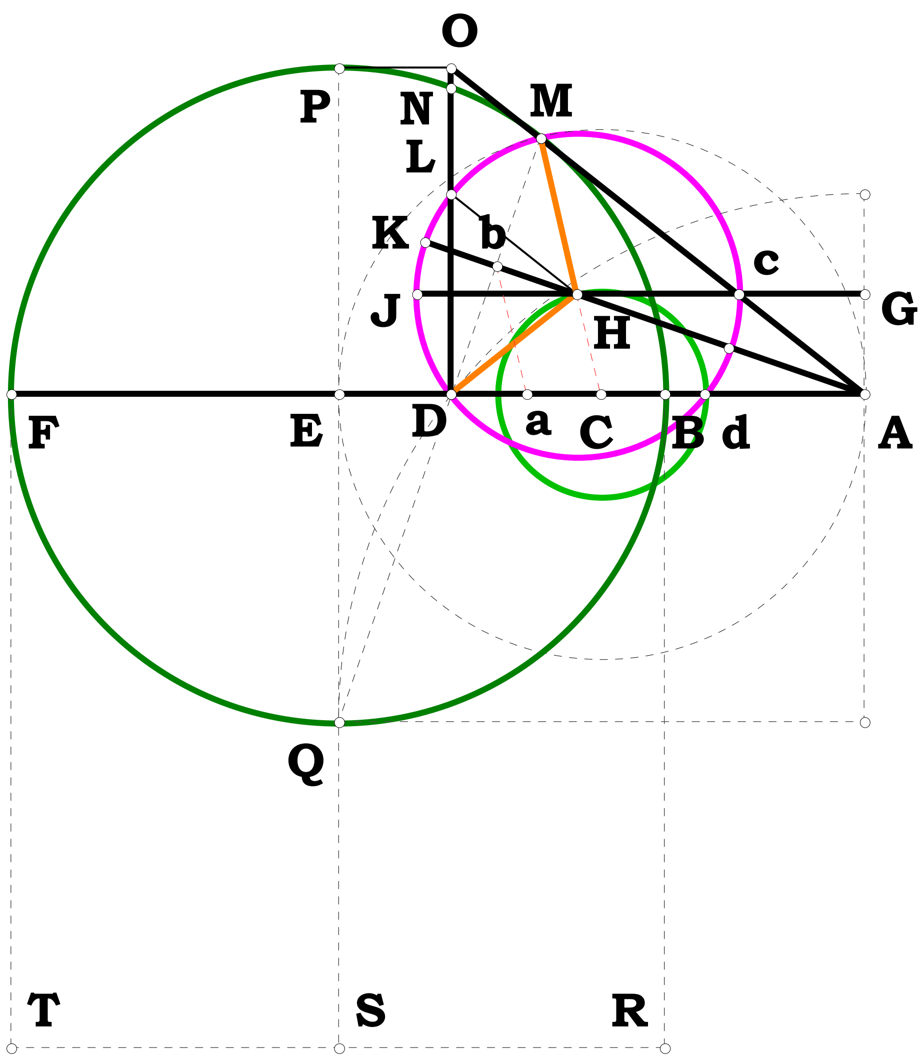
$$CM := AC \qquad ab := \frac{CM \cdot Db}{DM} \qquad CD := AD - AC$$

$$Ca := \frac{CD}{2} \qquad Aa := AC + Ca \qquad CH := \frac{ab \cdot AC}{Aa}$$

$$AM := AD \qquad Ac := \frac{AM \cdot CH}{CM} \qquad HM := CM - CH \qquad HM - Ac = 0$$

$$Ac - \frac{\sqrt{N_1 \cdot (N_1 + N_2)} \cdot (2 \cdot N_1 + N_2)}{2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}} = 0$$

$$HM - \frac{\sqrt{N_1^{\,2} + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2)}{2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}} = 0$$



Definitions:

$$AB - N_1 = 0 \qquad BF - N_2 = 0 \qquad AF - (N_1 + N_2) = 0 \qquad AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0 \qquad BE - \frac{N_2}{2} = 0$$

$$BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right] = 0 \qquad DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}}{2} = 0 \qquad AE - \frac{2 \cdot N_1 + N_2}{2} = 0$$

$$DQ - \frac{\left[(2 \cdot N_1 + N_2) \cdot \left(2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}\right)\right]^{\frac{1}{2}}}{\sqrt{2}} = 0 \qquad AC - \frac{2 \cdot N_1 + N_2}{4} = 0$$

$$QM - \frac{\sqrt{2} \cdot N_2^{\,2}}{2 \cdot \sqrt{(2 \cdot N_1 + N_2) \cdot \left(2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}\right)}} = 0 \qquad ab - \frac{2 \cdot N_1 + N_2}{8} = 0$$

$$DM - \frac{\sqrt{2} \cdot \left[\sqrt{N_1^{\,2} + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^{\,2}\right]}{\sqrt{(2 \cdot N_1 + N_2) \cdot \left[2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot (N_1 + N_2)}\right]}} = 0$$

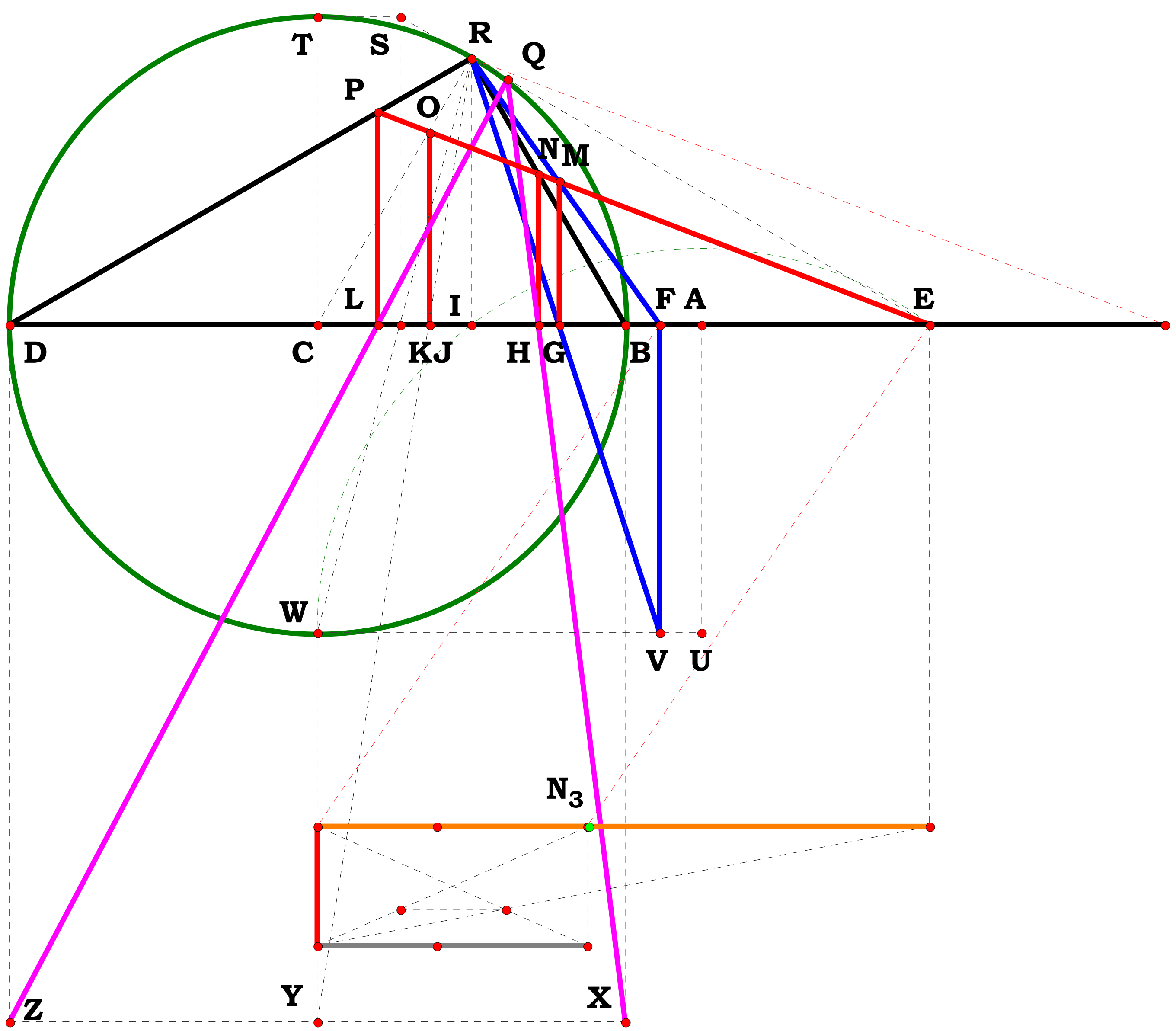
$$Db - \frac{\sqrt{2} \cdot \left[\sqrt{N_1^{\,2} + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^{\,2}\right]}{2 \cdot \sqrt{(2 \cdot N_1 + N_2) \cdot \left[2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot (N_1 + N_2)}\right]}} = 0$$

$$CD - \frac{4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1} - N_2 - 2 \cdot N_1}{4} = 0 \qquad Ca - \frac{4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1} - N_2 - 2 \cdot N_1}{8} = 0$$

$$Aa - \frac{2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}}{8} = 0 \qquad CH - \frac{(2 \cdot N_1 + N_2)^2}{4 \cdot \left(2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}\right)} = 0$$

On Gemini Roots 072499

CE is to EF as CY is to CW



$N_1 := 1.51292 \quad N_2 := 10.77334 \quad N_3 := 3$

$AB := N_1 \quad AD := N_2$

$BD := AD - AB \quad BC := \frac{BD}{2} \quad CW := BC \quad CT := BC$

$FV := BC \quad AI := \sqrt{AB \cdot AD} \quad BI := AI - AB \quad DI := BD - BI$

$IR := \sqrt{BI \cdot DI} \quad AE := AI \quad AC := AB + BC \quad ED := AD + AE$

$CE := AC + AE \quad EF := \frac{CE}{N_3} \quad BE := AE + AB \quad EI := AE + AI$

$FI := EI - EF \quad FG := \frac{FI \cdot FV}{FV + IR} \quad EG := EF + FG \quad GI := FI - FG$

$GM := \frac{FV \cdot GI}{FI} \quad Ia := \frac{EG \cdot IR}{GM} \quad EL := \frac{Ia \cdot ED}{Ia + DI} \quad BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI \quad BH := \frac{BI \cdot BE}{Ba} \quad EH := BE + BH \quad CI := AC - AI$

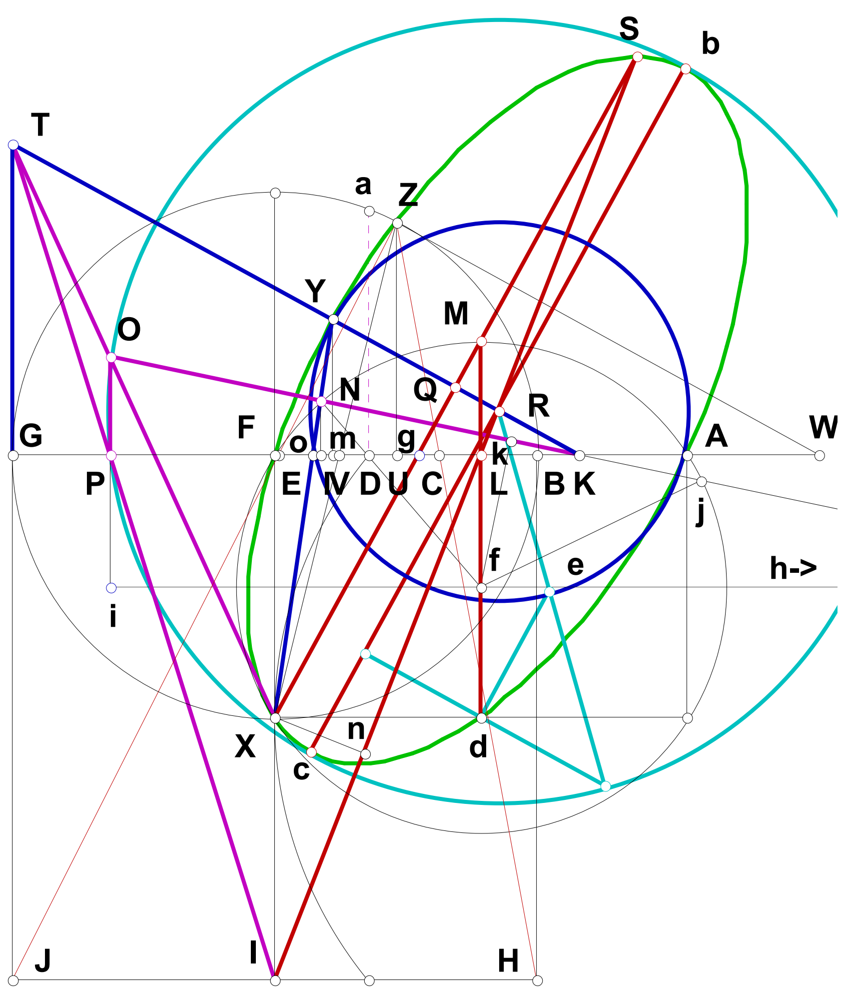
$JO := \frac{IR \cdot CE}{CI + Ia} \quad CJ := \frac{CI \cdot JO}{IR} \quad JI := CI - CJ \quad CY := \frac{IR \cdot CJ}{JI}$

$\frac{CY}{CW} - \frac{CE}{EF} = 0 \qquad \frac{CE}{EF} = 3 \qquad \frac{CY}{CW} = 3$

Note: Needs lipstick and a dress. Parse it. If I get around to it, I will redo the graphics.

A Delian Solution 081199

What are the minor and major axis for the ellipse that will give point Z for the cube root?



$AB := 1 \quad N := 16 \quad AG := AB \cdot N \quad BG := AG - AB$

$BF := \dfrac{BG}{2} \quad FG := BF \quad AF := AB + BF \quad FX := BF \quad Mf := \dfrac{\sqrt{AF^2 + FX^2}}{2}$

$Lf := \dfrac{FX}{2} \quad ML := Mf - Lf \quad FL := \dfrac{AF}{2} \quad Xd := FL \quad df := Lf \quad IX := FX$

$Md := Mf + df \quad MX := \sqrt{Xd^2 + Md^2} \quad SX := \dfrac{MX \cdot IX}{IX - ML} \quad Lg := \dfrac{FL \cdot ML}{ML + FX}$

$QX := \dfrac{SX}{2} \quad Fg := FL - Lg \quad Xg := \dfrac{MX \cdot Fg}{Xd} \quad Qg := QX - Xg$

$Kg := \dfrac{Xg \cdot Qg}{Fg} \quad GK := FG + Fg + Kg \quad GJ := BG \quad GT := \dfrac{Fg \cdot GK}{FX}$

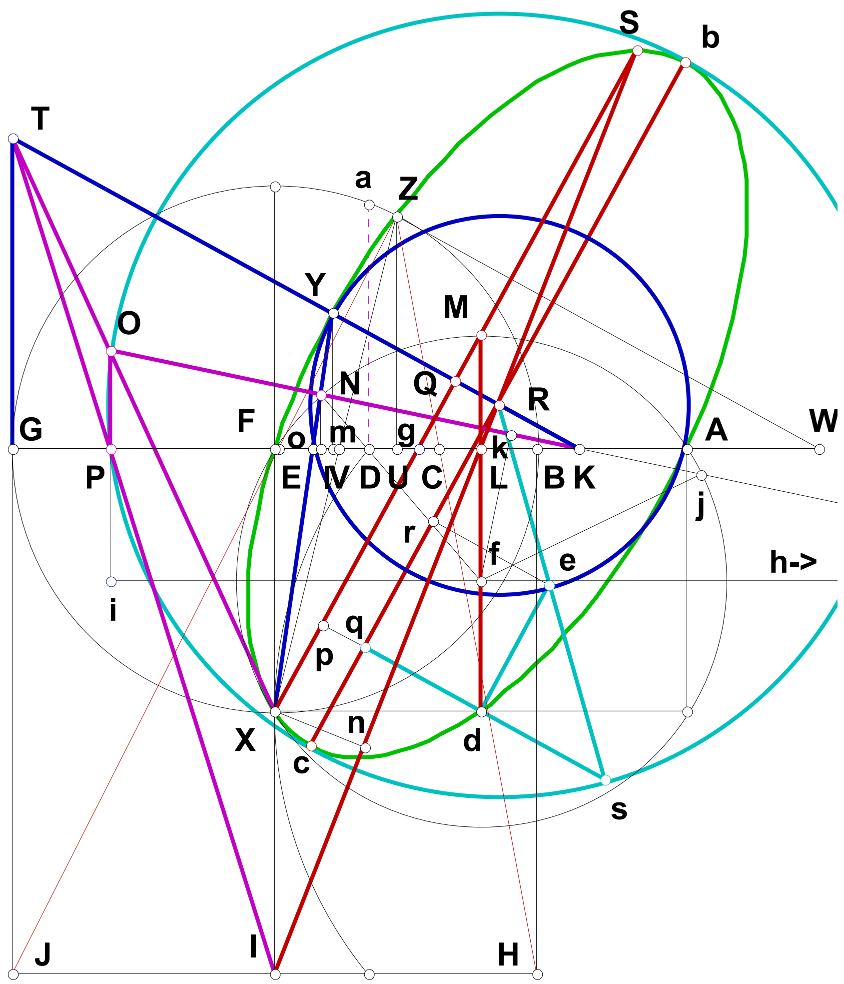
$JT := GJ + GT \quad IJ := BF \quad FP := \dfrac{IJ \cdot GJ}{JT} \quad OP := \dfrac{IX \cdot GT}{JT}$

$KP := Fg + Kg + FP \quad Pi := Lf \quad Oi := OP + Pi \quad hi := \dfrac{KP \cdot Oi}{OP} \quad fi := FP + FL$

$fh := hi - fi \quad KO := \sqrt{KP^2 + OP^2} \quad hk := \dfrac{KP \cdot fh}{KO} \quad fk := \dfrac{OP \cdot fh}{KO} \quad Nf := Mf$

$Nk := \sqrt{Nf^2 - fk^2} \quad Nh := hk + Nk \quad Oh := \dfrac{KO \cdot Oi}{OP} \quad NO := Oh - Nh$

$KN := KO - NO$



$$N1 := \frac{OP \cdot KN}{KO} \quad K1 := \frac{KP \cdot KN}{KO} \quad FK := KP - FP \quad F1 := FK - K1$$

$$NX := \sqrt{(FX + N1)^2 + F1^2} \quad XY := \frac{NX \cdot IX}{IX - N1} \quad Fm := \frac{F1 \cdot XY}{NX} \quad Km := FK - Fm$$

$$Fo := \frac{F1 \cdot FX}{FX + N1} \quad Xo := \frac{NX \cdot Fo}{F1} \quad mo := Fm - Fo \quad Ym := \frac{FX \cdot mo}{Fo}$$

$$FI := 2 \cdot BF \quad IL := \sqrt{FL^2 + FI^2} \quad IS := \frac{IL \cdot IX}{IX - ML} \quad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

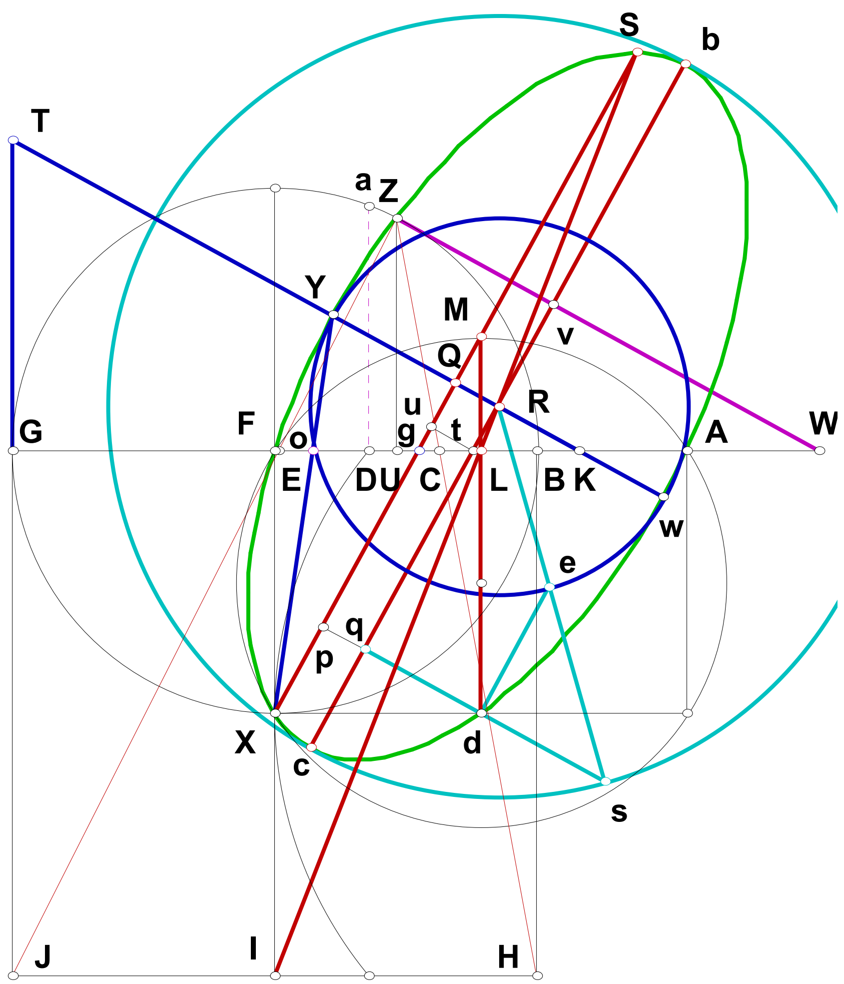
$$Xn := \sqrt{IX^2 - In^2} \quad QR := \frac{Xn \cdot QX}{IS - In} \quad KT := \sqrt{GK^2 + GT^2} \quad KY := \frac{KT \cdot Ym}{GT}$$

$$KQ := \frac{GK \cdot Qg}{GT} \quad RY := KY - KQ + QR \quad Xd := FL \quad dp := \frac{GK \cdot Xd}{KT}$$

$$pq := QR \quad dq := dp - pq \quad Xp := \frac{GT \cdot Xd}{KT} \quad Qp := QX - Xp \quad Rq := Qp$$

$$er := dq \quad Re := RY \quad Rr := \sqrt{Re^2 - er^2} \quad Rs := \frac{Re \cdot Rq}{Rr}$$

Is the segment Zv equal to the perpendicular for the ellipse?



$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BC := AC - AB \quad BE := AE - AB$$

$$CE := BE - BC \quad EG := BG - BE \quad CU := \frac{BC \cdot CE}{BC + EG} \quad BU := BC + CU$$

$$GU := BG - BU \quad UZ := \sqrt{BU \cdot GU} \quad UW := \frac{GK \cdot UZ}{GT} \quad Gg := GK - Kg$$

$$tu := QR \quad gt := \frac{KT \cdot tu}{GK} \quad Gt := Gg + gt \quad GW := GU + UW$$

$$Wt := GW - Gt \quad tv := \frac{GT \cdot Wt}{KT} \quad Kt := GK - Gt \quad Rt := \frac{GT \cdot Kt}{KT}$$

$$Rv := tv - Rt \quad bc := 2 \cdot Rs \quad Rc := Rs \quad cv := Rc + Rv \quad Yw := 2 \cdot RY$$

$$WZ := \frac{KT \cdot UZ}{GT} \quad Wv := \frac{GK \cdot tv}{GT} \quad Zv := \left|WZ - Wv\right|$$

$$N_1 := Yw \quad N_2 := bc \quad N_3 := cv \quad N_4 := bc$$

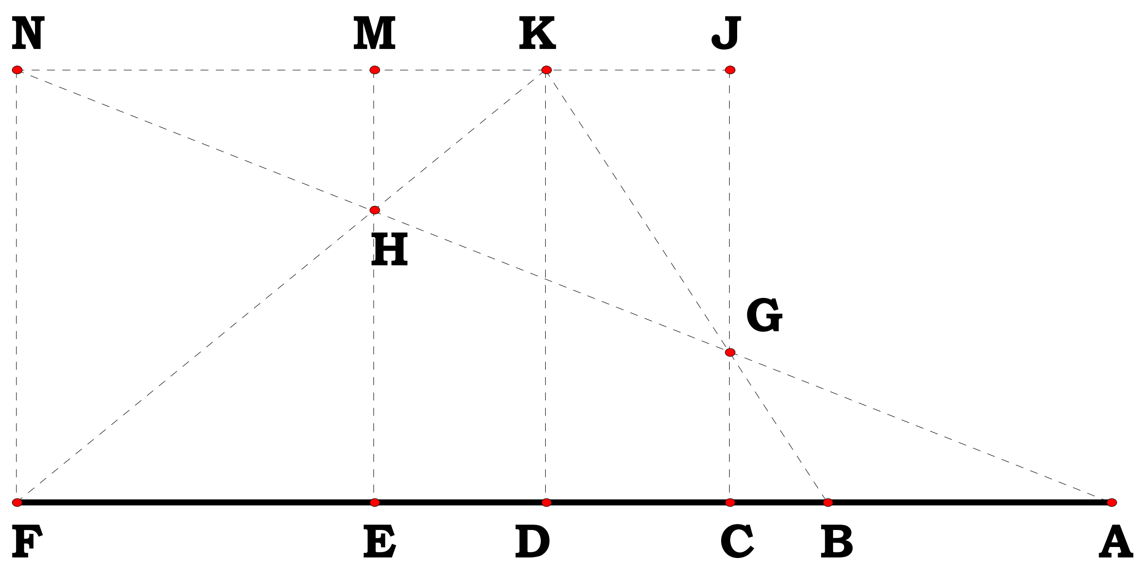
$$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2} - Zv = 0 \qquad \frac{AC}{2 \cdot AB} - 2^{\frac{1}{3}} = 0$$

$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2}$ is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.

081899

Promptly writing this up in 0816 2015

Exponential series by changing the unit, in other words, the same way as done inside a circle only this circle is getting smaller.



$$\left(\frac{AF}{AC}\right)^2 - \frac{BF}{BC} = 0 \qquad \sqrt{\frac{BF}{BC}} - \frac{AF}{AC} = 0$$

$$N_1 := 5.05354 \quad N_2 := 14.49917 \quad N_3 := 5.71500$$

$$AC := N_1 \quad AF := N_2 \quad FN := N_3 \quad CF := AF - AC$$

$$CG := \frac{FN \cdot AC}{AF} \quad EF := \frac{CF}{2} \quad EH := \frac{FN \cdot (AC + EF)}{AF}$$

$$KN := \frac{EF \cdot FN}{EH} \quad JK := CF - KN \quad BC := \frac{JK \cdot CG}{FN - CG} \quad BF := BC + CF$$

$$AD := AC + JK \quad BD := BC + JK$$

$$CF - (N_2 - N_1) = 0$$

$$CG - \frac{N_3 \cdot N_1}{N_2} = 0 \quad EF - \frac{N_2 - N_1}{2} = 0 \quad EH - \frac{N_3 \cdot (N_1 + N_2)}{2 \cdot N_2} = 0$$

$$KN - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \quad JK - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \quad BC - \frac{N_1^{\,2}}{N_1 + N_2} = 0$$

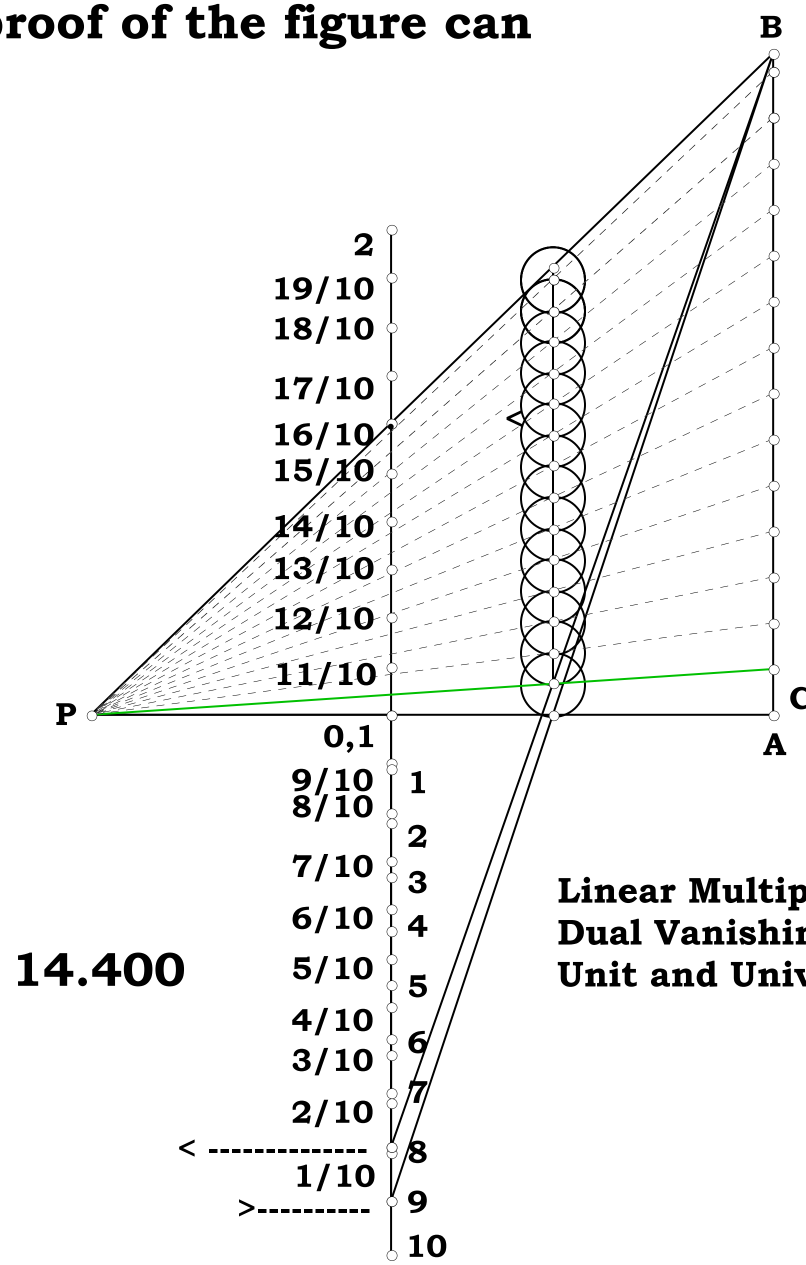
$$BF := \frac{N_2^{\,2}}{N_1 + N_2} \quad AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0 \quad BD - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$$

Since the figure only uses proportion, which has been proven any proof of the figure can be left for an exersize.

1
2
3
4
5
6
7
8
9
10

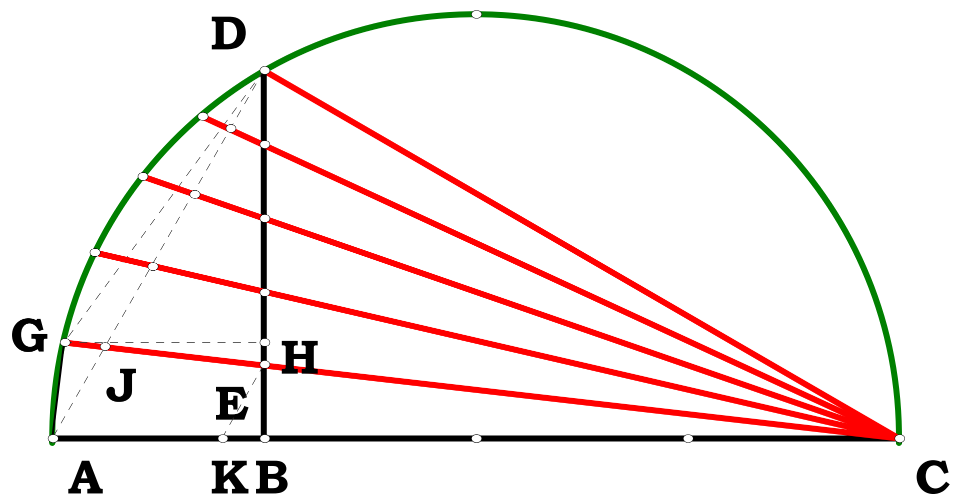
X 1/10
X 2/10
X 3/10
X 4/10
X 5/10
X 6/10
X 7/10
X 8/10
X 9/10
X 1
X 11/10
X 12/10
X 13/10
X 14/10
X 15/10
X 16/10
X 17/10
X 18/10
X 19/10
X 2

$$\frac{AB}{AC} = 14.400$$



Linear Multiplication and Division,
Dual Vanishing Point Method
Unit and Universe of Discourse.

In process. POR something or other.



$N_1 := 4 \quad N_2 := 5 \quad AC := 1 \quad \delta := 1 .. N_2$

$AB := \frac{AC}{N_1} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC} \quad BE_\delta := \frac{BD \cdot \delta}{N_2} \quad CE_\delta := \sqrt{(BE_\delta)^2 + BC^2}$

$CG_\delta := \frac{BC \cdot AC}{CE_\delta} \quad AD := \sqrt{AB^2 + BD^2} \quad AG_\delta := \frac{BE_\delta \cdot AC}{CE_\delta} \quad EG_\delta := CG_\delta - CE_\delta \quad EH_\delta := \frac{BE_\delta \cdot EG_\delta}{CE_\delta}$

$GH_\delta := \frac{BC \cdot EH_\delta}{BE_\delta} \quad BH_\delta := BE_\delta + EH_\delta \quad DH_\delta := BD - BH_\delta \quad DG_\delta := \sqrt{(GH_\delta)^2 + (DH_\delta)^2}$

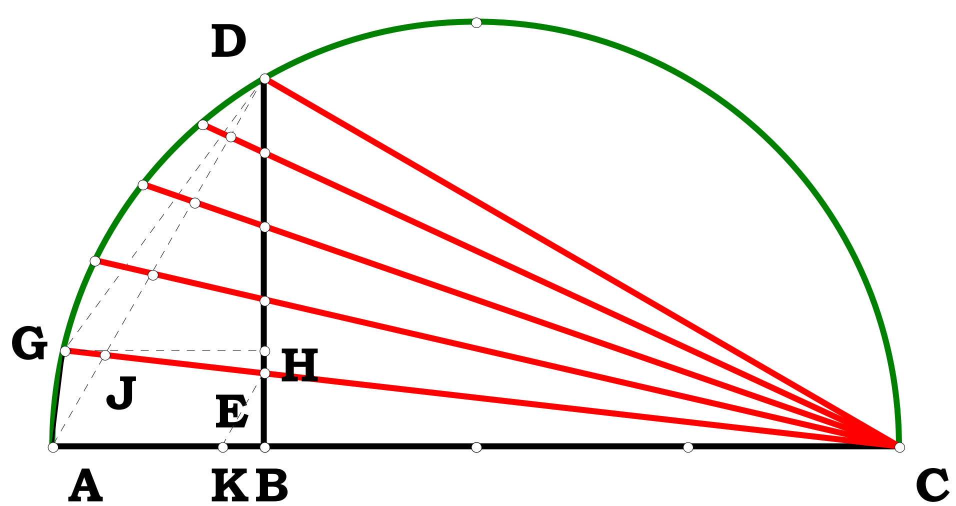
$BK_\delta := \frac{AB \cdot BE_\delta}{BD} \quad CK_\delta := BC + BK_\delta \quad CJ_\delta := \frac{CE_\delta \cdot AC}{CK_\delta} \quad EJ_\delta := CJ_\delta - CE_\delta$

$AG_\delta =$	$\frac{\delta}{\sqrt{\delta^2 + N_1 \cdot N_2^2 - N_2^2}} =$
0.114708	0.114708
0.225018	0.225018
0.327327	0.327327
0.419314	0.419314
0.5	0.5

$DG_\delta =$	$(N_2 - \delta) \cdot \sqrt{\frac{(N_1 - 1)}{\left[N_1 \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)\right]}} =$
0.39736	0.39736
0.292306	0.292306
0.188982	0.188982
0.090784	0.090784
0	0

$EG_\delta =$	$\frac{\left(N_2^2 - \delta^2\right) \cdot \sqrt{N_1 - 1}}{N_2 \cdot N_1 \cdot \sqrt{\delta^2 + N_2^2 \cdot N_1 - N_2^2}} =$
0.238416	0.238416
0.204614	0.204614
0.151186	0.151186
0.081706	0.081706
0	0

$CG_\delta =$	$N_2 \cdot \frac{(N_1 - 1)}{\sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}} =$
0.993399	0.993399
0.974355	0.974355
0.944911	0.944911
0.907841	0.907841
0.866025	0.866025



$$BE_\delta = \quad \sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} \cdot \frac{\delta}{N_2} =$$

BE_δ	$\sqrt{\frac{(N_1-1)}{(N_1 \cdot N_1)}} \cdot \frac{\delta}{N_2}$
0.086603	0.086603
0.173205	0.173205
0.259808	0.259808
0.34641	0.34641
0.433013	0.433013

$$EJ_\delta = \quad \frac{(N_2 - \delta) \cdot \sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}}{N_2 \cdot N_1 \cdot (N_2 \cdot N_1 - N_2 + \delta)} =$$

EJ_δ	$\frac{(N_2-\delta)\cdot\sqrt{(N_1-1)\cdot(\delta^2+N_2^2\cdot N_1-N_2^2)}}{N_2\cdot N_1\cdot(N_2\cdot N_1-N_2+\delta)}$
0.188746	0.188746
0.135837	0.135837
0.088192	0.088192
0.043481	0.043481
0	0

$$AD = 0.5 \sqrt{\frac{1}{N_1}} = 0.5$$

$$CJ_\delta = \quad \frac{\sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}}{(N_2 \cdot N_1 - N_2 + \delta)} =$$

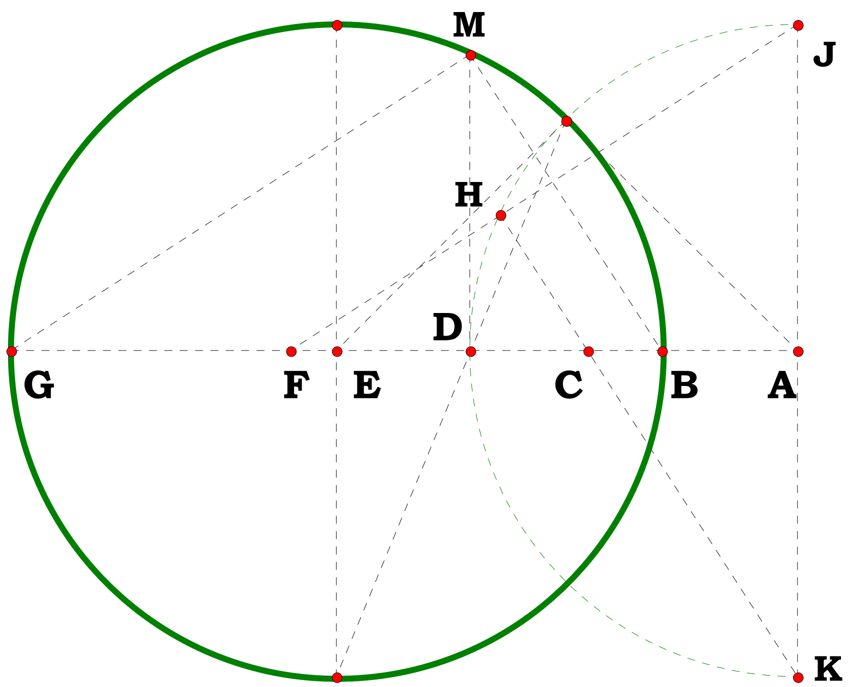
CJ_δ	$\frac{\sqrt{(N_1-1)\cdot(\delta^2+N_2^2\cdot N_1-N_2^2)}}{(N_2\cdot N_1-N_2+\delta)}$
0.943729	0.943729
0.905577	0.905577
0.881917	0.881917
0.869616	0.869616
0.866025	0.866025

$$\frac{AD}{AG_\delta} = \quad \frac{\sqrt{\frac{\left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}{N_1}}}{\delta} =$$

$\frac{AD}{AG_\delta}$	$\frac{\sqrt{\frac{(\delta^2+N_2^2\cdot N_1-N_2^2)}{N_1}}}{\delta}$
4.358899	4.358899
2.222049	2.222049
1.527525	1.527525
1.192424	1.192424
1	1

$$\frac{CG_\delta}{EG_\delta} = \quad \mathrm{if}\left(N_2^2 - \delta^2, \frac{N_2^2 \cdot N_1}{N_2^2 - \delta^2}, 0\right) = \mathrm{if}\left(EJ_\delta, \frac{CJ_\delta}{EJ_\delta}, 0\right) = \mathrm{if}\left(EJ_\delta, \frac{N_2 \cdot N_1}{N_2 - \delta}, 0\right) =$$

$\frac{CG_\delta}{EG_\delta}$	$\mathrm{if}\left(N_2^2-\delta^2, \frac{N_2^2\cdot N_1}{N_2^2-\delta^2}, 0\right)$	$\mathrm{if}\left(EJ_\delta, \frac{CJ_\delta}{EJ_\delta}, 0\right)$	$\mathrm{if}\left(EJ_\delta, \frac{N_2\cdot N_1}{N_2-\delta}, 0\right)$
4.166667	4.166667	5	5
4.761905	4.761905	6.666667	6.666667
6.25	6.25	10	10
11.111111	11.111111	20	20
$7.800463 \cdot 10^{15}$	0	0	0



07/09/00 Alternate Method Quad Roots

$N_1 := 1.79201 \quad N_2 := 10.41743$

$AB := N_1 \quad AG := N_2 \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

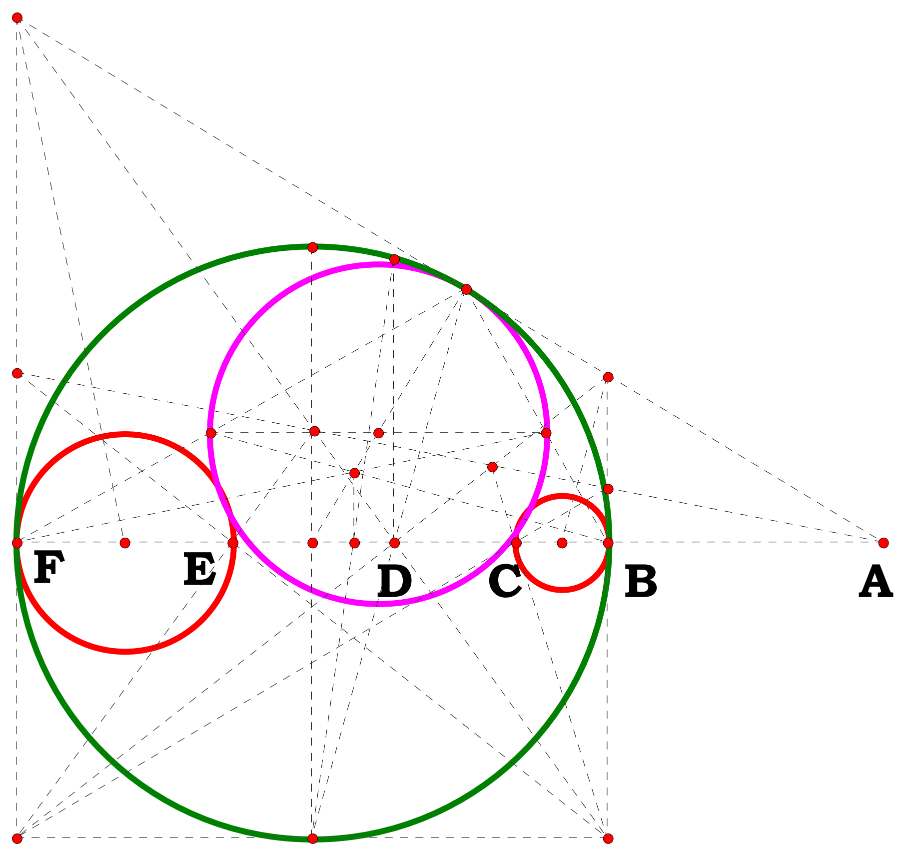
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \dfrac{GM \cdot AJ}{BM}$

$AC := \dfrac{BM \cdot AK}{GM}$

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \qquad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

000720a Quad Roots via Tangent Circles.



$N_1 := 3.73926 \qquad N_2 := 11.78259$

$AB := N_1 \qquad AF := N_2 \qquad AD := \sqrt{AB \cdot AF}$

$BF := AF - AB \qquad Bd := \dfrac{BF}{2}$

$BD := AD - AB \qquad Dd := Bd - BD$

$DH := \sqrt{Bd^2 + Dd^2} \qquad HL := \dfrac{Bd \cdot BF}{DH}$

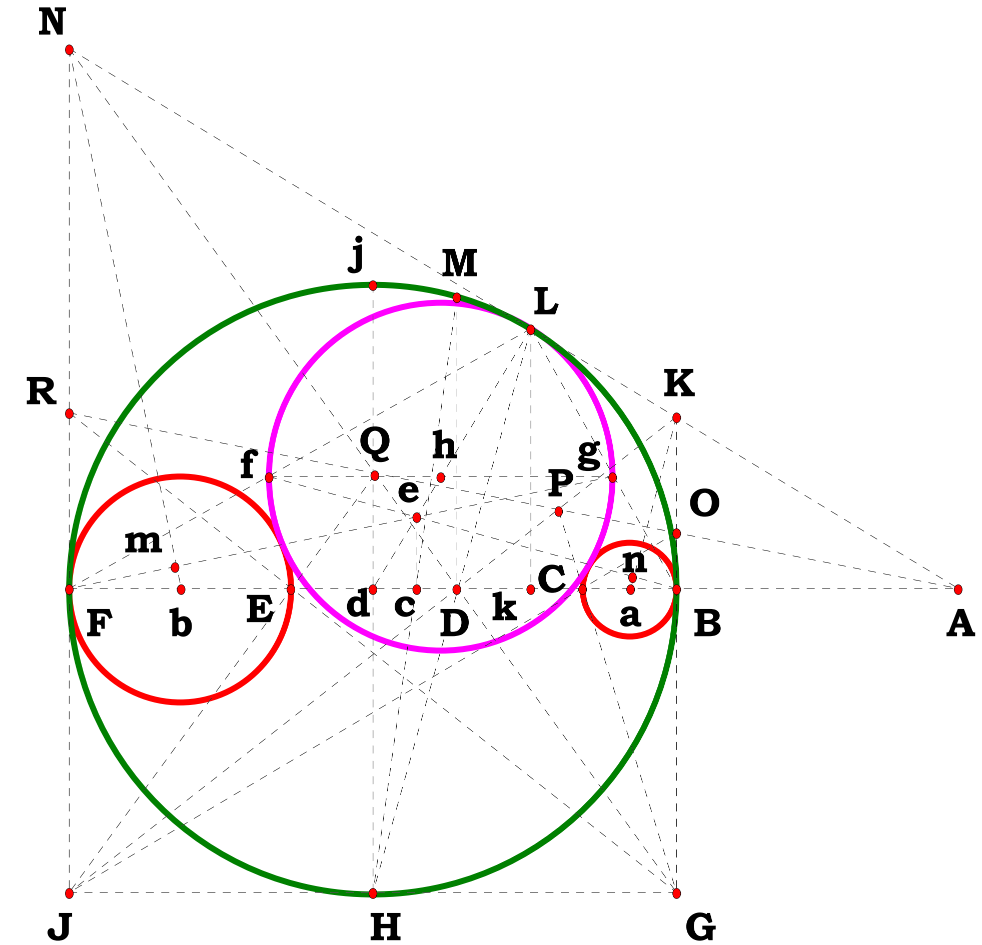
$DL := HL - DH \qquad Dk := \dfrac{Dd \cdot DL}{DH}$

$Bk := Bd - (Dd + Dk) \qquad Lk := \dfrac{Bd \cdot Dk}{Dd}$

$JN := \dfrac{Bd \cdot BF}{BD} \qquad GK := \dfrac{Bd \cdot BF}{(Bd + Dd)}$

$FN := JN - Bd \qquad BK := GK - Bd$

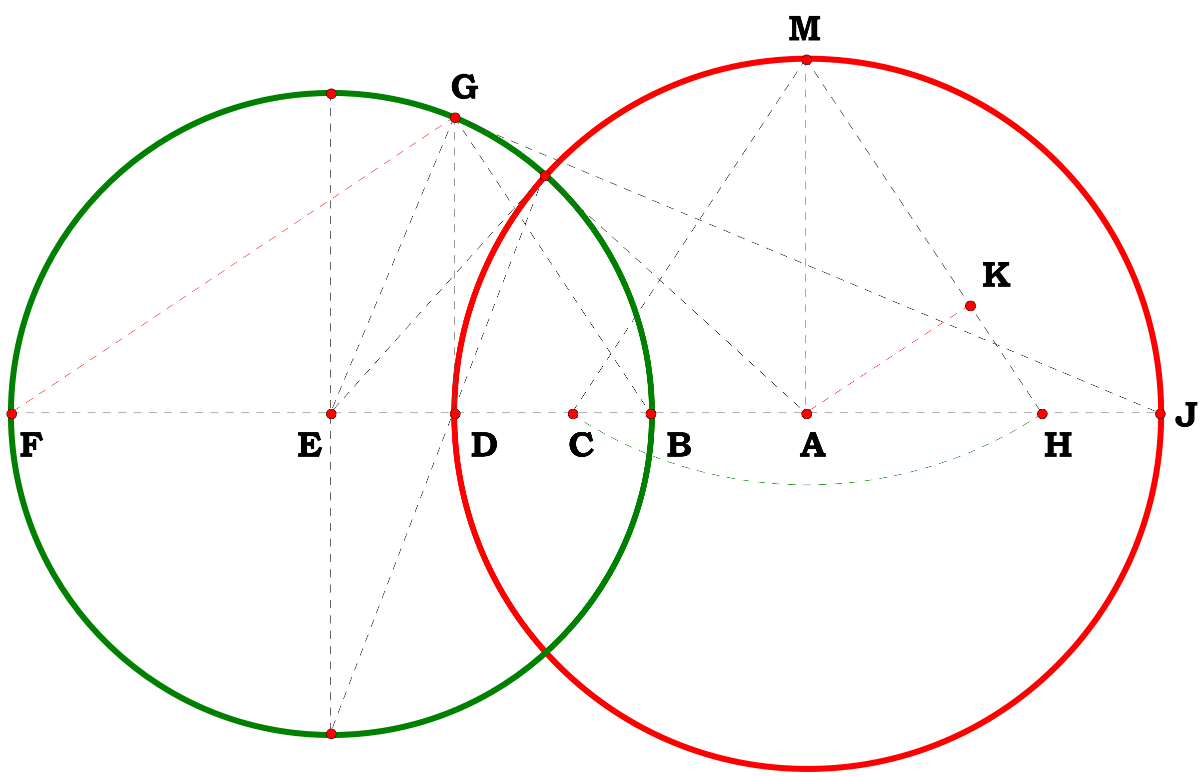
$Ak := Bk + AB \qquad AF - \dfrac{BF \cdot FN}{FN - BK} = 0 \qquad \dfrac{AF}{FN} - \dfrac{Ak}{Lk} = 0$ i.e., A, K, L and N are colinear.



$DF := Bd + Dd \qquad DM := \sqrt{BD \cdot DF} \qquad cd := \dfrac{Dd \cdot Bd}{Bd + DM} \qquad dk := Bd - Bk \qquad ce := \dfrac{Lk \cdot cd}{dk} \qquad Fb := \dfrac{ce \cdot FN}{Bd + cd} \qquad Ba := \dfrac{ce \cdot BK}{Bd - cd} \qquad AE := AF - 2 \cdot Fb$

$AC := AB + 2 \cdot Ba \qquad \left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$ etc., etc.

OOO720b Quad Roots by equal angles.



$N_1 := 2.07320$

$N_2 := 10.53987$

$AB := N_1 \quad AF := N_2$

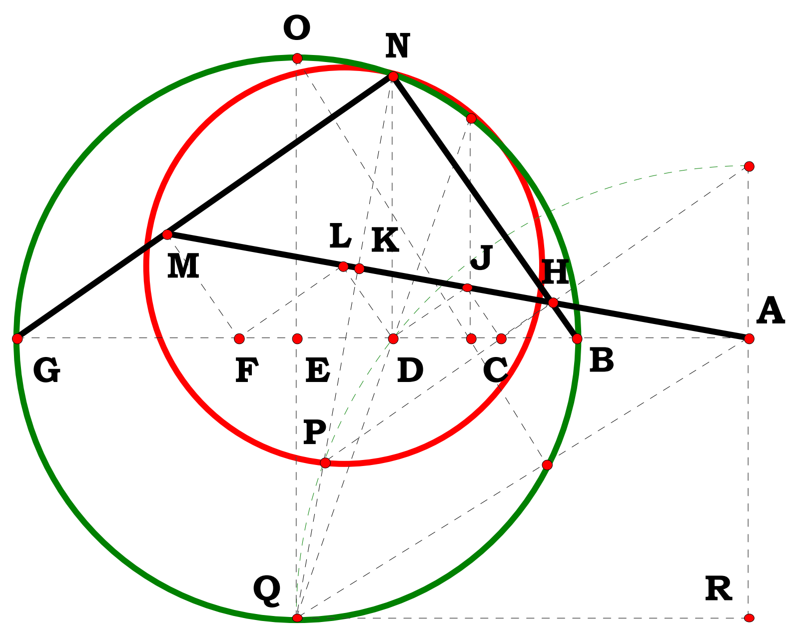
$AD := \sqrt{AB \cdot AF} \quad BF := AF - AB$

$AM := AD \quad DF := AF - AD$

$BD := AD - AB \quad DG := \sqrt{DF \cdot BD}$

$BG := \sqrt{DG^2 + BD^2} \quad AC := \dfrac{BD \cdot AD}{DG}$

$AC - \left(AB^3 \cdot AF\right)^{\frac{1}{4}} = 0$



08/01/00 Alternate Method Quad Roots

$\mathbf{N := 5 \quad AB := 1 \quad AG := AB \cdot N}$

$\mathbf{BG := AG - AB \quad BE := \dfrac{BG}{2} \quad AD := \sqrt{AB \cdot AG}}$

$\mathbf{AE := AB + BE \quad DE := AE - AD \quad NY := DE}$

$\mathbf{BD := AD - AB \quad DG := BG - BD \quad EQ := BE}$

$\mathbf{DN := \sqrt{BD \cdot DG} \quad NQ := \sqrt{DE^2 + (DN + EQ)^2}}$

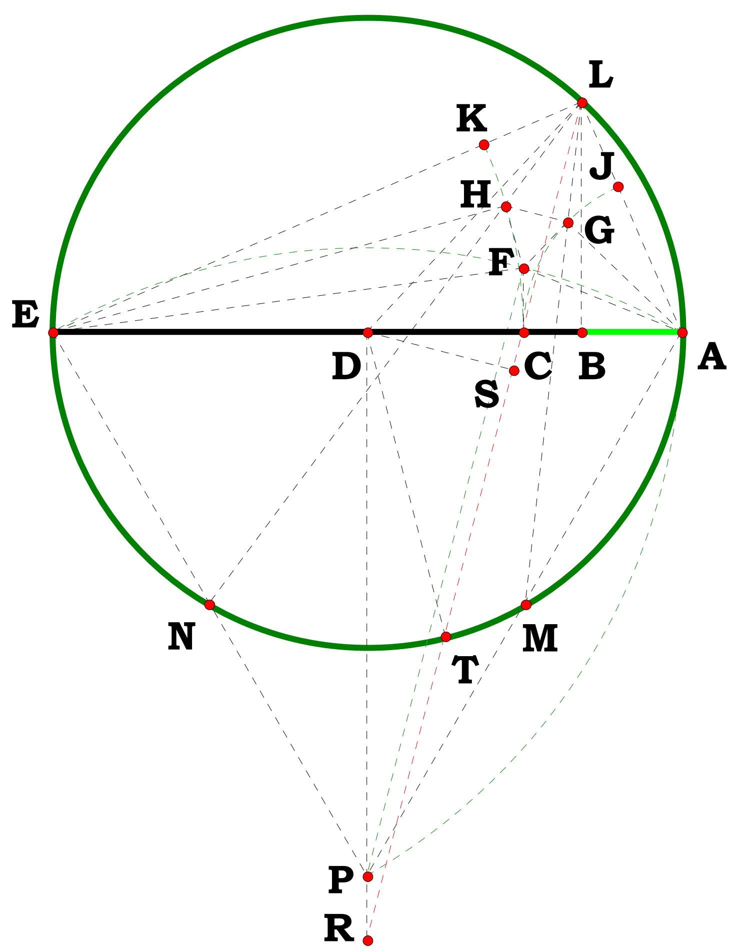
$\mathbf{QR := AE \quad OQ := BG \quad NO := \sqrt{OQ^2 - NQ^2}}$

$\mathbf{PQ := \dfrac{NO \cdot 2 \cdot QR}{OQ} \quad NP := NQ - PQ \quad MN := \sqrt{\dfrac{NP^2}{2}} \quad BN := \sqrt{BD^2 + DN^2} \quad GN := \sqrt{DG^2 + DN^2}}$

$\mathbf{GM := GN - MN \quad FG := \dfrac{BG \cdot GM}{GN} \quad AF := AG - FG \quad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0}$

In Trisection What Is AB? 08/02/00

In the trisection figure given and given AC as the Unit what is AB?



$AC := .884 \quad AE := 3.521 \quad AD := \frac{AE}{2} \quad EP := AE \quad DE := AD \quad DP := \sqrt{EP^2 - DE^2}$

$FP := EP \quad CE := AE - AC \quad CD := CE - DE \quad CF := \sqrt{FP^2 - CD^2} - DP \quad PR := CF \quad DR := DP + PR$

$CR := \sqrt{CD^2 + DR^2} \quad CS := \frac{CD^2}{CR} \quad DS := \sqrt{CD^2 - CS^2} \quad DL := AD \quad LS := \sqrt{DL^2 - DS^2} \quad RS := CR - CS$

$LR := RS + LS \quad BD := \frac{CD \cdot LR}{CR} \quad AB := AD - BD \quad ST := LS \quad RT := RS - ST$

In trisection the length RT to the similarity point is equal to the radius of the circle.

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

$$AD - \frac{AE}{2} = 0 \quad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \quad CF - \left(\frac{\sqrt{4 \cdot AC \cdot AE - 4 \cdot AC^2 + 3 \cdot AE^2}}{2} - \frac{\sqrt{3} \cdot AE}{2}\right) = 0$$

$$CE - (AE - AC) = 0 \quad CD - \left(\frac{1}{2} \cdot AE - AC\right) = 0 \quad DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0$$

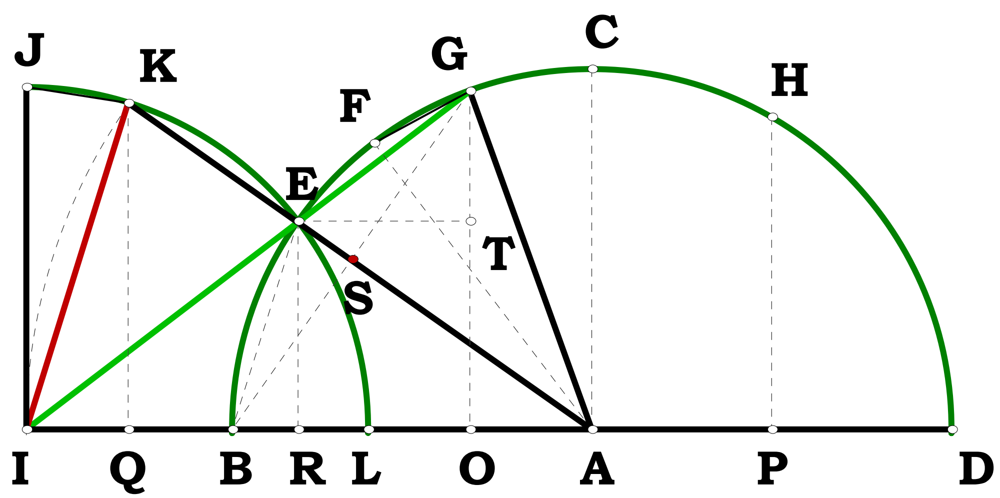
$$CR - AE = 0 \quad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \quad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \quad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0 \quad RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0$$

$$BD - \left(\frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3\right) = 0 \quad AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \quad AB \cdot AE^2 - AC^2 (3 \cdot AE - 2 \cdot AC) = 0$$

080300 Trisection

If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.



$$N := 3 \qquad BD := 2$$

$$AB := \frac{BD}{2} \quad AD := AB \quad AP := \frac{AD}{2} \quad BP := AB + AP \quad BO := \frac{BP}{N} \quad AE := AB$$

$$DO := BD - BO \quad GO := \sqrt{BO \cdot DO} \quad BG := \sqrt{GO^2 + BO^2} \quad BS := \frac{BG}{2} \quad ER := BS \quad TO := ER$$

$$GT := GO - TO \quad AS := \sqrt{AB^2 - BS^2} \quad ES := AE - AS \quad BR := ES \quad OR := BO - BR$$

$$ET := OR \quad IO := \frac{ET \cdot GO}{GT} \quad BI := IO - BO \quad AI := BI + AB \quad BE := \sqrt{ER^2 + BR^2}$$

$$GE := BE \quad GI := \frac{GE \cdot GO}{GT} \quad EI := GI - GE \quad AK := AI \quad IK := EI \quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$$

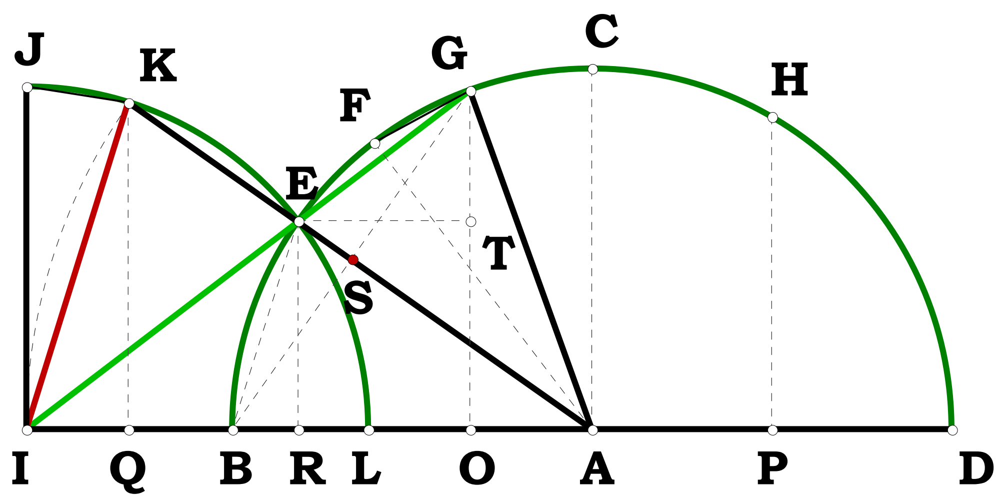
$$EK := AK - AE \quad \frac{EK}{IQ} = 2$$

Some Algebraic Names:

$$\frac{3 \cdot BD}{4 \cdot N} - BO = 0 \quad \frac{BD}{4} \cdot \frac{(4 \cdot N - 3)}{N} - DO = 0 \quad \frac{BD}{(4 \cdot N)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N - 3} - GO = 0 \quad \frac{BD}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BG = 0 \quad \frac{BD}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BS = 0$$

$$\frac{BD}{4} \cdot \sqrt{3} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - \sqrt{\frac{1}{N}} \cdot N\right)}{N} - GT = 0 \quad \frac{BD}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - AS = 0 \quad \frac{BD}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N - 3)}{N}}\right] - ES = 0 \quad \frac{BD}{4} \cdot \frac{\left[3 - 2 \cdot N + \sqrt{\frac{(4 \cdot N - 3)}{N}} \cdot N\right]}{N} - OR = 0$$

$$\frac{-BD}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N-3}\cdot\sqrt{N}-2\cdot\sqrt{4\cdot N-3}\cdot N^{\left(\frac{3}{2}\right)}+4\cdot N^{2}-3\cdot N\right]}{\left[N^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)\right]}-IO=0$$



$$\frac{-BD}{2}\cdot\frac{\left(\sqrt{4\cdot N-3}-2\cdot\sqrt{N}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-BI=0$$

$$\frac{-BD}{\left(-2\cdot\sqrt{4\cdot N-3}+2\cdot\sqrt{N}\right)}\cdot\sqrt{N}-AI=0$$

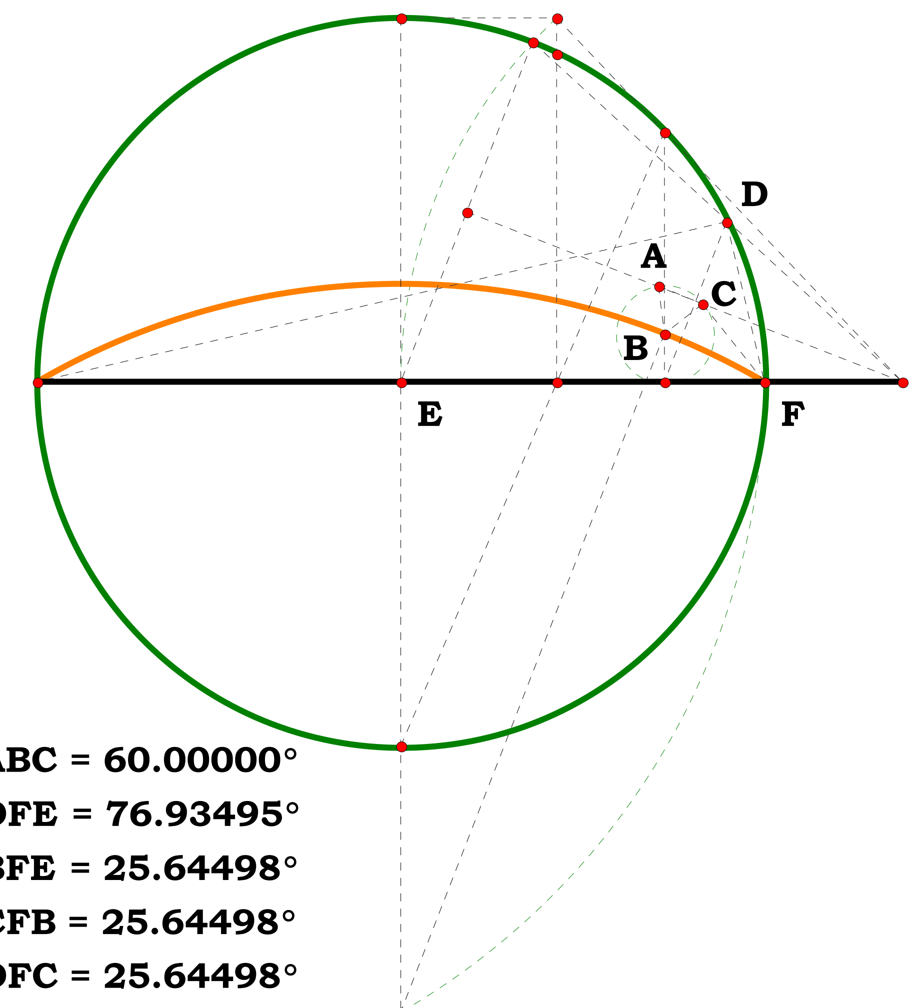
$$\frac{BD}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N}}\cdot\sqrt{4\cdot N-3}}-BE=0$$

$$\frac{BD}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N}+\sqrt{4\cdot N-3}\right)}{\sqrt{N}}}\cdot\frac{\sqrt{4\cdot N-3}}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-GI=0$$

$$\frac{-BD}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N-3}}{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)}-EI=0$$

$$\frac{BD}{4}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-IQ=0 \qquad \frac{BD}{2}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-EK=0$$

000804A Trisection and Square Roots



With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?

I am going to figure this out and then I am going to order the equations a little different at the start to see what happens to all the definitions.

$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

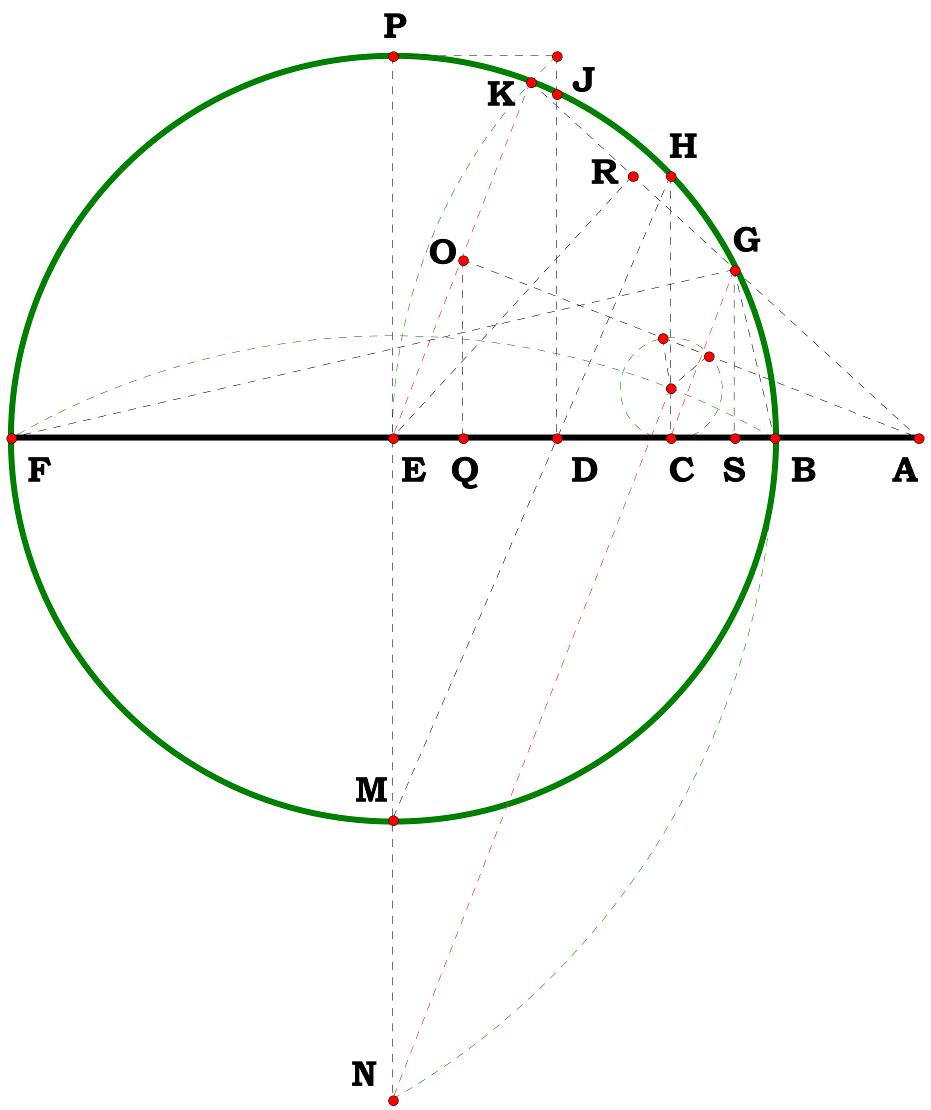
$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$



$N_1 := 1.90557 \qquad N_2 := 12.01265$

$AB := N_1 \qquad AF := N_2 \qquad AD := \sqrt{AB \cdot AF} \qquad BD := AD - AB$

$BF := AF - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

$BE := \dfrac{BF}{2} \qquad EO := \dfrac{BE}{2} \qquad AE := BE + AB \qquad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \dfrac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

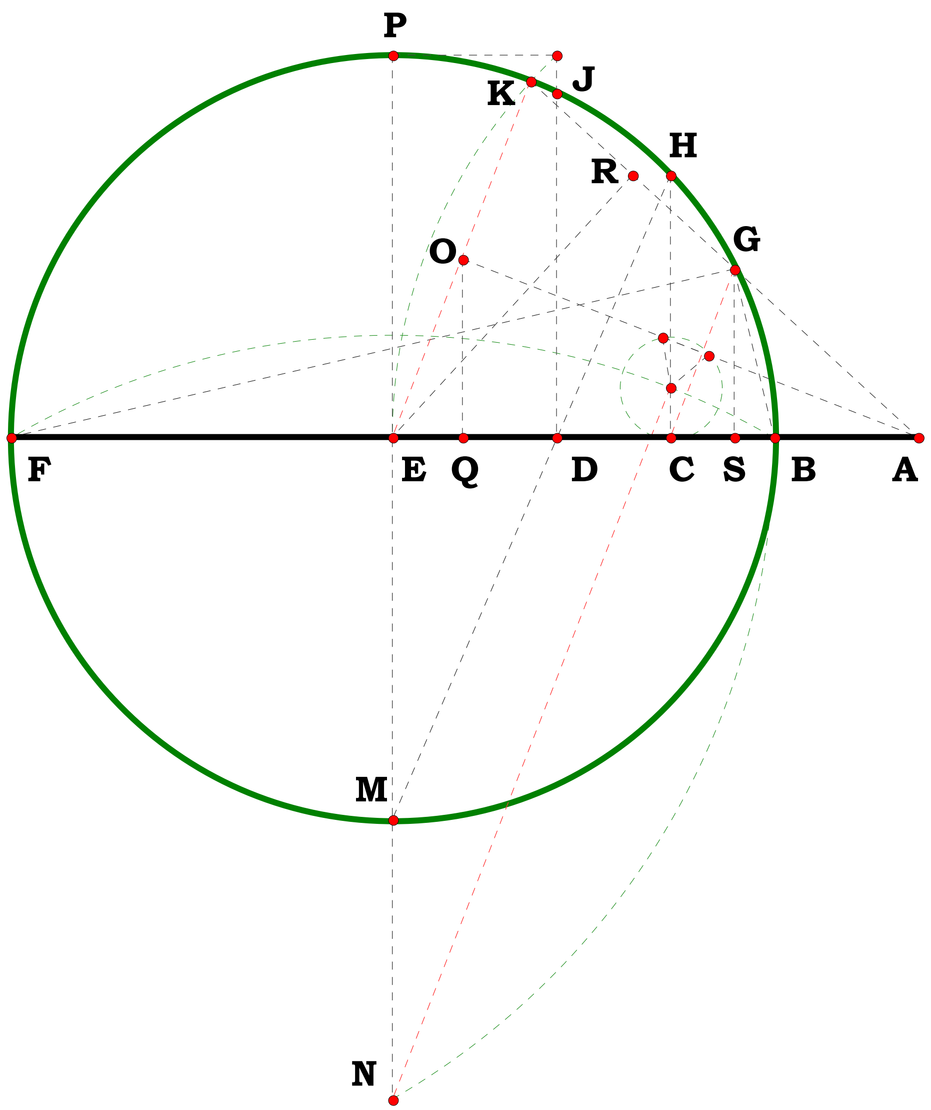
$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$

Definitions:

$AB - N_1 = 0 \qquad AF - N_2 = 0 \qquad AD - \sqrt{N_1 \cdot N_2} = 0 \qquad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0$

$BF - \left(N_2 - N_1\right) = 0 \qquad DF - \left(N_2 - \sqrt{N_1 \cdot N_2}\right) = 0 \qquad DJ - \sqrt{\sqrt{N_1 \cdot N_2} \cdot \left(N_1 + N_2\right) - 2 \cdot N_1 \cdot N_2} = 0$

$BE - \dfrac{N_2 - N_1}{2} \qquad EO - \dfrac{N_2 - N_1}{4} = 0 \qquad AE - \dfrac{N_1 + N_2}{2} = 0 \qquad EQ - \dfrac{\left(N_1 - N_2\right)^2}{8 \cdot \left(N_1 + N_2\right)} = 0$



$$DE - \frac{N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}}{2} = 0 \qquad DM - \frac{\sqrt{\left[N_1^{\,2} + N_2^{\,2} + 2\cdot N_1\cdot N_2 - 2\cdot\sqrt{N_1\cdot N_2}\cdot\left(N_1 + N_2\right)\right]}}{\sqrt{2}} = 0$$

$$HM - \frac{\sqrt{2}\cdot\left(N_1 - N_2\right)^2}{2\cdot\sqrt{N_1^{\,2} + N_2^{\,2} - 2\cdot\sqrt{N_1\cdot N_2}\cdot\left(N_1 + N_2\right) + 2\cdot N_1\cdot N_2}} = 0$$

$$DH - \frac{\left(N_1 + N_2\right)\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{\sqrt{\left(N_1 + N_2\right)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}\right)}} = 0$$

$$CD - \frac{\sqrt{N_1\cdot N_2}\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right) - 4\cdot N_1^{\,2}\cdot N_2 - 4\cdot N_1\cdot N_2^{\,2}}{\left(N_1 + N_2\right)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}\right)} = 0$$

$$CE - \frac{\left(N_1 - N_2\right)^2}{2\cdot\left(N_1 + N_2\right)} = 0 \qquad BC - \frac{N_1\cdot\left(N_2 - N_1\right)}{N_1 + N_2} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{\left(N_1 - N_2\right)^2}}{2} = 0$$

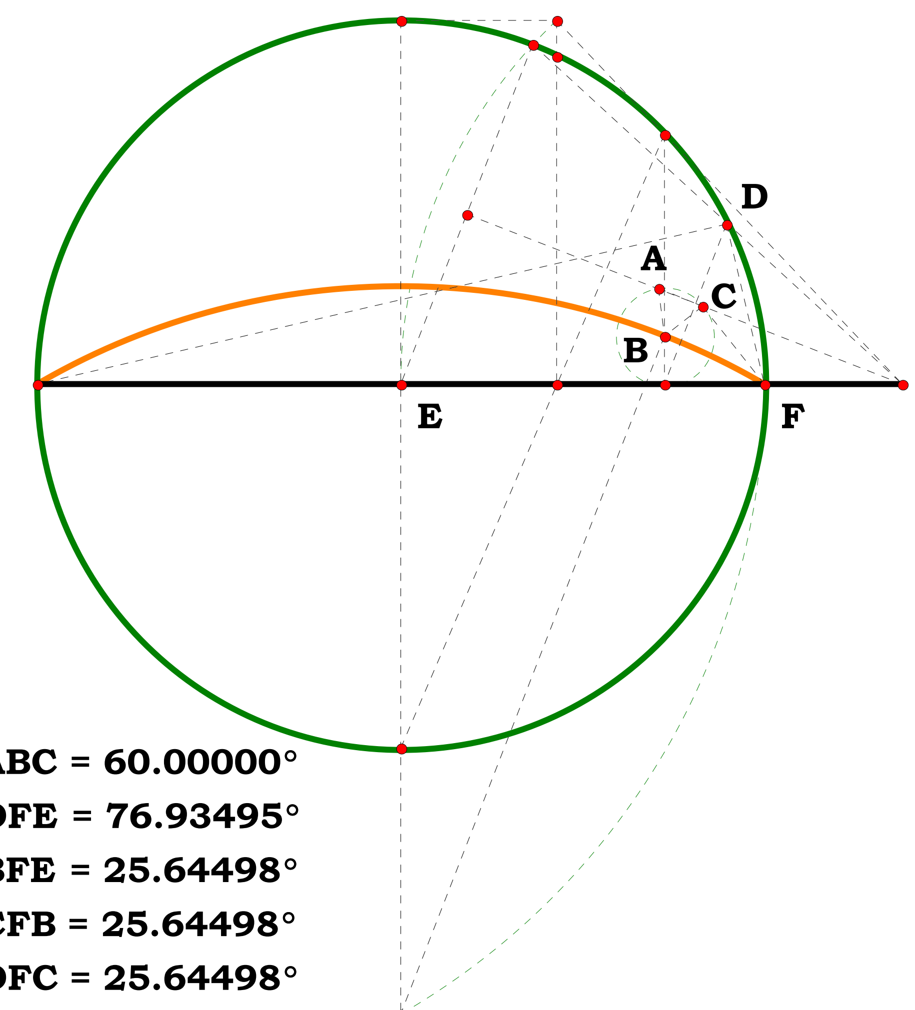
$$KG - \frac{\left(N_1 - N_2\right)^2}{2\cdot\left(N_1 + N_2\right)} = 0 \qquad AG - \frac{2\cdot N_1\cdot N_2}{N_1 + N_2} = 0 \qquad CS - \frac{N_1\cdot N_2\cdot\left(N_1 - N_2\right)^2}{\left(N_1 + N_2\right)^3} = 0$$

$$AS - \frac{N_1\cdot N_2\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right)}{\left(N_1 + N_2\right)^3} = 0$$

$$BS - \frac{N_1^{\,2}\cdot\left(N_2 - N_1\right)\cdot\left(N_1 + 3\cdot N_2\right)}{\left(N_1 + N_2\right)^3} = 0$$

000804B Trisection and Square Roots

With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?



m∠ABC = 60.00000°

m∠DFE = 76.93495°

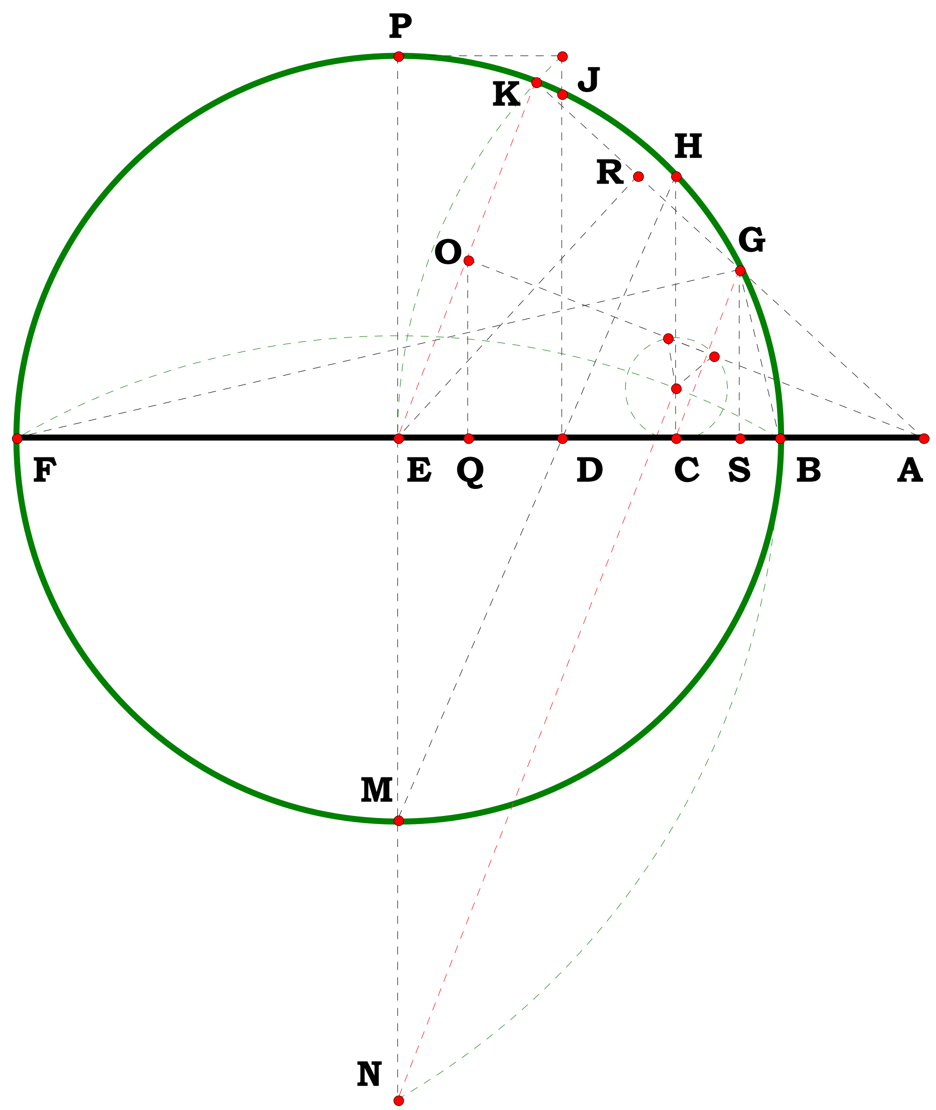
m∠BFE = 25.64498°

m∠CFB = 25.64498°

m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°



$N_1 := 1.90557 \quad N_2 := 10.10708$

$AB := N_1 \quad BF := N_2 \quad AF := AB + BF \quad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB \quad DF := AF - AD \quad DJ := \sqrt{BD \cdot DF}$

$BE := \dfrac{BF}{2} \quad EO := \dfrac{BE}{2} \quad AE := BE + AB \quad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \quad DM := \sqrt{DE^2 + BE^2} \quad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \quad CD := \dfrac{DE \cdot DH}{DM} \quad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \quad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \quad AG := AE - KG$

$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \quad AS := AE - (DE + CD + CS) \quad BS := AS - AB$

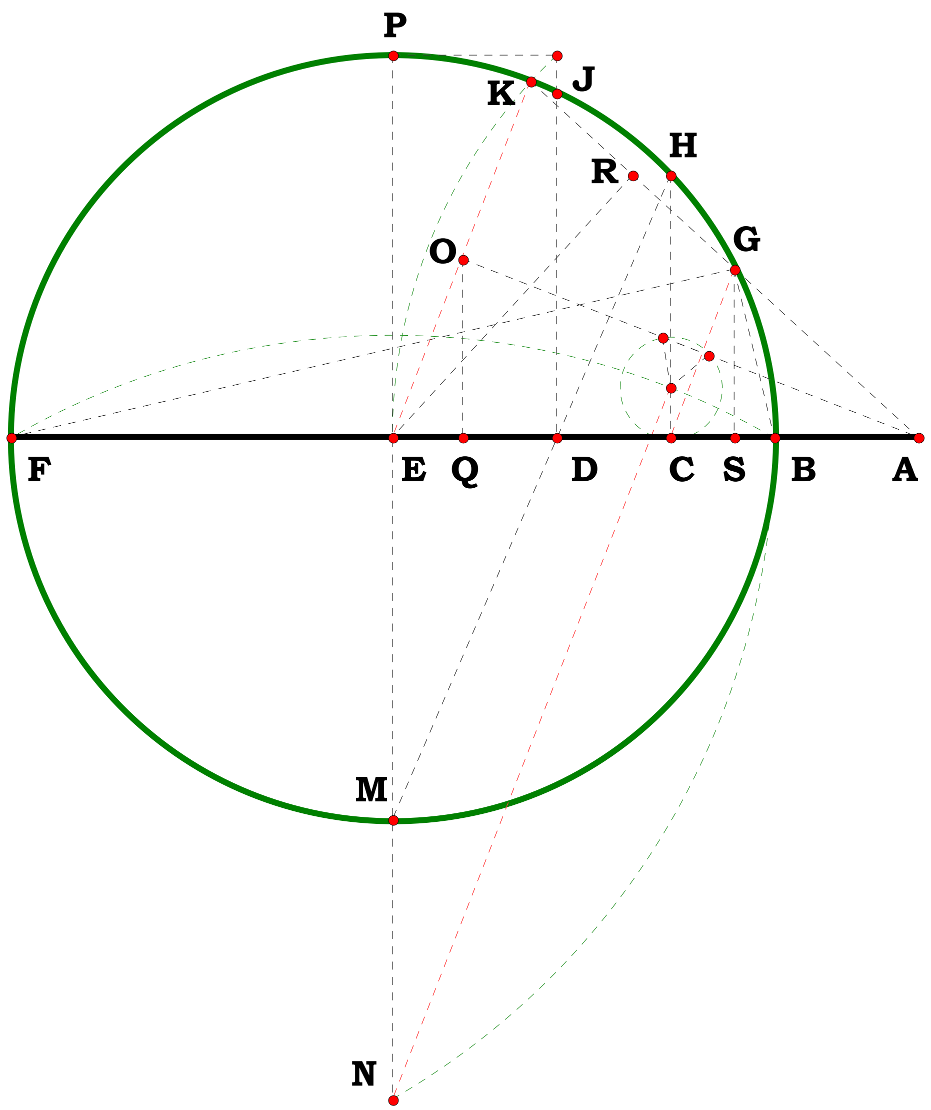
Definitions:

$AB - N_1 = 0 \quad BF - N_2 = 0 \quad AF - (N_1 + N_2) = 0$

$AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0 \quad BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right] \quad DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot (N_1 + N_2)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0 \quad BE - \dfrac{N_2}{2} = 0 \quad EO - \dfrac{N_2}{4} = 0$

$AE - \dfrac{2 \cdot N_1 + N_2}{2} = 0 \quad EQ - \dfrac{N_2^2}{8 \cdot (2 \cdot N_1 + N_2)} = 0 \quad DE - \dfrac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$



$$DM - \frac{\sqrt{2\cdot(2\cdot N_1+N_2)^2 - 4\cdot\sqrt{N_1^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)}}{2} = 0$$

$$HM - \frac{N_2^2}{\sqrt{2\cdot(2\cdot N_1+N_2)^2 - 4\cdot\sqrt{N_1^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)}} = 0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{N_1^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2) - 2\cdot N_1^2 - 2\cdot N_1\cdot N_2\right]}{\sqrt{(2\cdot N_1+N_2)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot(N_1+N_2)}\right]}} = 0$$

$$CD - \frac{\sqrt{N_1^2+N_2\cdot N_1}\cdot\left(8\cdot N_1^2+8\cdot N_1\cdot N_2+N_2^2\right) - 4\cdot N_1\cdot(2\cdot N_1+N_2)\cdot(N_1+N_2)}{(2\cdot N_1+N_2)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{N_1^2+N_2\cdot N_1}\right)} = 0$$

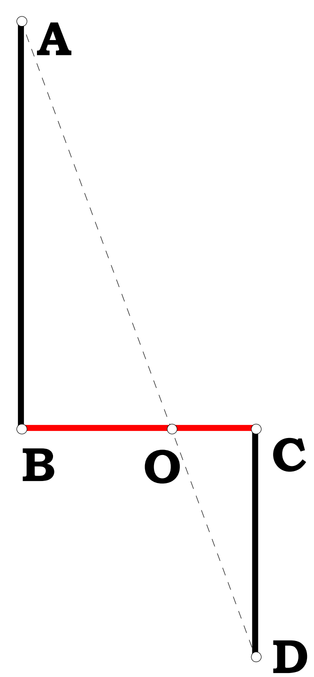
$$CE - \frac{N_2^2}{2\cdot(2\cdot N_1+N_2)} = 0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{N_2^2}}{2} = 0$$

$$KG - \frac{N_2^2}{2\cdot(2\cdot N_1+N_2)} = 0 \qquad AG - \frac{2\cdot N_1\cdot(N_1+N_2)}{2\cdot N_1+N_2} = 0 \qquad CS - \frac{N_1\cdot N_2^2\cdot(N_1+N_2)}{(2\cdot N_1+N_2)^3} = 0$$

$$AS - \frac{N_1\cdot(N_1+N_2)\cdot\left(8\cdot N_1^2+8\cdot N_1\cdot N_2+N_2^2\right)}{(2\cdot N_1+N_2)^3} = 0$$

$$BS - \frac{N_1^2\cdot N_2\cdot(4\cdot N_1+3\cdot N_2)}{(2\cdot N_1+N_2)^3} = 0$$

08/07/00 Proportion Series II



Divide BC into the same ratio as AB:CD.

$$N_1 := 9 \qquad N_2 := 2 \qquad N_3 := 5$$

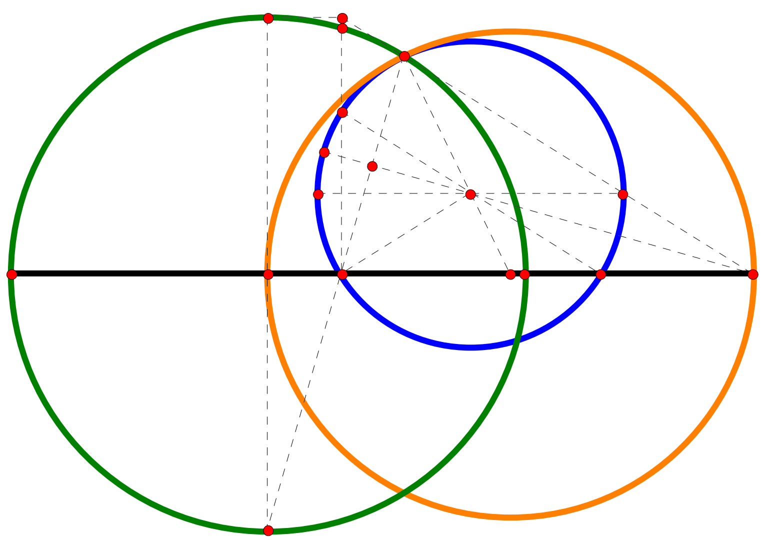
$$AB := N_1 \qquad CD := N_2 \qquad BC := N_3$$

$$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$$

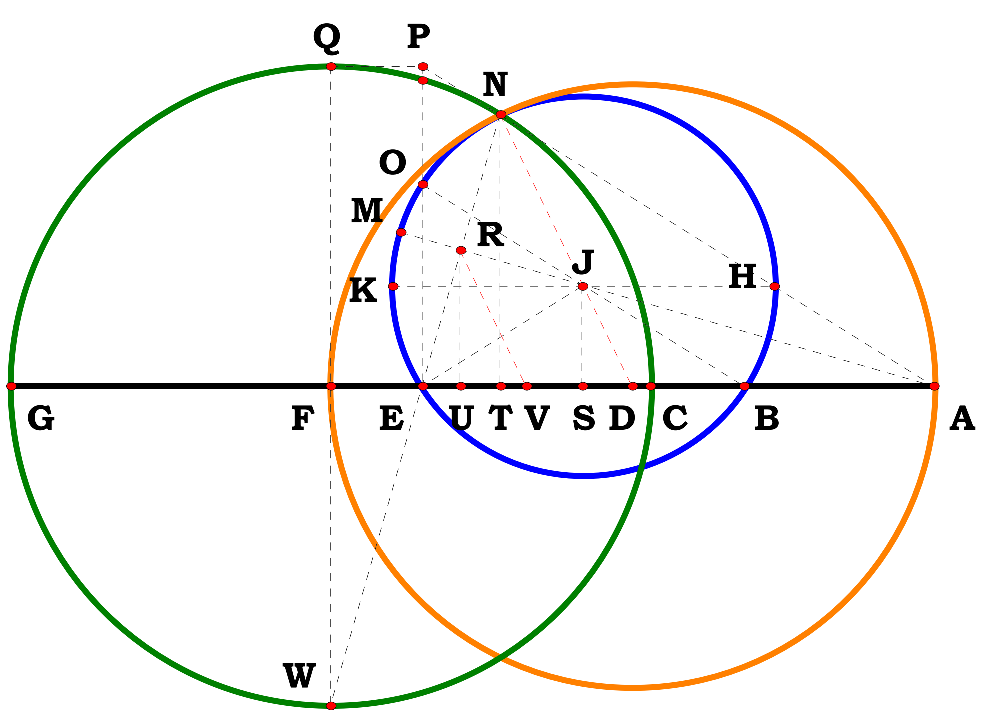
$$BO + CO - BC = 0$$

$$\frac{AB}{CD} - \frac{BO}{CO} = 0 \qquad BO - \frac{N_1 \cdot N_3}{N_1 + N_2} = 0 \qquad CO - \frac{N_2 \cdot N_3}{N_1 + N_2} = 0$$

000822 Square Root and the Archimedean Paper Trisecter.



This square root figure affords another approach to proofing the Archimedean Paper Trisecter.



$N_1 := 3.76152 \qquad N_2 := 12.22818$

$AC := N_1 \qquad AG := N_2 \qquad AE := \sqrt{AC \cdot AG} \qquad CG := AG - AC$

$CF := \dfrac{CG}{2} \qquad AF := AC + CF \qquad AD := \dfrac{AF}{2} \qquad CE := AE - AC$

$EF := CF - CE \qquad EW := \sqrt{EF^2 + CF^2} \qquad NW := \dfrac{CF \cdot CG}{EW}$

$EN := NW - EW \qquad EU := \dfrac{EF \cdot EN}{2 \cdot EW} \qquad AU := AE - EU$

$NT := \dfrac{CF \cdot EN}{EW} \qquad UV := \sqrt{\left(\dfrac{AD}{2}\right)^2 - \left(\dfrac{NT}{2}\right)^2} \qquad AV := AU - UV$

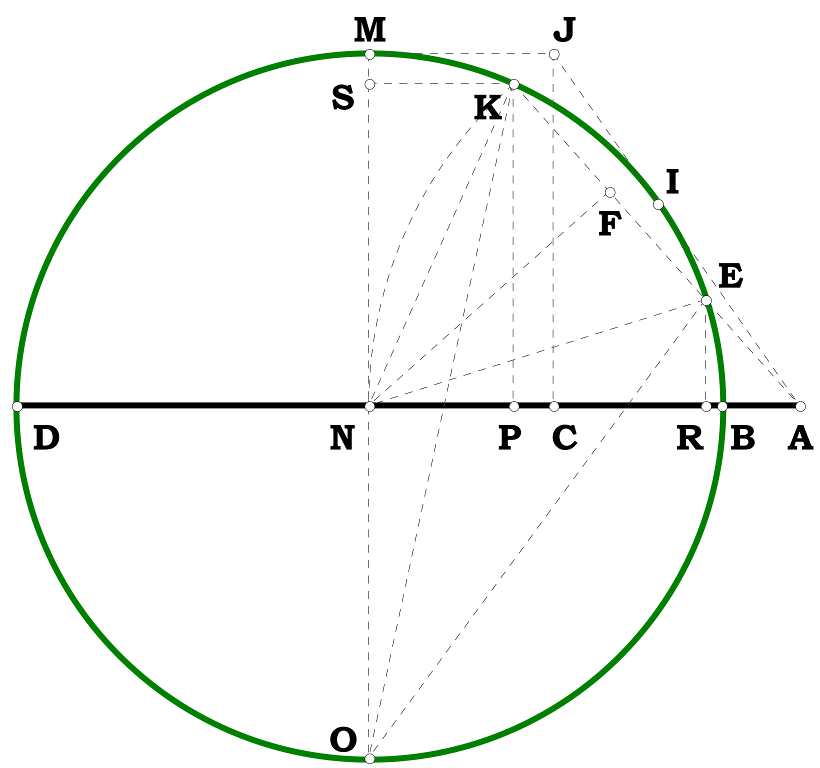
$JS := \dfrac{NT \cdot AD}{2 \cdot AV} \qquad DJ := \dfrac{AD \cdot JS}{NT} \qquad BJ := AD - DJ$

$DS := \dfrac{2 \cdot UV \cdot JS}{NT} \qquad AS := AD + DS \qquad BS := \sqrt{BJ^2 - JS^2} \qquad AB := AS - BS$

$AB - BJ = 0$

08/23/00 Trisection In A Square Root Figure

Given the square root figure drawn for trisection, what is AR given AB and AD? A slightly different apprach than the one on 04.



$N := 4 \quad AB := 2 \quad AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD}$

$BD := AD - AB \quad BN := \dfrac{BD}{2} \quad KN := BN$

$AN := AB + BN \quad AK := AN \quad AP := \dfrac{AK^2 + AN^2 - KN^2}{2 \cdot AN}$

$AF := \dfrac{AP \cdot AN}{AK} \quad FK := AK - AF \quad EF := FK \quad AE := AK - 2 \cdot (EF)$

$AR := \dfrac{AP \cdot AE}{AK} \quad AB \cdot N \cdot \dfrac{\left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} - AR = 0 \quad BR := AR - AB$

$AB \cdot (3 \cdot N + 1) \cdot \dfrac{(N-1)}{(N+1)^3} - BR = 0$

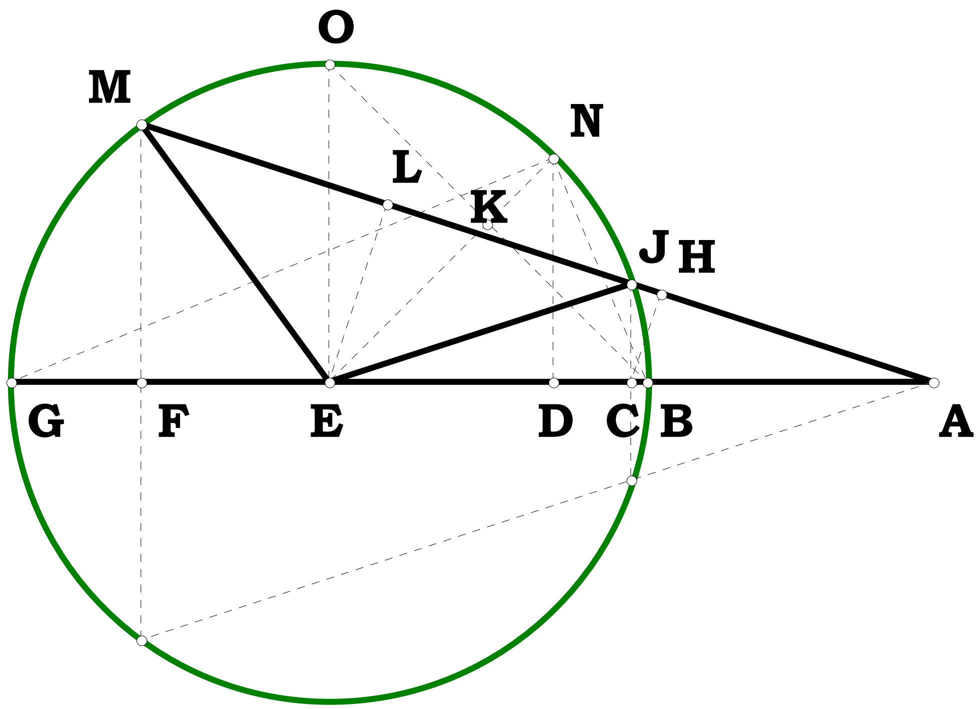
Does KS = FK?

$BP := AP - AB \quad DP := BD - BP \quad NP := BN - BP \quad KS := NP \quad KS - FK = 0$

$\dfrac{AB \cdot (N-1)^2}{4 \cdot (N+1)} - KS = 0$

09/03/00 Ratios In Trisection

How does BF vary with BC? How does DF vary with BC?



$N_1 := 4 \quad N_2 := 8$

$BG := 1 \quad BE := \frac{BG}{2} \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2} \quad EN := BE \quad EK := \frac{BE \cdot BE}{BO}$

$KN := EN - EK \quad BK := \frac{BO}{2} \quad BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE \quad AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC$

$AE := AB + BE \quad JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB \quad DF := BF - BD$

$$BF - \frac{1}{8} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{{N_2}^3} = 0$$

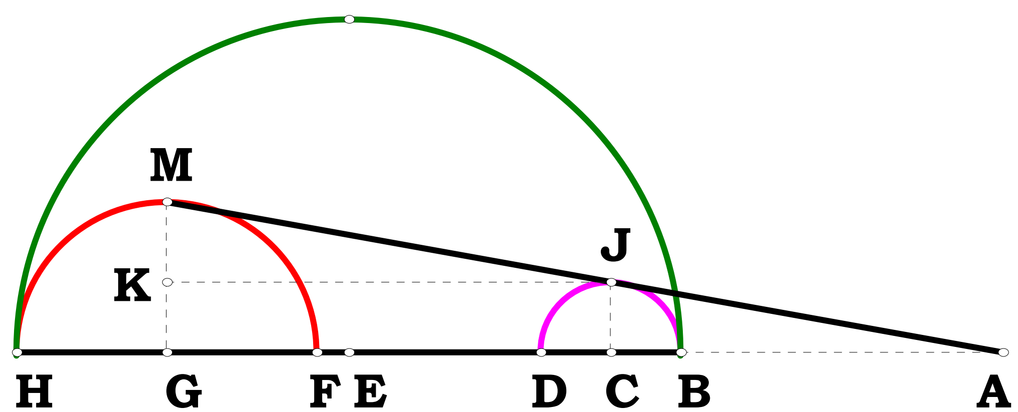
$$\frac{\left(3 - 2 \cdot \sqrt{2}\right) \cdot \left(2 \cdot N_2 - 2 \cdot N_1 + \sqrt{2} \cdot N_2\right)^2 \cdot \left(4 \cdot N_2 - N_1 + 2 \cdot \sqrt{2} \cdot N_2\right)}{2 \cdot N_1 \cdot {N_2}^2} - \frac{BF}{BC} = 0$$

$$\frac{N_1 \cdot \left(2 - \sqrt{2}\right)}{4 \cdot N_2} - BC = 0$$

$$\frac{\left(N_1 - N_2\right) \cdot \left[\left(7 \cdot \sqrt{2} - 10\right) \cdot \left(12 \cdot \sqrt{2} \cdot {N_2}^2 + 17 \cdot {N_2}^2 + 2 \cdot {N_1}^2 - 10 \cdot N_1 \cdot N_2 - 6 \cdot N_1 \cdot \sqrt{2} \cdot N_2\right)\right]}{4{N_2}^3} - DF = 0$$

$$\frac{\left(N_1 - N_2\right) \cdot \left[\left(2 \cdot \sqrt{2} - 3\right) \cdot \left(2 \cdot {N_1}^2 - 10 \cdot N_1 \cdot N_2 - 6 \cdot N_1 \cdot \sqrt{2} \cdot N_2 + 17 \cdot {N_2}^2 + 12 \cdot \sqrt{2} \cdot {N_2}^2\right)\right]}{N_1 \cdot {N_2}^2} - \frac{DF}{BC} = 0$$

Midpoints and Similarity Points 09/18/00



What is AE given the radius of the two circles and the difference between their centers? (External Unit).

$N_1 := 2 \quad N_2 := 3 \quad N_3 := 8 \quad BC := N_1 \quad GH := N_2 \quad CG := N_3$

$CJ := BC \quad GM := GH \quad GK := CJ \quad BH := BC + CG + GH \quad BE := \dfrac{BH}{2}$

$KM := GM - GK \quad JK := CG \quad AG := \dfrac{JK \cdot GM}{KM} \quad AH := AG + GH$

$AB := AH - BH \quad AE := AB + BE \quad AE - \dfrac{N_3 \cdot N_2 - 2 \cdot N_1 \cdot N_2 + N_1{}^2 + N_3 \cdot N_1 + N_2{}^2}{2 \cdot (N_2 - N_1)} = 0$

What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).

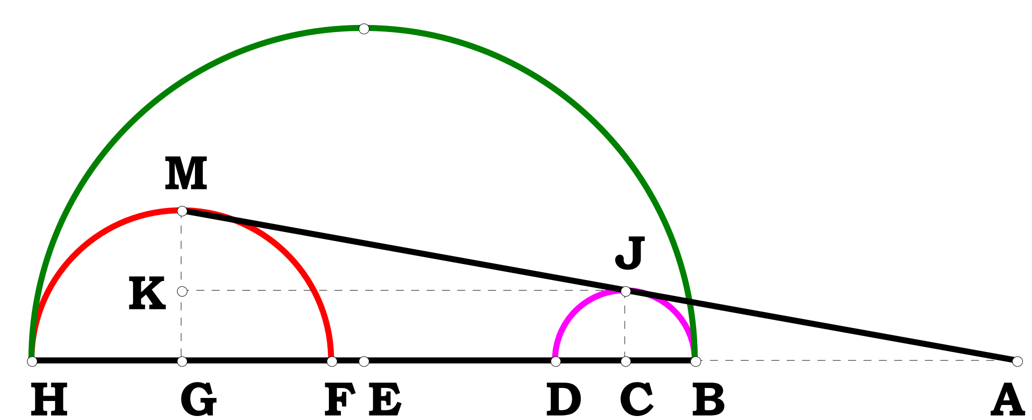
$N_1 := 3 \quad N_2 := 7 \quad N_3 := 4 \quad N_4 := 9$

$BH := 1 \quad BE := \dfrac{BH}{2} \quad BC := BH \cdot \dfrac{N_1}{N_2} \quad GH := BH \cdot \dfrac{N_3}{N_4} \quad CG := BH - (BC + GH)$

$CJ := BC \quad GM := GH \quad GK := CJ \quad KM := GM - GK \quad JK := CG \quad AG := \dfrac{JK \cdot GM}{KM}$

$AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

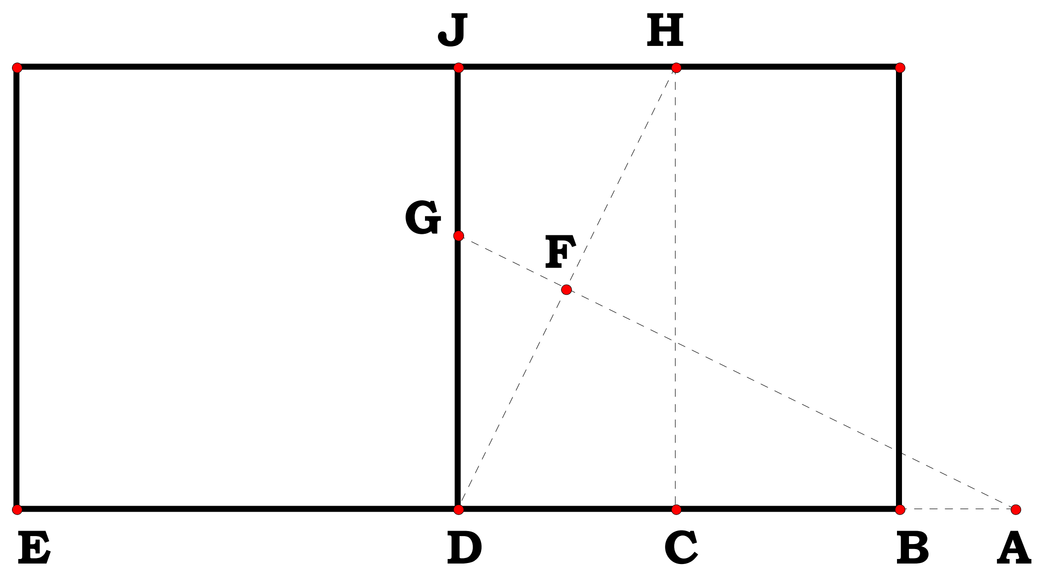
$$AE - \frac{(N_3 \cdot N_2 - 4 \cdot N_3 \cdot N_1 + N_1 \cdot N_4)}{2 \cdot (N_3 \cdot N_2 - N_1 \cdot N_4)} = 0$$



000920 Squaring

Is AC the square root of AB x AE?

Given BC, find AB such that AB x AE is the square root.



$N_1 := 11.69458 \quad N_2 := 2.96916$

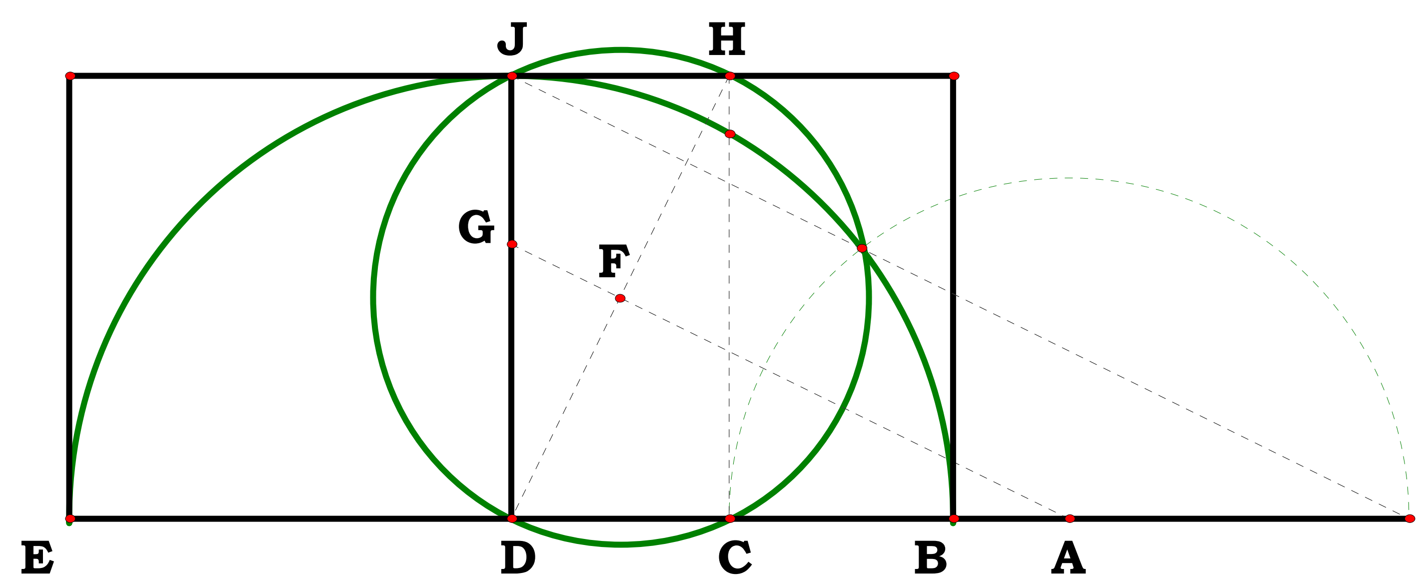
$BE := N_1 \quad BC := N_2 \quad BD := \frac{BE}{2} \quad CD := BD - BC$

$CH := BD \quad DH := \sqrt{BD^2 + CD^2} \quad DG := \frac{DH^2}{2BD}$

$AD := \frac{BD \cdot DG}{CD} \quad AE := AD + BD \quad AB := AE - BE$

$AC := BC + AB$

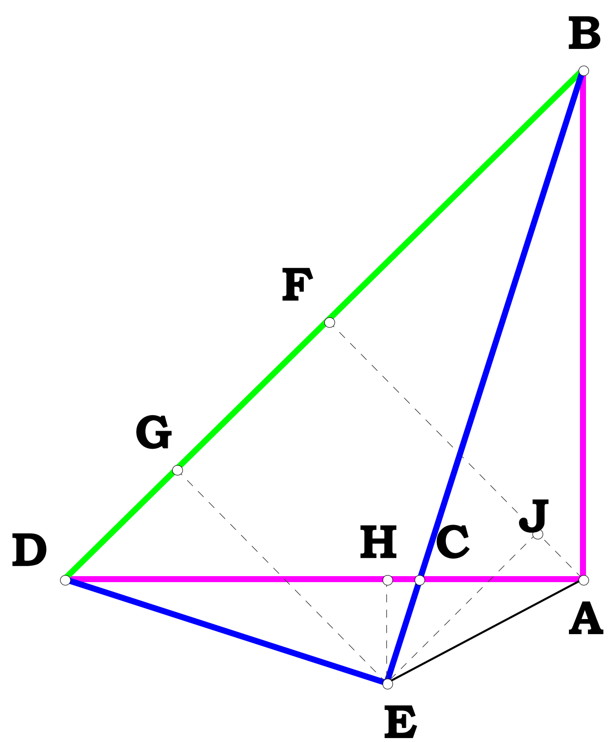
$AC - \sqrt{AB \cdot AE} = 0$



11/13/00 For Two Right Triangles.

Given AB, DE, AD find BE, AC, CD, CE, BC.

BAD and BED are right.



$N_1 := 3 \quad N_2 := 5 \quad N_3 := 1$

$AB := N_1 \quad AD := N_2 \quad DE := N_3$

$BD := \sqrt{AB^2 + AD^2} \quad BF := \dfrac{AB^2}{BD} \quad DG := \dfrac{DE^2}{BD} \quad BE := \sqrt{BD^2 - DE^2}$

$AF := \sqrt{AB^2 - BF^2} \quad EG := \sqrt{DE^2 - DG^2} \quad FG := BD - (BF + DG)$

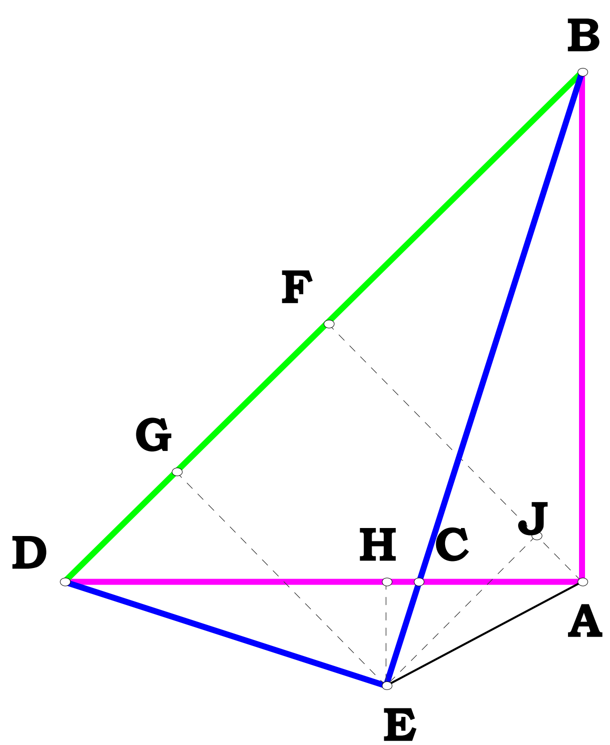
$EJ := FG \quad FJ := EG \quad AJ := AF - FJ \quad AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD \quad S_2 := DE \quad S_3 := AE \quad AH := \dfrac{{S_3}^2 + {S_1}^2 - {S_2}^2}{2 \cdot S_1}$

$EH := \sqrt{AE^2 - AH^2} \quad CH := \dfrac{EH \cdot AH}{AB + EH} \quad AC := AH - CH$

$CE := \dfrac{AC \cdot DE}{AB} \quad CD := AD - AC \quad BC := BE - CE$

Definitions:



$$BE - \sqrt{N_1^2 + N_2^2 - N_3^2} = 0$$

$$\frac{N_1 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - AC = 0$$

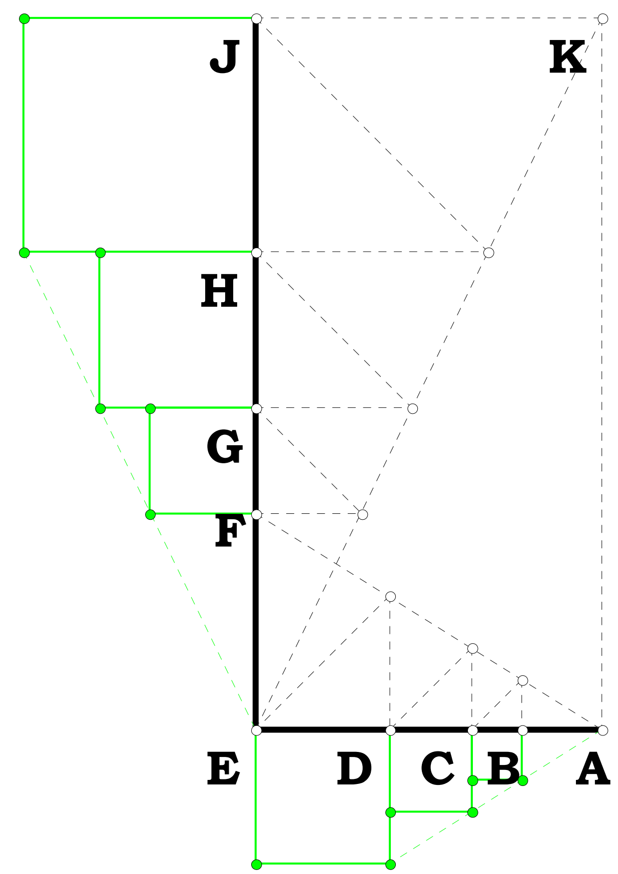
$$N_2 + \frac{N_1 \cdot N_2 \cdot N_3^2 - N_2 \cdot \left(N_1^3 + N_1 \cdot N_2^2\right) + N_1^2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CD = 0$$

$$\frac{N_3 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CE = 0$$

$$BC - \left(\sqrt{N_1^2 + N_2^2 - N_3^2} + \frac{N_2 \cdot N_3^3 - N_2^3 \cdot N_3 - N_1^2 \cdot N_2 \cdot N_3 + N_1 \cdot N_3^2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}}\right) = 0$$

Means On Means 11/28/00

Modify 02/28/98 for Mean proportionals between E and J.



$AE := 1 \quad N := 3 \quad EJ := AE \cdot N \quad JK := AE$

$HJ := \dfrac{JK \cdot EJ}{JK + EJ} \quad EH := EJ - HJ \quad GH := \dfrac{EH \cdot HJ}{EH + HJ}$

$EG := EH - GH \quad FG := \dfrac{EG \cdot GH}{EG + GH} \quad EF := EG - FG$

$DE := \dfrac{EF \cdot AE}{EF + AE} \quad AD := AE - DE \quad CD := \dfrac{AD \cdot DE}{AD + DE}$

$AC := AD - CD \quad BC := \dfrac{AC \cdot CD}{AC + CD} \quad AB := AC - BC$

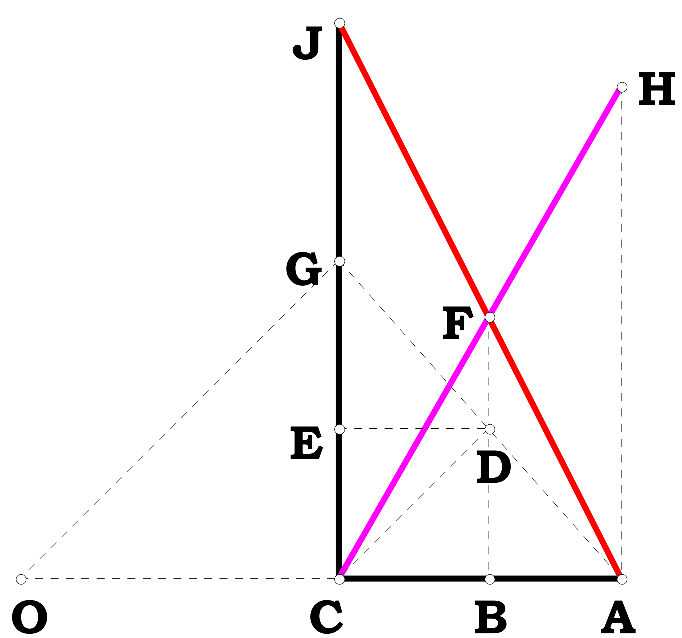
$M := 0 .. 3 \quad P := 0 .. 3 \qquad AEAB_{M,P} := \left[\dfrac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

$$AEAB = \begin{pmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.5625 & 34.328125 \\ 1 & 2.6875 & 7.222656 & 19.410889 \\ 1 & 2.265625 & 5.133057 & 11.629581 \end{pmatrix}$$

$AEAB_{3,3} - \dfrac{AE}{AB} = 0 \quad AEAB_{3,2} - \dfrac{AE}{AC} = 0 \quad AEAB_{3,1} - \dfrac{AE}{AD} = 0 \quad AEAB_{3,0} - \dfrac{AE}{AE} = 0$

Multiplication and Division-Line By A Line 11/29/00

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



$$\mathbf{AC} := 1 \quad \mathbf{N_1} := 3 \quad \mathbf{N_2} := 12 \quad \mathbf{AH} := \mathbf{N_1}$$

$$\mathbf{CJ} := \mathbf{N_2} \quad \mathbf{AB} := \frac{\mathbf{AH}}{(\mathbf{CJ} + \mathbf{AH})} \cdot \mathbf{AC}$$

$$\mathbf{BC} := \mathbf{AC} - \mathbf{AB} \quad \mathbf{BD} := \mathbf{BC} \quad \mathbf{CG} := \frac{\mathbf{BD} \cdot \mathbf{AC}}{\mathbf{AB}}$$

$$\mathbf{CG} - \frac{\mathbf{N_2}}{\mathbf{N_1}} = 0 \qquad \mathbf{CG} = 4$$

For Multiplication:

$$\mathbf{AC} := 1 \quad \mathbf{N_1} := 5 \quad \mathbf{N_2} := 7 \quad \mathbf{AH} := \mathbf{N_1}$$

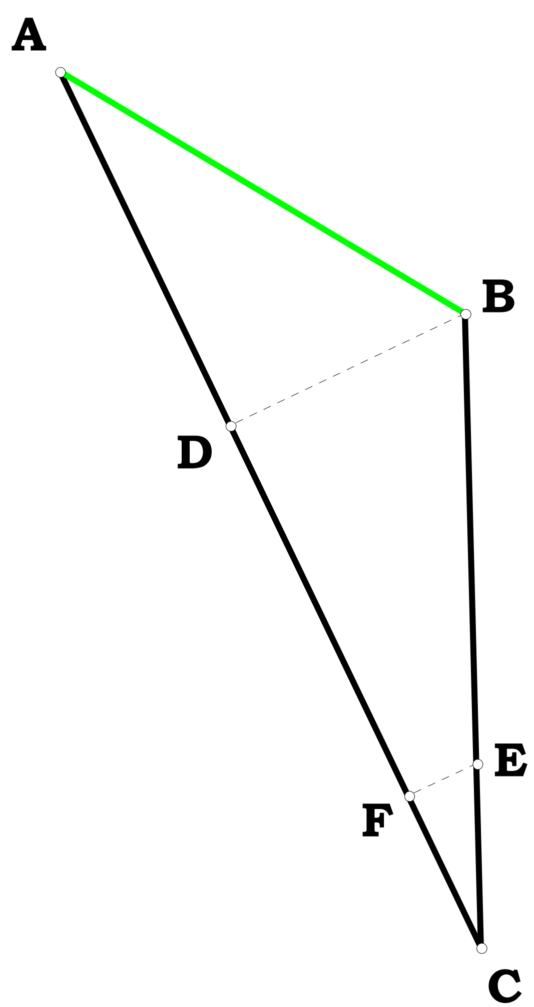
$$\mathbf{CG} := \mathbf{N_2} \quad \mathbf{CO} := \mathbf{CG} \quad \mathbf{BD} := \frac{\mathbf{CG} \cdot \mathbf{AC}}{\mathbf{AC} + \mathbf{CO}}$$

$$\mathbf{BC} := \mathbf{BD} \quad \mathbf{AB} := \mathbf{AC} - \mathbf{BC} \quad \mathbf{BF} := \frac{\mathbf{AH} \cdot \mathbf{BC}}{\mathbf{AC}}$$

$$\mathbf{CJ} := \mathbf{BF} \cdot \frac{\mathbf{AC}}{\mathbf{AB}} \quad \mathbf{CJ} - \mathbf{N_1} \cdot \mathbf{N_2} = 0 \qquad \mathbf{CJ} = 35$$

120500

From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.



$$N_1 := 5 \quad N_2 := 25 \quad N_3 := 1 \quad N_4 := .5$$

$$BC := N_1 \quad AC := N_2 \quad CE := N_3 \quad EF := N_4$$

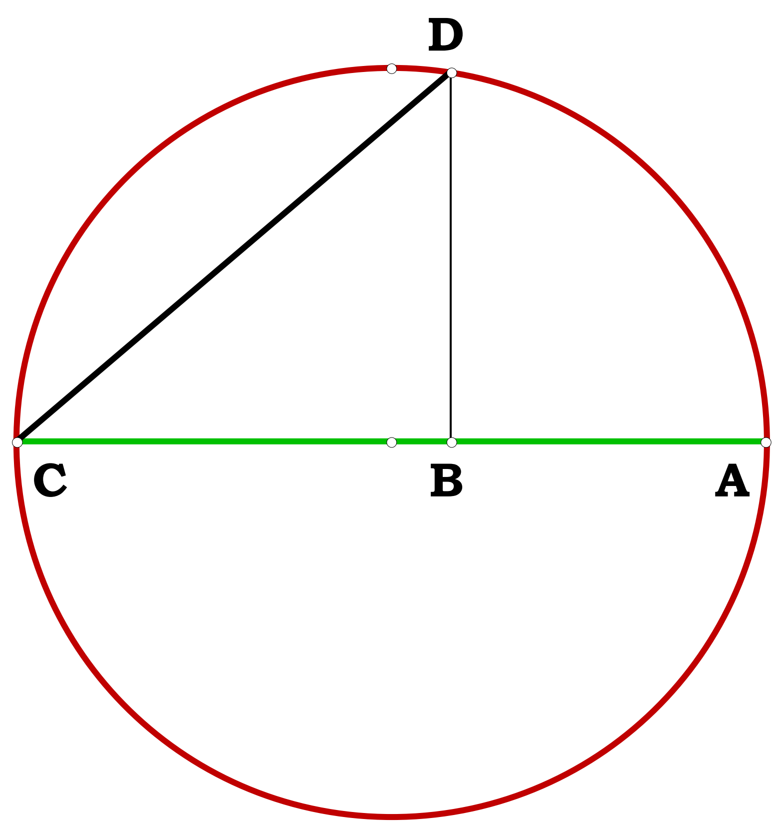
$$BD := \frac{EF \cdot BC}{CE} \quad CF := \sqrt{CE^2 - EF^2} \quad CD := \frac{CF \cdot BC}{CE}$$

$$AD := AC - CD \quad AB := \sqrt{BD^2 + AD^2}$$

$$AB - \frac{\sqrt{N_1^2 \cdot N_3 + N_2^2 \cdot N_3 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_3^2 - N_4^2}}}{\sqrt{N_3}} = 0$$

010101 Square Root, common segment common endpoint.

Alternate method for common segment common endpoint square root. $\sqrt{AC \cdot BC} = CD$



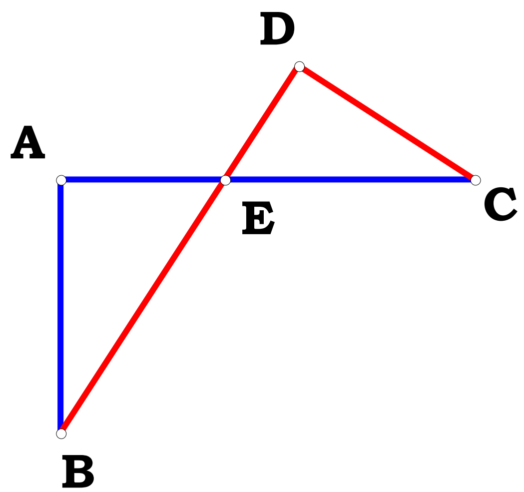
$$N_1 := 5 \quad N_2 := 3 \quad AC := N_1 \quad BC := N_2$$

$$AB := AC - BC \quad BD := \sqrt{AB \cdot BC} \quad CD := \sqrt{BD^2 + BC^2}$$

$$\sqrt{N_2 \cdot N_1} - CD = 0$$

Three Given Five Taken 042101

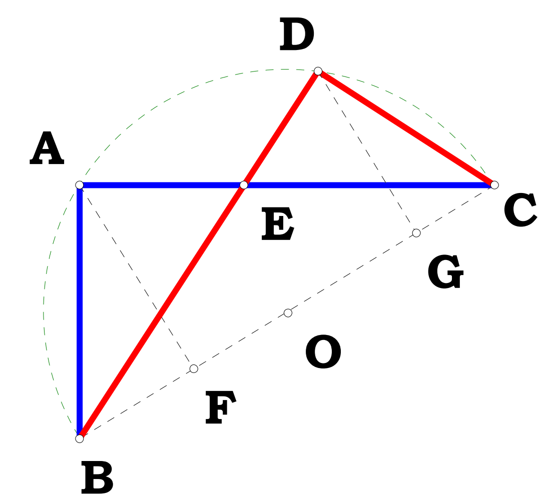
Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?



$$N_1 := 2 \quad N_2 := 3 \quad N_3 := 4$$

$$AB := N_1 \quad CD := N_2 \quad AC := N_3$$

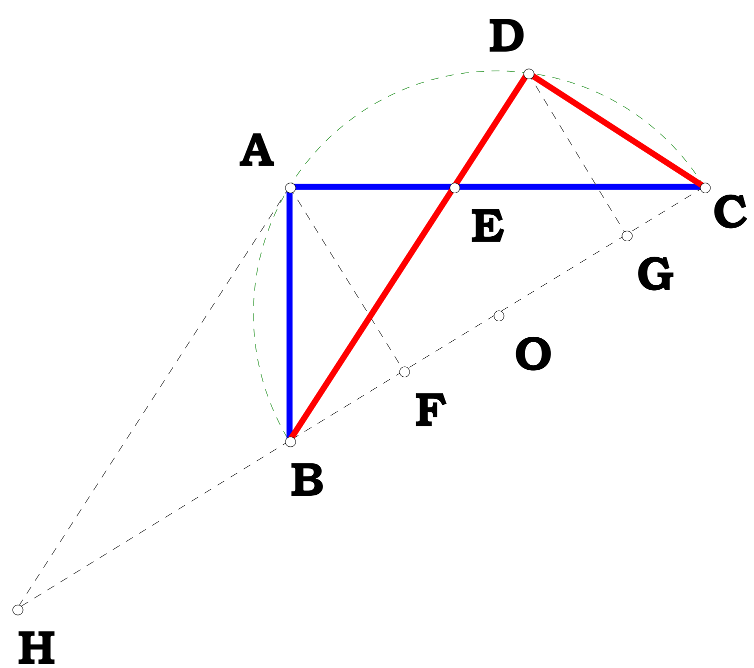
$$BC := \sqrt{AB^2 + AC^2} \quad CG := \frac{CD^2}{BC} \quad BF := \frac{AB^2}{BC}$$



$$BG := BC - CG \quad CF := BC - BF$$

$$AF := \sqrt{AB^2 - BF^2} \quad DG := \sqrt{CD^2 - CG^2}$$

$$FH := \frac{BG \cdot AF}{DG} \quad CH := CF + FH$$

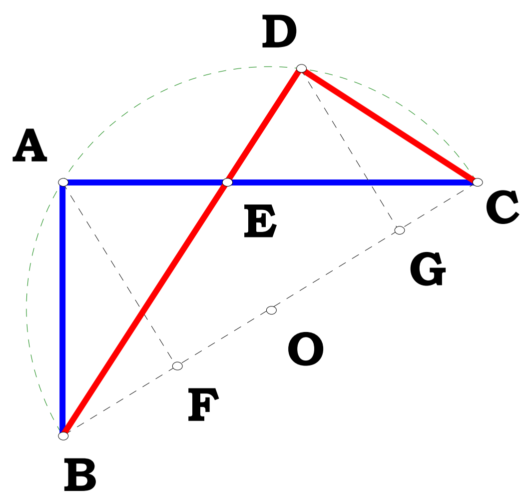


$$BD := \sqrt{BG^2 + DG^2}$$

$$AH := \frac{BD \cdot FH}{BG} \quad BE := \frac{AH \cdot BC}{CH}$$

$$DE := BD - BE \quad CE := \frac{AC \cdot BC}{CH}$$

$$AE := AC - CE$$



Some Algebraic Names:

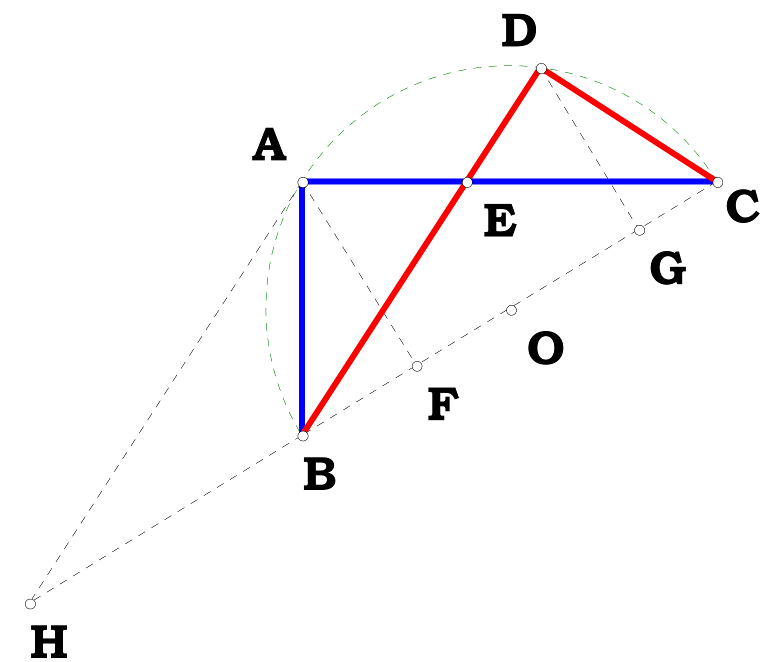
$$\sqrt{N_1^2+N_3^2}-BC=0 \qquad \frac{N_2^2}{\sqrt{N_1^2+N_3^2}}-CG=0$$

$$\frac{N_1^2}{\sqrt{N_1^2+N_3^2}}-BF=0 \qquad \frac{\left(N_1^2+N_3^2-N_2^2\right)}{\sqrt{N_1^2+N_3^2}}-BG=0$$

$$\frac{N_3^2}{\sqrt{N_1^2+N_3^2}}-CF=0$$

$$\frac{N_1\cdot N_3}{\sqrt{N_1^2+N_3^2}}-AF=0$$

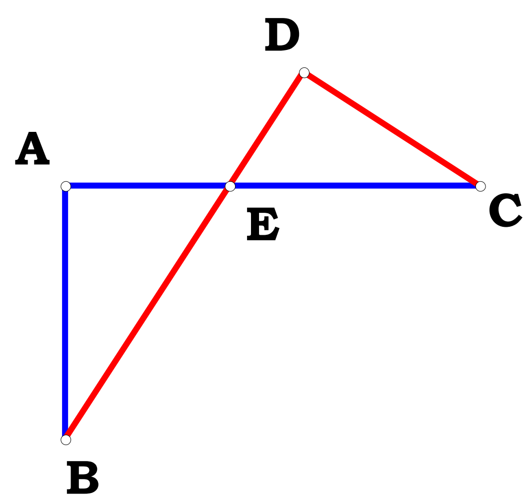
$$N_2\cdot\sqrt{\frac{\left(N_1^2+N_3^2-N_2^2\right)}{\left(N_1^2+N_3^2\right)}}-DG=0$$



$$\frac{N_3\cdot N_1\cdot\left(N_1^2+N_3^2-N_2^2\right)}{N_2\cdot\sqrt{\left(N_1^2+N_3^2-N_2^2\right)\cdot\left(N_1^2+N_3^2\right)}}-FH=0$$

$$\left[\frac{N_3^2}{\sqrt{N_1^2+N_3^2}}+\frac{N_3\cdot N_1\cdot\left(N_1^2+N_3^2-N_2^2\right)}{N_2\cdot\sqrt{\left(N_1^2+N_3^2-N_2^2\right)\cdot\left(N_1^2+N_3^2\right)}}\right]-CH=0 \qquad N_3\cdot\frac{N_1}{N_2}-AH=0$$

The Five Sought:



$$\sqrt{N_1^2+N_3^2-N_2^2}-BD=0$$

$$N_2\cdot\frac{\left(\sqrt{N_1^2+N_3^2-N_2^2}\cdot N_3-N_1\cdot N_2\right)}{\left(N_3\cdot N_2+\sqrt{N_1^2+N_3^2-N_2^2}\cdot N_1\right)}-DE=0$$

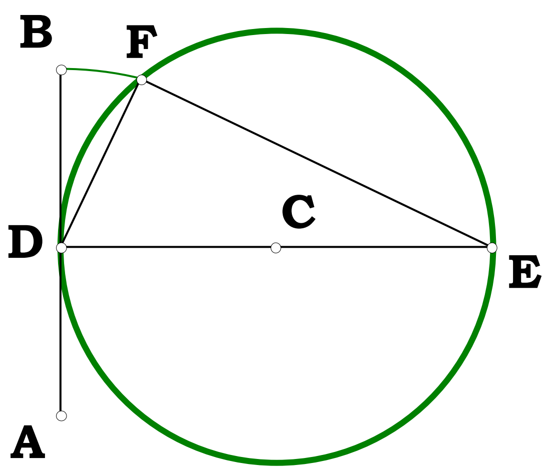
$$N_1\cdot\frac{\left(N_1^2+N_3^2\right)}{\left(N_3\cdot N_2+\sqrt{N_1^2+N_3^2-N_2^2}\cdot N_1\right)}-BE=0$$

$$\frac{N_2\cdot\left(N_1^2+N_3^2\right)}{N_3\cdot N_2+\sqrt{N_1^2+N_3^2-N_2^2}\cdot N_1}-CE=0$$

$$N_3-\frac{N_2\cdot\left(N_1^2+N_3^2\right)}{\left(N_3\cdot N_2+\sqrt{N_1^2+N_3^2-N_2^2}\cdot N_1\right)}-AE=0$$

042201

Given AB as unit, AD and DC, what is EF and DF?



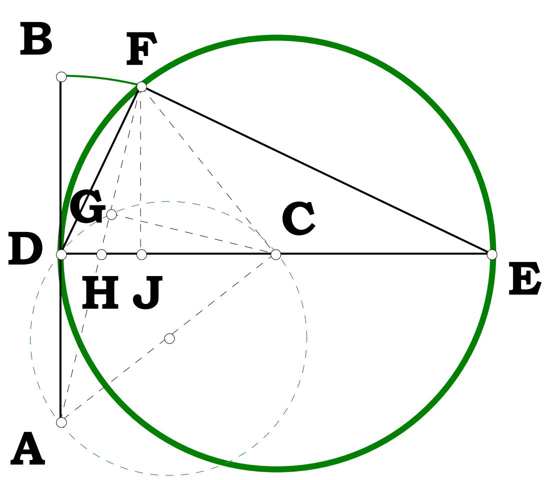
$N_1 := 2.052 \quad N_2 := .62 \quad AB := 1.802$

$AD := \frac{AB}{N_1} \quad CD := AB \cdot N_2 \quad DE := 2CD \quad AF := AB \quad CF := CD$

$AC := \sqrt{AD^2 + CD^2} \quad S_1 := AF \quad S_2 := AC \quad S_3 := CF$

$AG := \frac{S_2{}^2 + S_1{}^2 - S_3{}^2}{2 \cdot S_1} \quad CG := \sqrt{AC^2 - AG^2}$

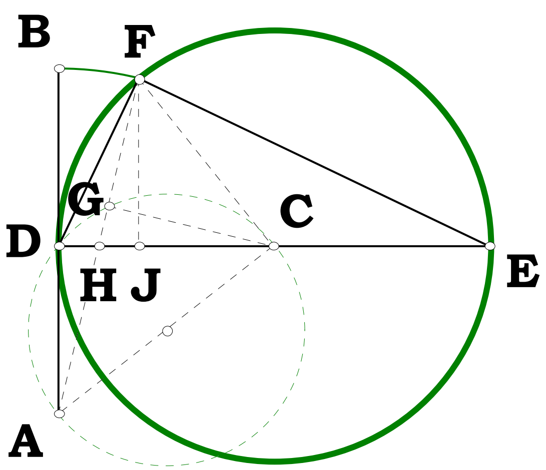
$L_1 := AD \quad L_2 := CG \quad L_3 := CD$



$$DH := L_3 - \frac{L_2 \cdot \left(L_1{}^2 + L_3{}^2\right)}{L_3 \cdot L_2 + \sqrt{L_1{}^2 + L_3{}^2 - L_2{}^2} \cdot L_1}$$

$$AH := L_1 \cdot \frac{\left(L_1{}^2 + L_3{}^2\right)}{\left(L_3 \cdot L_2 + \sqrt{L_1{}^2 + L_3{}^2 - L_2{}^2} \cdot L_1\right)} \quad FH := AF - AH \quad HJ := \frac{DH \cdot FH}{AH}$$

$$DJ := DH + HJ \quad EJ := DE - DJ \quad FJ := \frac{AD \cdot FH}{AH} \quad EF := \sqrt{FJ^2 + EJ^2} \quad DF := \sqrt{DJ^2 + FJ^2}$$



Some Algebraic Names:

$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0$$

$$(2AB) \cdot N_2 - DE = 0 \qquad AB \cdot \frac{\sqrt{\left(1 + N_2{}^2 \cdot N_1{}^2\right)}}{N_1} - AC = 0$$

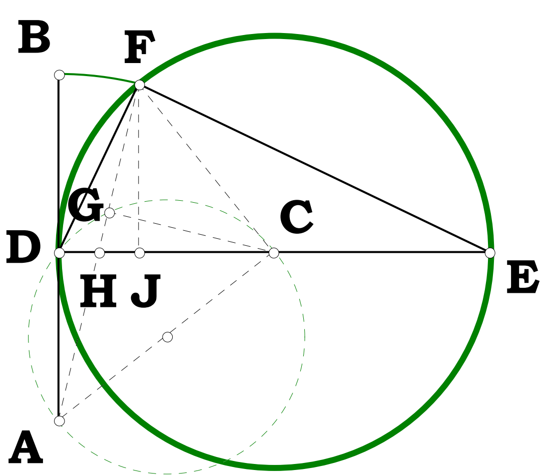
$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1{}^2\right)}{N_1{}^2} - AG = 0$$

$$AB \cdot \frac{\sqrt{\left(2 \cdot N_1{}^2 \cdot N_2 + N_1{}^2 - 1\right) \cdot \left(2 \cdot N_1{}^2 \cdot N_2 - N_1{}^2 + 1\right)}}{2 \cdot N_1{}^2} - CG = 0$$

$$AB \cdot \frac{N_2 \cdot N_1{}^3 + N_2 \cdot N_1 - \sqrt{2 \cdot N_1{}^2 + 4 \cdot N_1{}^4 \cdot N_2{}^2 - 1 - N_1{}^4}}{N_1{}^3 + N_1{}^2 \cdot N_2 \cdot \sqrt{2 \cdot N_1{}^2 + 4 \cdot N_1{}^4 \cdot N_2{}^2 - 1 - N_1{}^4} + N_1} - DH = 0$$

$$2 \cdot AB \cdot \frac{\left(1 + N_2{}^2 \cdot N_1{}^2\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1{}^2 + 4 \cdot N_1{}^4 \cdot N_2{}^2 - 1 - N_1{}^4} + 1 + N_1{}^2\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2{}^2 \cdot N_1{}^2\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1{}^2 + 4 \cdot N_1{}^4 \cdot N_2{}^2 - 1 - N_1{}^4} + 1 + N_1{}^2\right)} - FH = 0$$



$$\frac{-1}{2}\cdot AB\cdot\left(\frac{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}\cdot N_2^{\,2}+1}{N_1^{\,2}\cdot N_2+N_1^{\,4}\cdot N_2+N_1^{\,3}\cdot N_2^{\,2}\cdot\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}-\frac{N_1^{\,4}\cdot N_2^{\,2}+3\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+1}{N_1^{\,4}\cdot N_2^{\,3}+N_1^{\,2}\cdot N_2}\right)-HJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-DJ=0$$

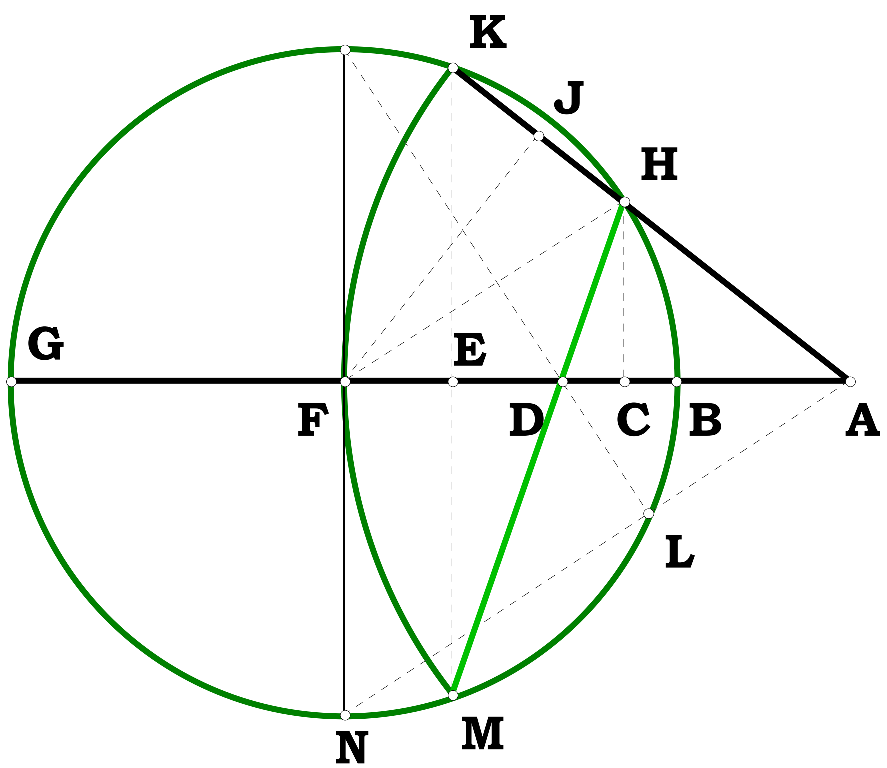
$$\frac{1}{2}\cdot AB\cdot\frac{4\cdot N_1^{\,3}\cdot N_2^{\,3}-N_1^{\,3}\cdot N_2+3\cdot N_1\cdot N_2+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-EJ=0$$

$$\frac{-1}{2}\cdot AB\cdot\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}+1-N_1^{\,2}+2\cdot N_2^{\,2}\cdot N_1^{\,2}\right)}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}-FJ=0$$

$$AB\cdot\frac{\sqrt{N_2\cdot\left[N_1\cdot N_2\cdot\left(4\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+3\right)+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right]}}{\sqrt{N_1^{\,3}\cdot N_2^{\,2}+N_1}}-EF=0$$

$$AB\cdot\frac{\sqrt{\left[N_2\cdot\left(N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right)\right]}}{\sqrt{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}}-DF=0$$

042401



Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?

$$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$$

$$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF \qquad AK := AF$$

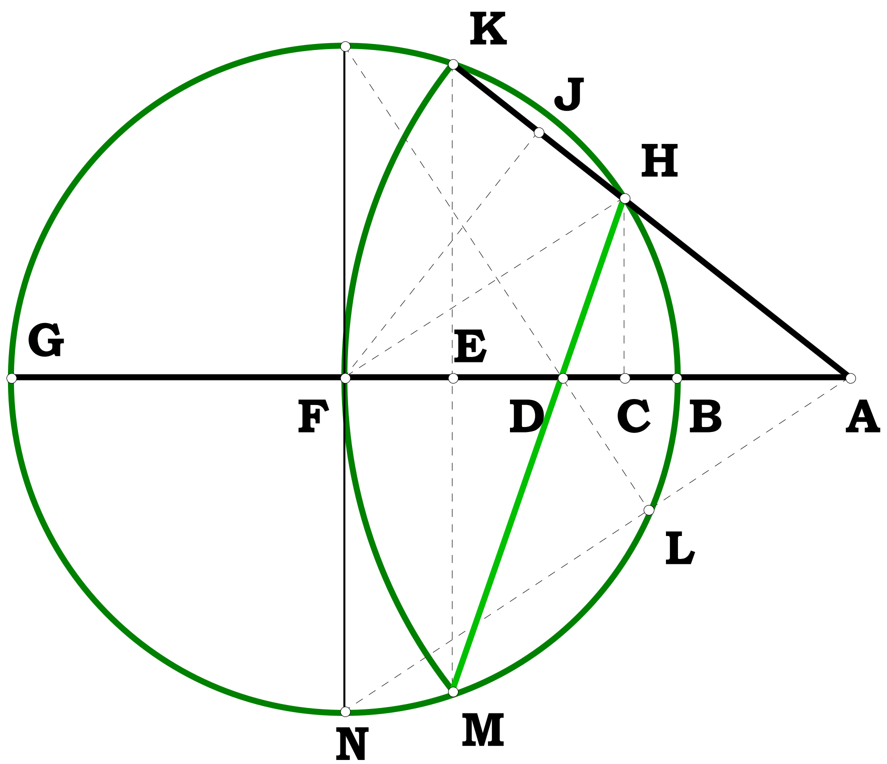
$$FK := BF \qquad AE := \frac{2AK^2 - FK^2}{2AF} \qquad AJ := AE \qquad JK := AK - AJ \qquad HJ := JK$$

$$AH := AK - (JK + HJ) \qquad AC := \frac{AE \cdot AH}{AK} \qquad CE := AE - AC$$

$$BE := AE - AB \qquad EG := BG - BE \qquad EK := \sqrt{BE \cdot EG} \qquad BC := AC - AB$$

$$CG := BG - BC \qquad CH := \sqrt{BC \cdot CG} \qquad DE := \frac{CE \cdot EK}{EK + CH}$$

$$DF := 2 \cdot DE \qquad HM := \sqrt{CE^2 + (EK + CH)^2}$$



Some Algebraic Names:

$$AB\cdot N - AB - BG = 0 \qquad \frac{1}{2}\cdot AB\cdot N - \frac{1}{2}\cdot AB - BF = 0$$

$$\frac{1}{2}\cdot AB + \frac{1}{2}\cdot AB\cdot N - AF = 0 \qquad \frac{1}{4}\cdot AB\cdot\frac{\left(N^2 + 6\cdot N + 1\right)}{(1+N)} - AE = 0$$

$$\frac{1}{4}\cdot AB\cdot\frac{\left(1 - 2\cdot N + N^2\right)}{(1+N)} - JK = 0 \qquad 2\cdot AB\cdot\frac{N}{(1+N)} - AH = 0$$

$$AB\cdot\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^3}\cdot N - AC = 0 \qquad \frac{1}{4}\cdot AB\cdot\left(N^2 + 6\cdot N + 1\right)\cdot\frac{(N-1)^2}{(1+N)^3} - CE = 0$$

$$\frac{1}{4}\cdot AB\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} - BE = 0 \qquad \frac{1}{4}\cdot AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)} - EG = 0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3} - BC = 0 \qquad \frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} - EK = 0$$

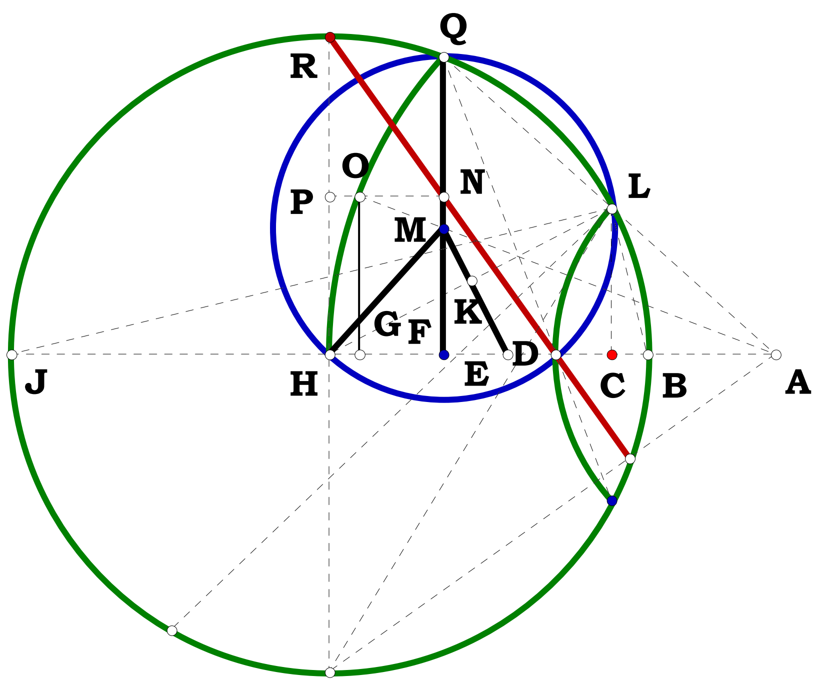
$$AB\cdot N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3} - CG = 0 \qquad AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3} - CH = 0$$

$$\frac{1}{4}\cdot AB\cdot\frac{(N-1)^2}{(1+N)} - DE = 0 \qquad \frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(N+1)} - DF = 0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^2} - HM = 0$$

042501

What is the Algebraic name of the circle HM? Does point N divide DR in half?



$$N := 5.768 \quad AB := .583 \quad AJ := AB \cdot N$$

$$BJ := AJ - AB \quad BH := \frac{BJ}{2} \quad HR := BH$$

$$HP := \frac{HR}{2} \quad GO := HP \quad AH := AB + BH$$

$$AO := AH \quad AG := \sqrt{AO^2 - GO^2}$$

$$HQ := BH \quad AQ := AH \quad FH := \frac{HQ^2}{2 \cdot AH}$$

$$AF := AH - FH \quad FM := \frac{GO \cdot AF}{AG}$$

$$HJ := BH \quad FJ := FH + HJ \quad BF := BJ - FJ$$

$$FQ := \sqrt{BF \cdot FJ} \quad MQ := FQ - FM \quad HM := \sqrt{FH^2 + FM^2} \quad HM - MQ = 0$$

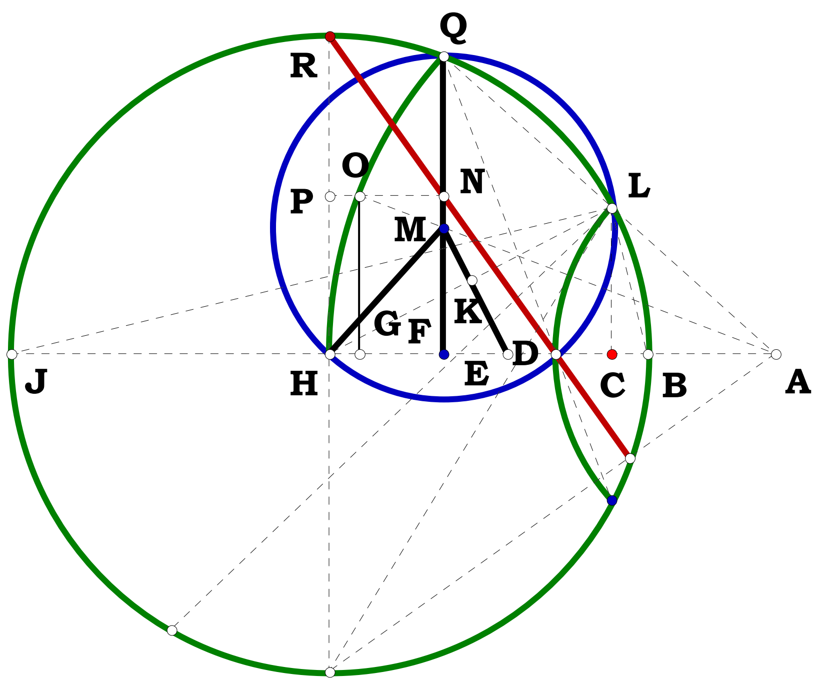
$$DH := \frac{HR^2}{AH} \quad \frac{DH}{2} - FH = 0$$

Some Algebraic Names:

$$AB \cdot N - AB - BJ = 0 \quad \frac{1}{2} \cdot AB \cdot (N - 1) - BH = 0 \quad \frac{1}{4} \cdot AB \cdot (N - 1) - HP = 0$$

$$\frac{1}{2} \cdot AB \cdot (1 + N) - AH = 0 \quad \frac{1}{4} \cdot AB \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - AG = 0 \quad \frac{1}{4} \cdot AB \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$$

$$\frac{1}{4} \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - AF = 0 \quad \frac{1}{4} \cdot (N - 1) \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} - FM = 0$$



$$\frac{1}{4}\cdot\mathbf{AB}\cdot(\mathbf{N}+3)\cdot\frac{(\mathbf{N}-1)}{(1+\mathbf{N})}-\mathbf{BF}=0 \qquad \frac{1}{4}\cdot\mathbf{AB}\cdot(\mathbf{N}-1)\cdot\frac{(3\cdot\mathbf{N}+1)}{(1+\mathbf{N})}-\mathbf{FJ}=0$$

$$\frac{1}{4}\cdot\mathbf{AB}\cdot\sqrt{(\mathbf{N}+3)\cdot(3\cdot\mathbf{N}+1)}\cdot\frac{(\mathbf{N}-1)}{(1+\mathbf{N})}-\mathbf{FQ}=0$$

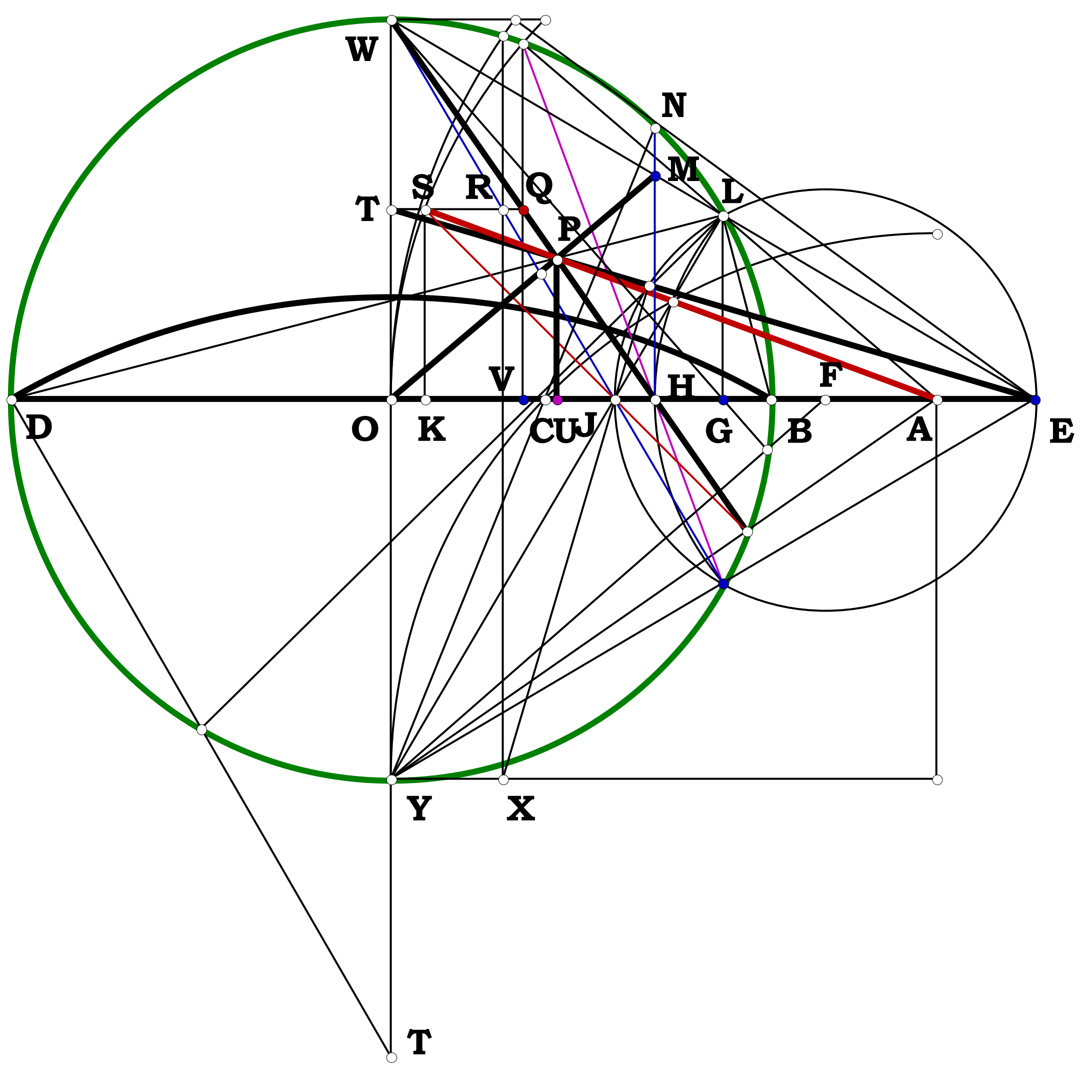
$$\frac{1}{2}\cdot(1+\mathbf{N})\cdot\mathbf{AB}\cdot\frac{(\mathbf{N}-1)}{\sqrt{(\mathbf{N}+3)\cdot(3\cdot\mathbf{N}+1)}}-\mathbf{MQ}=0 \qquad \frac{1}{2}\cdot\mathbf{AB}\cdot(\mathbf{N}-1)\cdot\frac{(1+\mathbf{N})}{\sqrt{(\mathbf{N}+3)\cdot(3\cdot\mathbf{N}+1)}}-\mathbf{HM}=0$$

$$\mathbf{HM}-\mathbf{MQ}=0$$

$$\frac{1}{2}\cdot\mathbf{AB}\cdot\frac{(\mathbf{N}-1)^2}{(1+\mathbf{N})}-\mathbf{DH}=0 \qquad \frac{\mathbf{DH}}{2}-\mathbf{FH}=0$$

Four Lines To A Point 042901

Does the difference OU and PU each have but one Algebraic name?



$N := 5 \qquad AB := 1 \qquad AD := AB \cdot N$

$BD := AD - AB \qquad BO := \dfrac{BD}{2} \qquad OW := BO$

$OY := BO \qquad DO := BO \qquad AO := AB + BO$

$HO := \dfrac{OY^2}{AO} \qquad AH := AO - HO \qquad AL := AH$

$OL := BO \qquad GO := \dfrac{OL^2 + AO^2 - AL^2}{2AO}$

$BG := BO - GO \qquad DG := GO + DO$

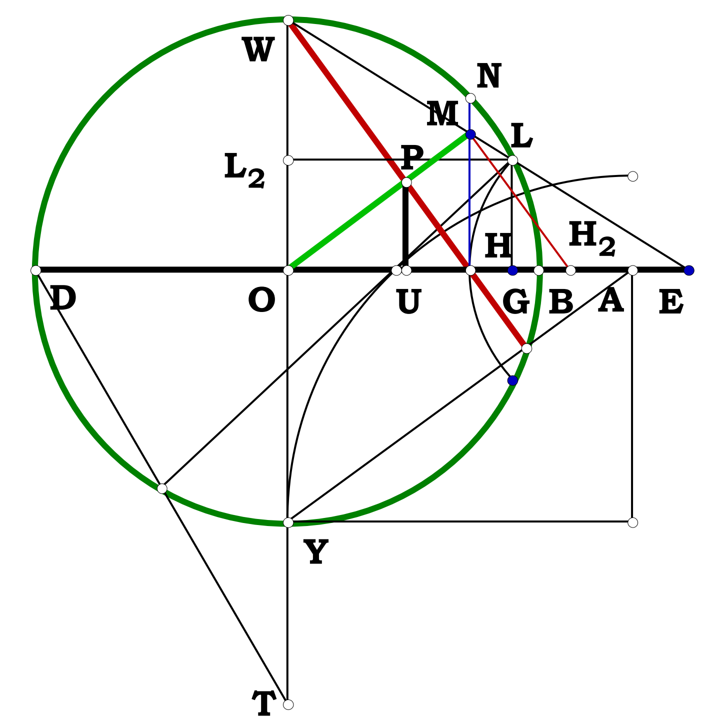
$GL := \sqrt{BG \cdot DG} \qquad EO := \dfrac{GO \cdot OW}{OW - GL}$

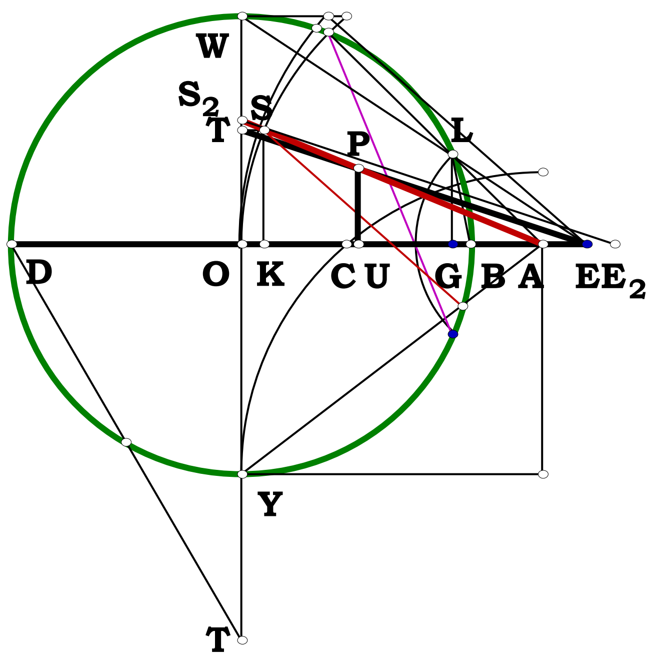
$BE := EO - BO \qquad BH := BO - HO$

$EG := BE + BG \qquad EH := BE + BH$

$HM := \dfrac{GL \cdot EH}{EG} \qquad HH_2 := \dfrac{HO \cdot HM}{OW} \qquad OU := \dfrac{HO^2}{HH_2 + HO}$

$UP := \dfrac{HM \cdot OU}{HO}$





$$OT := \frac{OW}{2} \quad KS := OT \quad AS := AO$$

$$AK := \sqrt{AS^2 - KS^2} \quad OS_2 := \frac{KS \cdot AO}{AK}$$

$$OE_2 := \frac{EO \cdot OS_2}{OT} \quad AS_2 := \sqrt{AO^2 + OS_2^{\,2}}$$

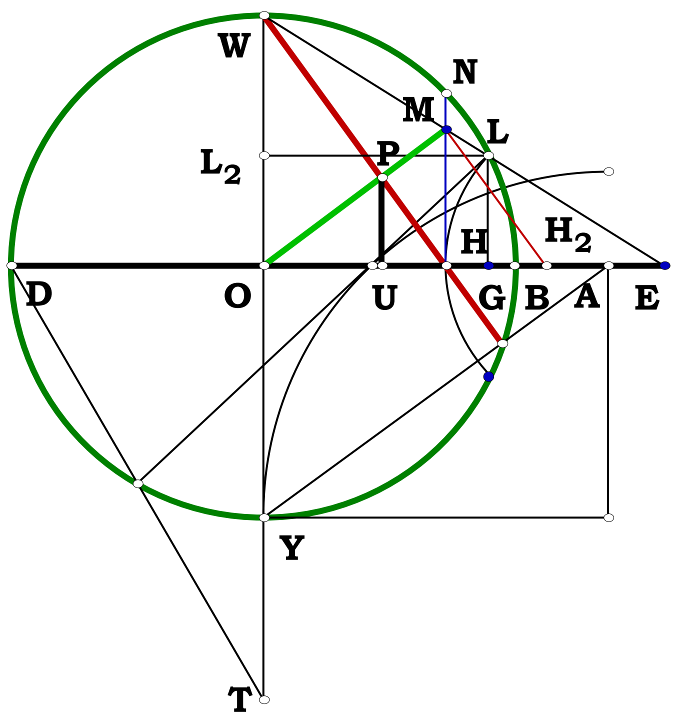
$$AE_2 := OE_2 - AO \quad AE := EO - AO$$

$$AP := \frac{AS_2 \cdot AE}{AE_2} \quad AU := \frac{AO \cdot AE}{AE_2}$$

$$UO := AO - AU \quad PU := \frac{OS_2 \cdot AU}{AO}$$

$$UO - OU = 0$$

$$PU - UP = 0$$



Some Algebraic Names:

$$AB\cdot(N-1)-BD=0 \qquad \frac{AB\cdot(N-1)}{2}-BO=0 \qquad \frac{1}{2}\cdot AB\cdot(1+N)-AO=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-HO=0 \qquad 2\cdot AB\cdot\frac{N}{(1+N)}-AH=0 \qquad \frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3}-GO=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BG=0 \qquad AB\cdot(N-1)\cdot N^2\cdot\frac{(N+3)}{(1+N)^3}-DG=0$$

$$\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^3}-GL=0$$

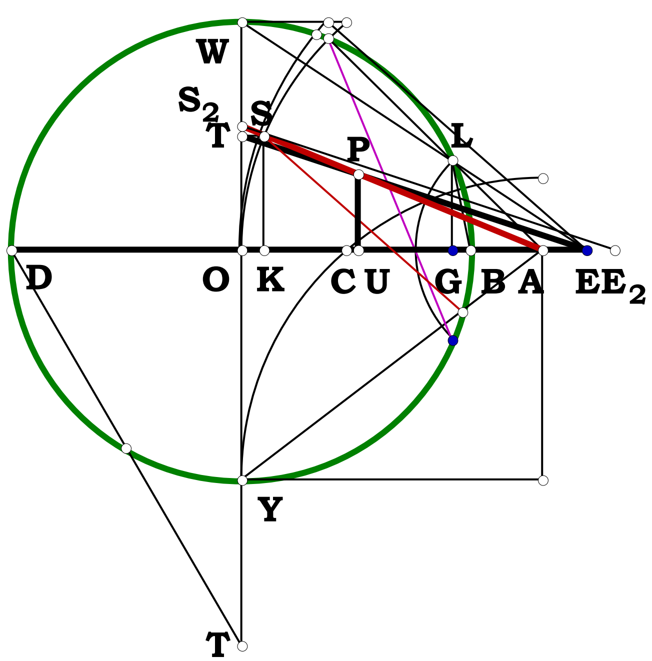
$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-EO=0$$

$$AB\cdot(N-1)\cdot\frac{\left[\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N-3\cdot N-1\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-BE=0 \qquad AB\cdot\frac{(N-1)}{(1+N)}-BH=0$$

$$AB\cdot\frac{(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\left(N^2+4\cdot N+1\right)}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)^3\right]}-EG=0$$

$$AB\cdot(N-1)^2\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)\right]}-EH=0 \qquad N\cdot(N-1)\cdot\frac{AB}{(1+N)}\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HM=0$$

$$AB\cdot\frac{(N-1)^2}{(1+N)^2}\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HH_2=0 \qquad \frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-OU=0$$



$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UP=0$$

$$\frac{1}{4}\cdot AB\cdot(N-1)-OT=0 \qquad \frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-AK=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)\cdot(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OS_2=0$$

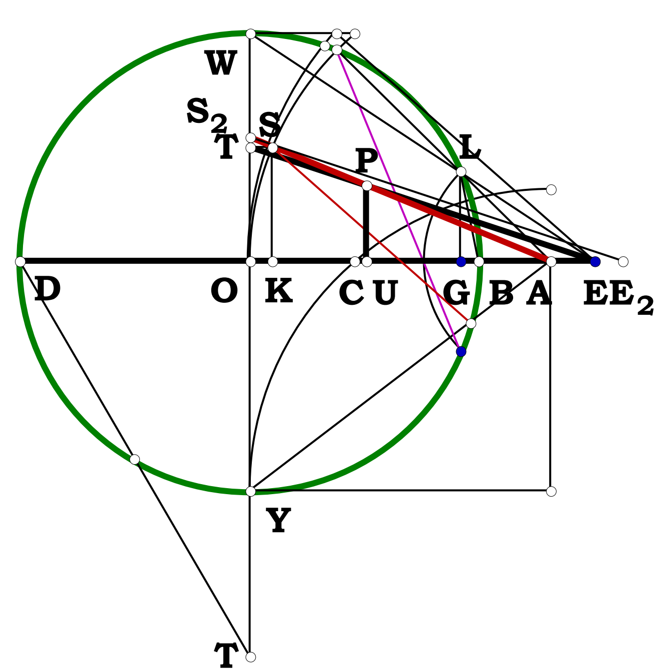
$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OE_2=0$$

$$(1+N)^2\cdot\frac{AB}{\sqrt{3+10\cdot N+3\cdot N^2}}-AS_2=0$$

$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-\frac{1}{2}\cdot AB\cdot(1+N)-AE_2=0$$

$$-AB\cdot N\cdot\frac{\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-AE=0$$

$$\frac{-2\cdot AB\cdot N\cdot(1+N)\cdot\left[N^2+6\cdot N-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots\\ +-\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3}-AP=0$$



$$-\frac{AB\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[6\cdot N-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{10\cdot N-3\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+2+\left(8\cdot N^2+10\cdot N^3+2\cdot N^4\right)}-AU=0$$

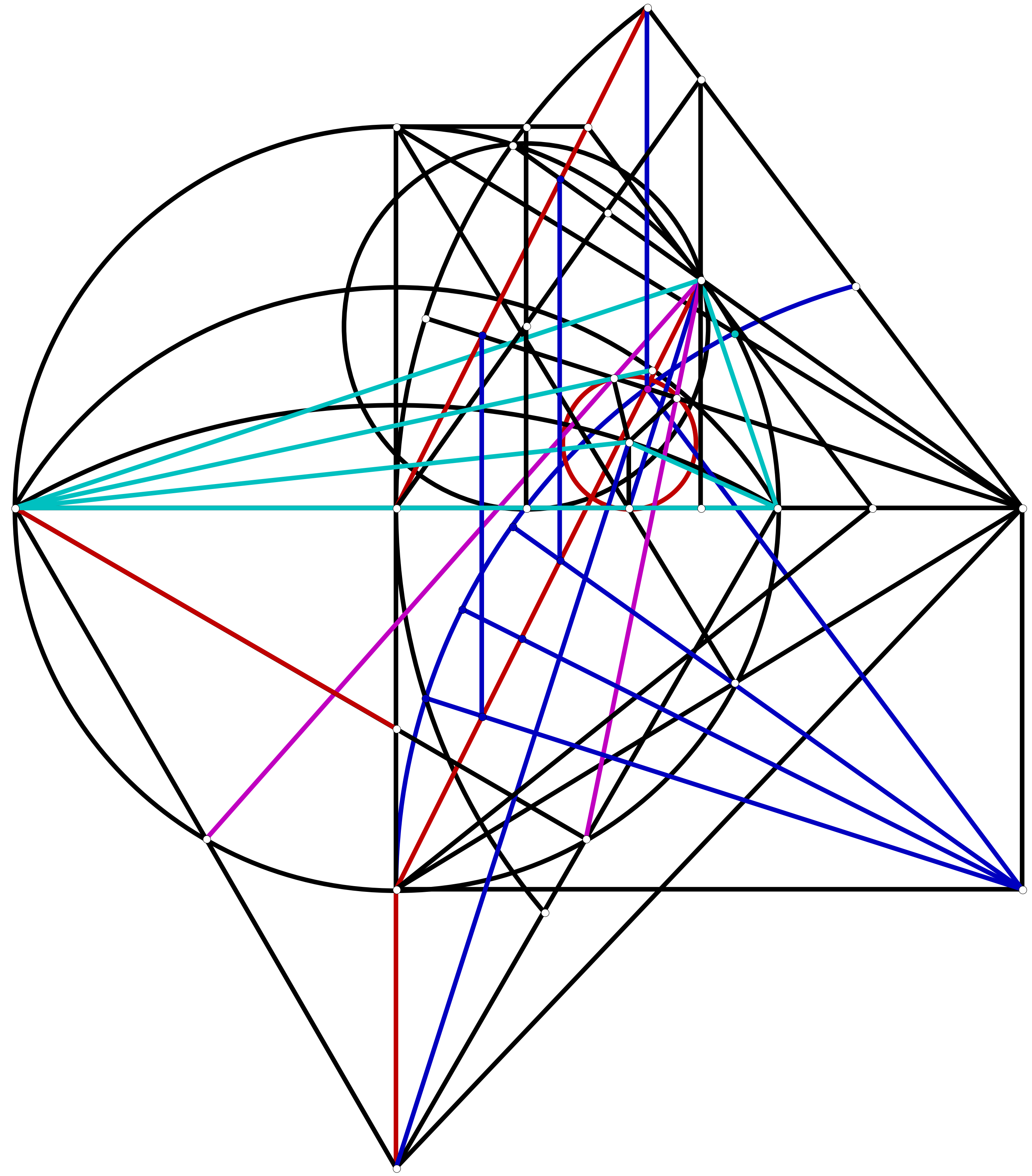
$$-\frac{AB\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[6\cdot N-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{10\cdot N-3\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+8\cdot N^2+10\cdot N^3+2\cdot N^4-\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+2}-AU=0$$

$$\frac{-AB\cdot N\cdot\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{\begin{array}{l}\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots\\ +\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3\end{array}}-AU=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UO=0$$

$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-PU=0$$

0105 01 Just an Illustration



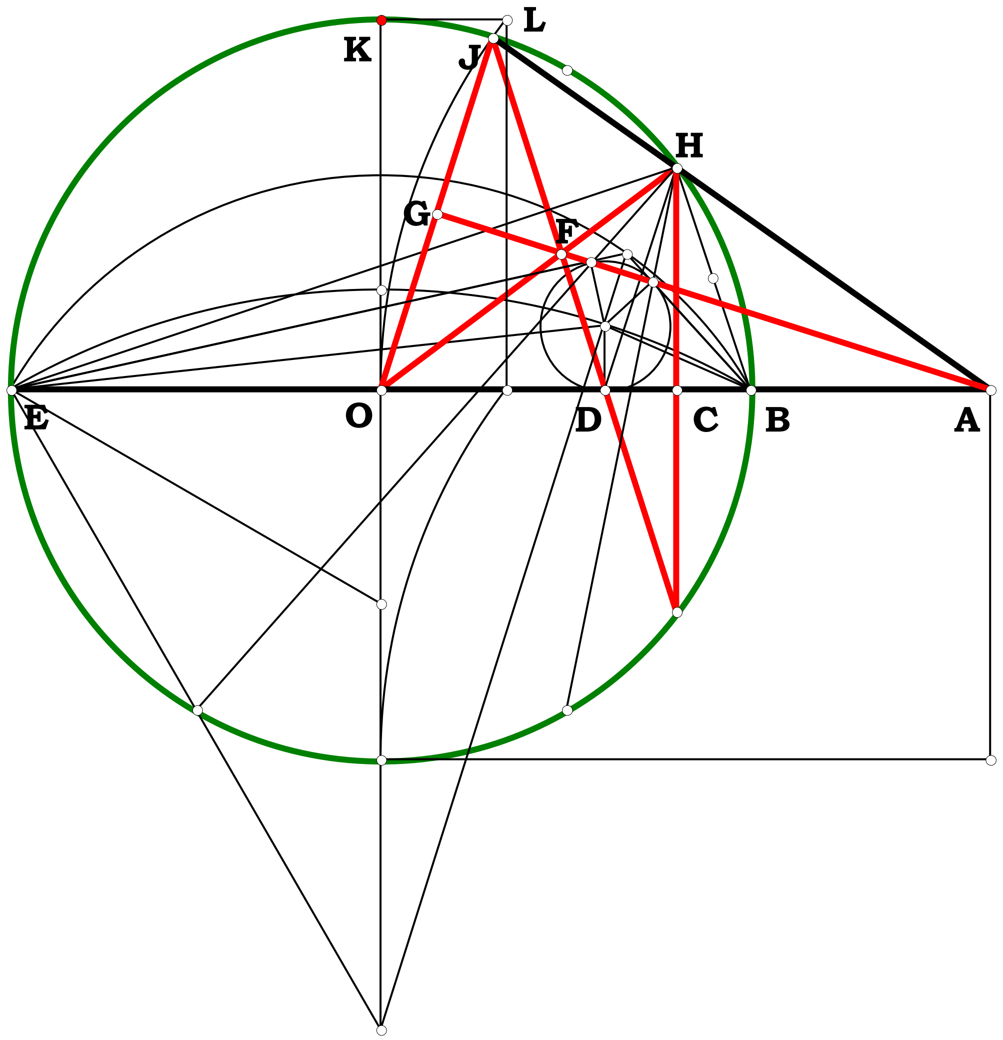
050601.MCD

Just some Algebraic Names

$AB := .818 \quad N := 4.082$

$AE := AB \cdot N \quad BE := AE - AB \quad BO := \frac{BE}{2} \quad AO := AB + BO$

$AJ := AO \quad JO := BO \quad GO := \frac{JO}{2} \quad AG := \sqrt{AO^2 - GO^2}$



$AP := \frac{AG^2}{AO} \quad OP := AO - AP$

$NO := 2 \cdot OP \quad AN := AO - NO$

$JM := NO \quad HO := BO$

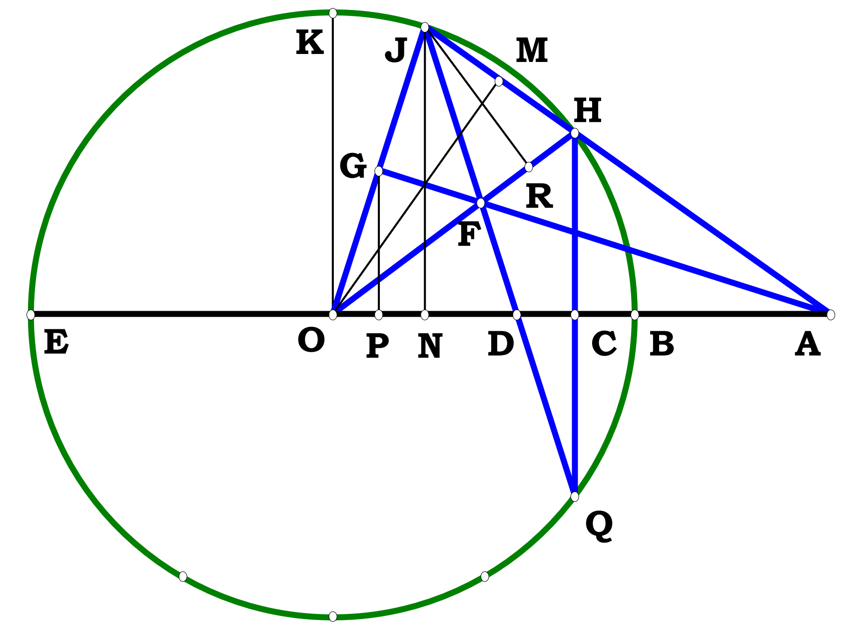
$HJ := 2 \cdot JM \quad AH := AJ - HJ$

$AC := \frac{AN \cdot AH}{AJ} \quad CH := \sqrt{AH^2 - AC^2}$

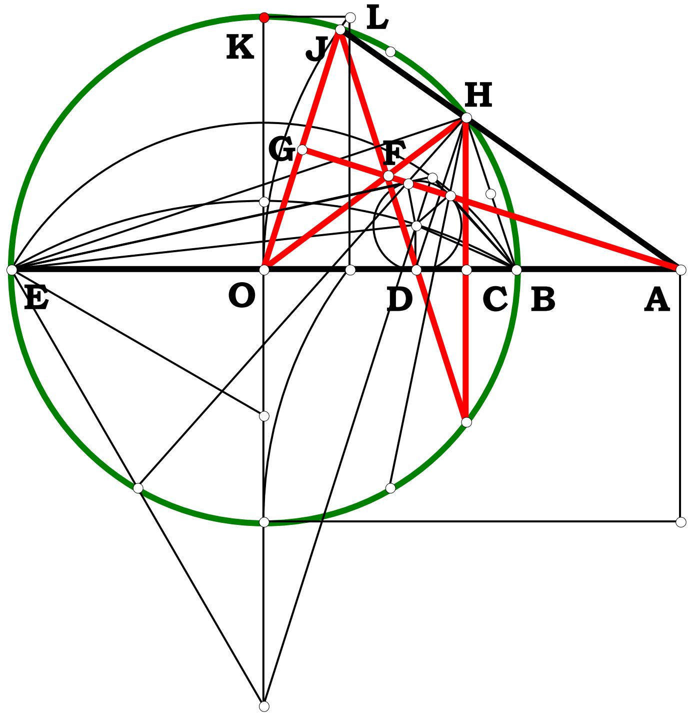
$HQ := 2 \cdot CH \quad CN := AN - AC$

$JN := \frac{CH \cdot AJ}{AH} \quad CQ := CH$

$JQ := \sqrt{(CQ + JN)^2 + CN^2} \quad OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO} \quad JR := \sqrt{JO^2 - OR^2}$



$FO := \frac{JO \cdot GO}{OR} \quad FJ := FO \quad DQ := \frac{JQ \cdot CQ}{CQ + JN} \quad DF := JQ - (DQ + FJ) \quad FH := HO - FO \quad FG := \frac{JR \cdot GO}{OR} \quad AF := AG - FG$



$$AE - AB\cdot N = 0 \qquad BE - (AB\cdot N - AB) = 0 \qquad BO - \frac{1}{2}\cdot AB\cdot (N-1) = 0$$

$$AO - \frac{1}{2}\cdot AB\cdot (1+N) = 0 \qquad AG - \frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} = 0$$

$$AP - \frac{AB}{8}\cdot (N+3)\cdot\frac{(3\cdot N+1)}{(1+N)} = 0 \qquad OP - \frac{AB}{8}\cdot\frac{(N-1)^2}{(1+N)} = 0 \qquad NO - \frac{AB}{4}\cdot\frac{(N-1)^2}{(1+N)} = 0$$

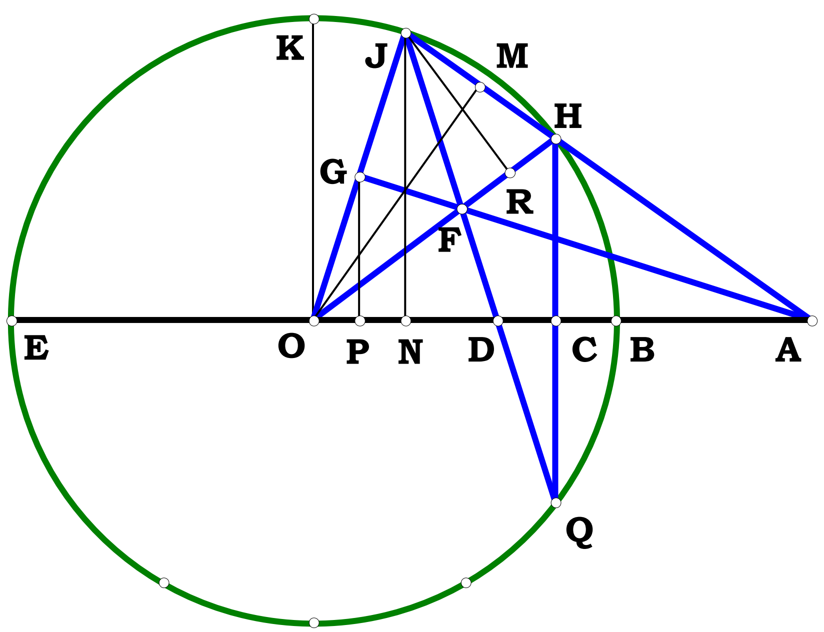
$$AN - \frac{AB}{4}\cdot\frac{\left(1+6\cdot N+N^2\right)}{(1+N)} = 0 \qquad HJ - \frac{AB}{2}\cdot\frac{(N-1)^2}{(1+N)} = 0 \qquad AH - 2\cdot AB\cdot\frac{N}{(1+N)} = 0$$

$$AC - AB\cdot\left(1+6\cdot N+N^2\right)\cdot\frac{N}{(1+N)^3} = 0 \qquad CH - AB\cdot N\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot\frac{(N-1)}{(1+N)^3} = 0$$

$$HQ - 2\cdot AB\cdot N\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot\frac{(N-1)}{(1+N)^3} = 0 \qquad CN - \frac{AB}{4}\cdot\left(N^2+6\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3} = 0$$

$$JN - \frac{AB}{4}\cdot (N-1)\cdot\sqrt{3\cdot N+1}\cdot\frac{\sqrt{N+3}}{(1+N)} = 0 \qquad JQ - \frac{AB}{2}\cdot (N-1)\cdot\frac{\left(N^2+6\cdot N+1\right)}{(1+N)^2} = 0$$

$$OR - \frac{AB}{4}\cdot\left(N^2+6\cdot N+1\right)\cdot\frac{(N-1)}{(1+N)^2} = 0 \qquad JR - \frac{AB}{4}\cdot (N-1)^2\cdot\sqrt{N+3}\cdot\frac{\sqrt{3\cdot N+1}}{(1+N)^2} = 0$$



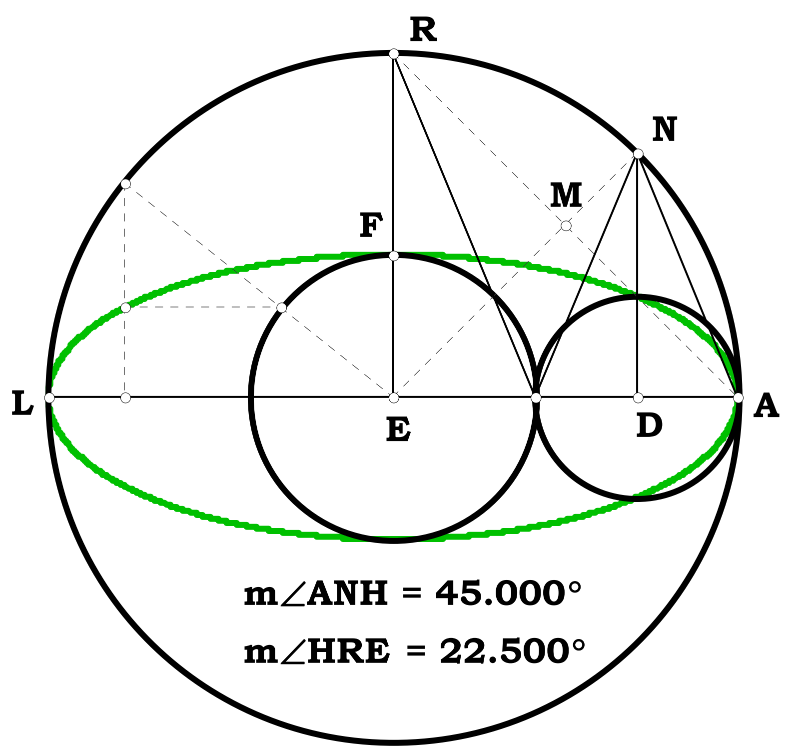
$$FO - \frac{AB}{2}\cdot (1+N)^2\cdot\frac{(N-1)}{\left(N^2+6\cdot N+1\right)} = 0 \qquad DQ - 2\cdot N\cdot (N-1)\cdot\frac{AB}{(1+N)^2} = 0$$

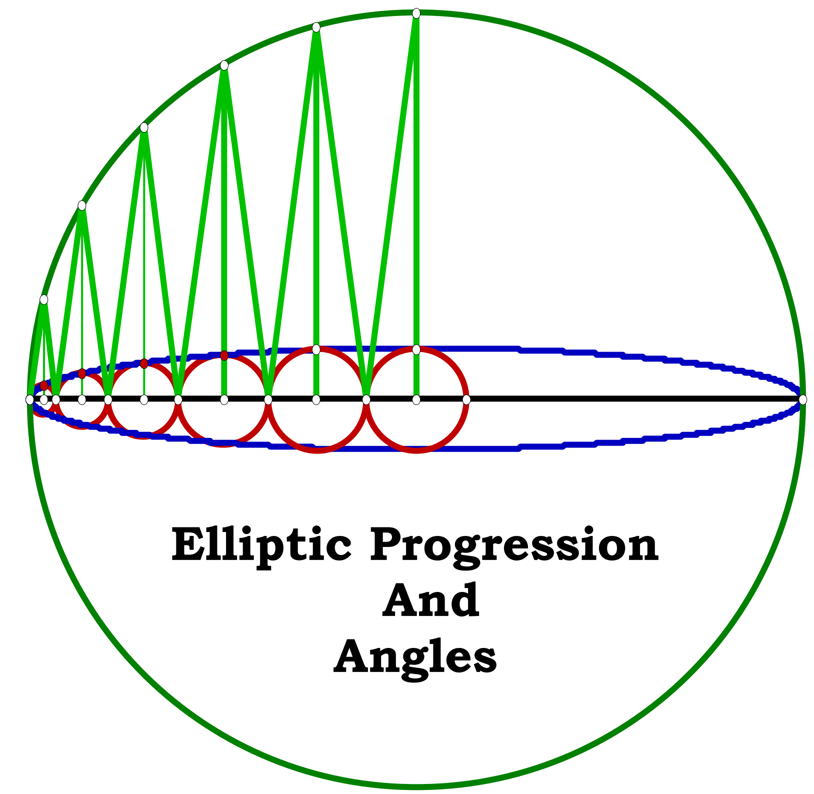
$$DF - 2\cdot N\cdot (N-1)\cdot\frac{AB}{\left(N^2+6\cdot N+1\right)} = 0 \qquad FH - 2\cdot N\cdot (N-1)\cdot\frac{AB}{\left(N^2+6\cdot N+1\right)} = 0$$

$$FG - \frac{AB}{4}\cdot (N-1)^2\cdot\sqrt{N+3}\cdot\frac{\sqrt{3\cdot N+1}}{\left(N^2+6\cdot N+1\right)} = 0 \qquad GO - \frac{AB}{4}\cdot (N-1) = 0$$

$$AF - \left[\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} - \frac{1}{4}\cdot AB\cdot (N-1)^2\cdot\sqrt{N+3}\cdot\frac{\sqrt{3\cdot N+1}}{\left(N^2+6\cdot N+1\right)}\right] = 0$$

Angles by Ellipse: 0507013





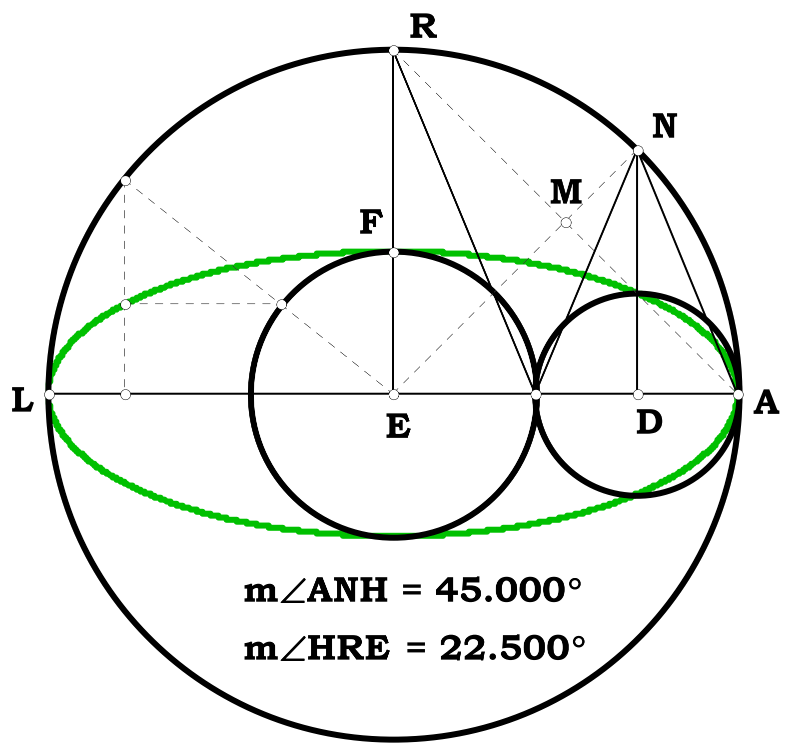
$N := 1$

$AL := N \quad AE := \frac{AL}{2} \quad AR := \sqrt{2 \cdot AE^2}$

$AM := \frac{AR}{2} \quad EN := AE \quad EM := AM$

$MN := EN - EM \quad AN := \sqrt{AM^2 + MN^2}$

$AD := \frac{AN^2}{AL} \quad EF := \frac{AL - (4 \cdot AD)}{2}$



Some Algebraic Names.

$$AR - \frac{N}{2}\cdot\sqrt{2} = 0 \quad AM - \frac{N}{4}\cdot\sqrt{2} = 0 \quad MN - \frac{N}{4}\cdot\left(2-\sqrt{2}\right) = 0 \quad AN - \frac{N}{2}\cdot\sqrt{2-\sqrt{2}} = 0$$

$$AD - \frac{N\cdot\left(2-\sqrt{2}\right)}{4} = 0 \quad EF - \left(\frac{1}{2}\cdot N\cdot\sqrt{2} - \frac{1}{2}\cdot N\right) = 0 \quad (2\cdot AD - EF) - \left(\frac{3}{2}\cdot N - N\cdot\sqrt{2}\right) = 0$$

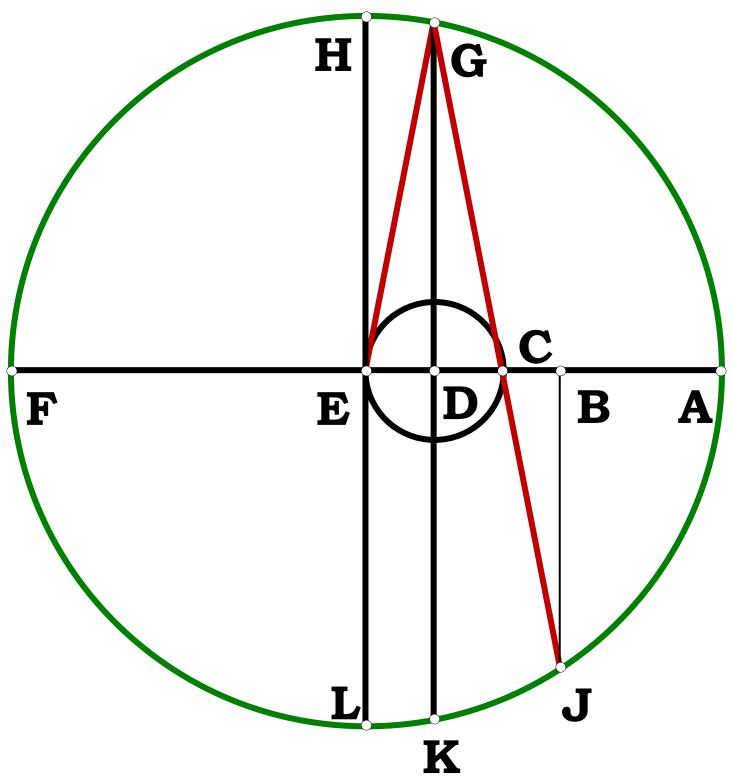
Some Algebraic Names Where N = 1.

$$AR - \frac{\sqrt{2}}{2} = 0 \quad AM - \frac{\sqrt{2}}{4} = 0 \quad MN - \frac{2-\sqrt{2}}{4} = 0 \quad AN - \frac{\sqrt{2-\sqrt{2}}}{2} = 0$$

$$AD - \frac{2-\sqrt{2}}{4} = 0 \quad EF - \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = 0 \quad (2\cdot AD - EF) - \left(\frac{3}{2} - \sqrt{2}\right) = 0$$

Elliptic Progression Outtake One 0507011

A method of trisection Algebraically.



$$N := M$$

$$N \ge 4 = 1 \quad AF := 1 \quad AE := \frac{AF}{2} \quad DE := \frac{AF}{N} \quad AD := AE - DE \quad DF := AF - AD$$

$$DG := \sqrt{AD \cdot DF} \quad CD := DE \quad EG := AE \quad CO := \frac{CD^2}{EG} \quad CG := EG \quad CJ := CG - 4 \cdot CO$$

$$BC := \frac{CD \cdot CJ}{CG} \quad AB := AE - (2 \cdot DE + BC) \quad BJ := \frac{DG \cdot BC}{CD} \quad BD := BC + CD$$

$$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2} \qquad \frac{JK}{2 \cdot DE} = 1$$

Some Algebraic Names,

Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.

$$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0 \quad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0 \quad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$$

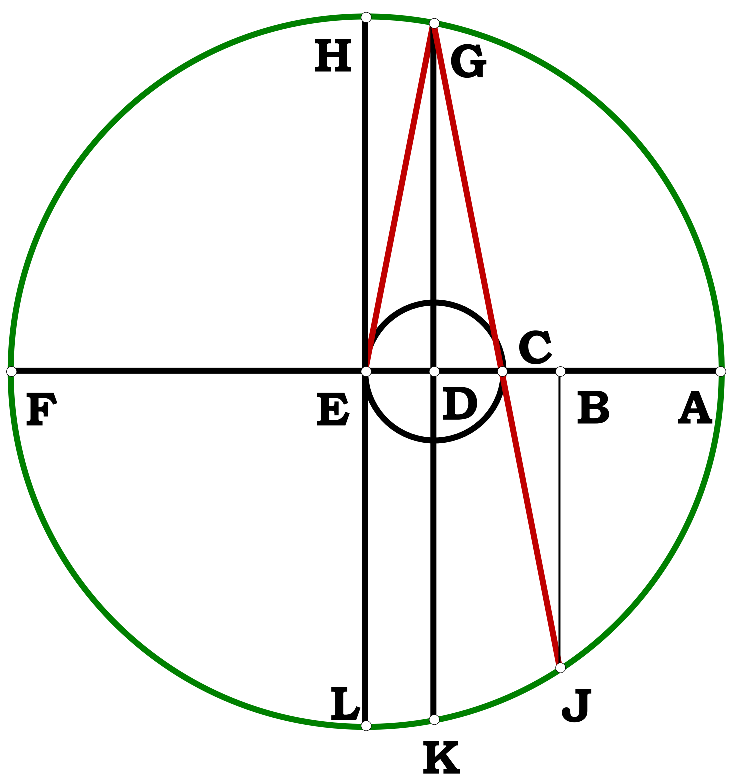
$$\frac{2AF}{N^2} - CO = 0 \quad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0 \quad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$$

$$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$$

One of the meanings of trisection is solving for the following equation when given AF and AB.

$$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0 \quad AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0$$

$$AF \cdot \frac{2 \cdot \left(N^2 - 8\right)}{N^3} - BD = 0 \quad \frac{2 \cdot AF}{N} - JK = 0$$



$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2}-\frac{AF}{AB}=0 \qquad \frac{AF}{AB}=6$$

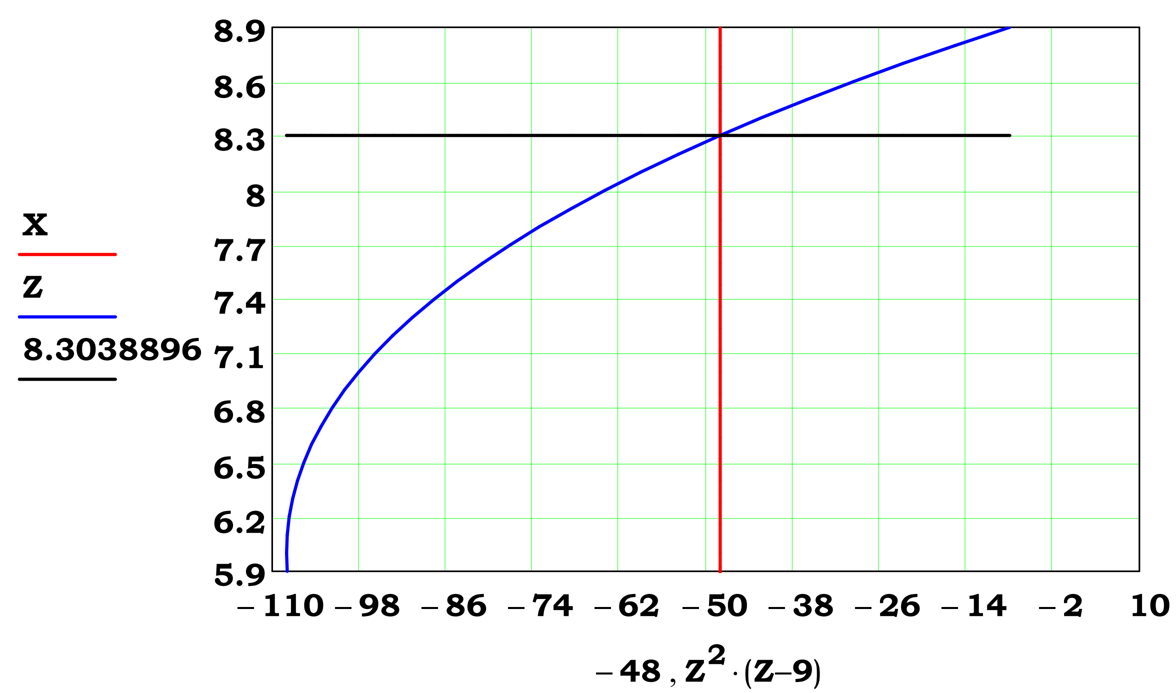
$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2}-6=0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2}=6$$

$$9M^2-M^3-48=0$$

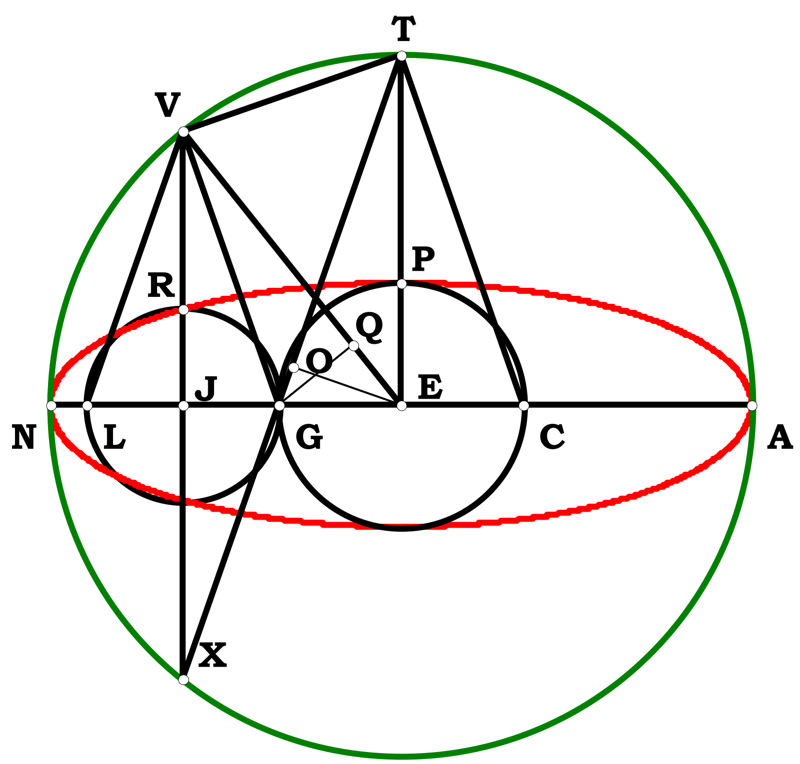
$$DE-\frac{AF}{M}=0$$

$$M \equiv 8.303889634816388 \qquad Z := 5.9\,,6\,..\,8.9$$



Elliptic Progression Outtake Two 0507012

Angles TEV and EVJ equals CTG.



$$N := 6.381 \quad AN := 2.792$$

$$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$$

$$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$$

$$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$$

$$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$$

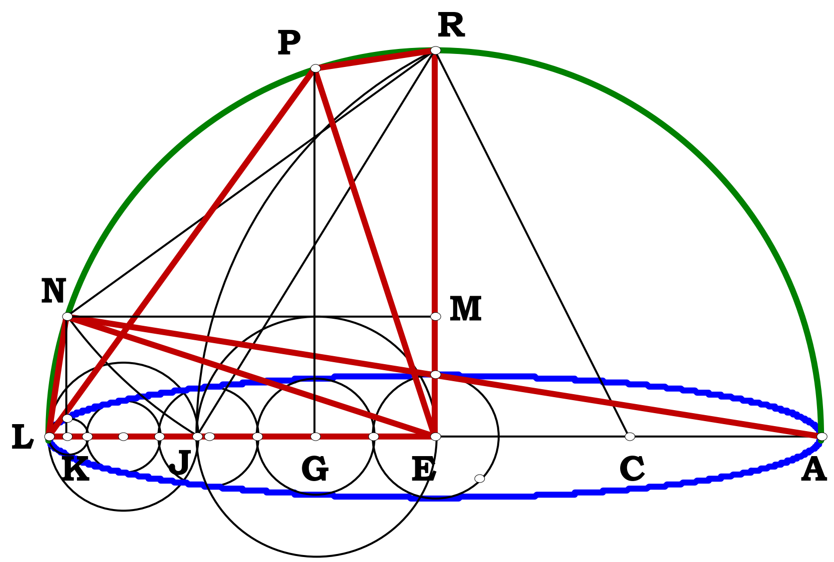
$$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \quad JV := JX$$

$$EJ := EG + GJ$$

$$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$$

$$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \quad EQ := \frac{EJ \cdot EG}{EV} \quad GQ := \frac{JV \cdot EQ}{EJ} \quad GQ - GJ = 0 \quad TV - \frac{2 \cdot AN}{\sqrt{N^2 + 4}} = 0$$

Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013



$$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2} \qquad CE := AC \qquad ER := AE \qquad CR := \sqrt{CE^2 + ER^2}$$

$$CJ := CR \qquad EJ := CJ - CE \qquad JR := \sqrt{EJ^2 + ER^2} \qquad NR := JR \qquad EN := AE \qquad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$$

$$KN := EM \qquad EK := \sqrt{EN^2 - KN^2} \qquad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$$

$$EG := \frac{EJ}{2} \qquad GL := EL - EG \qquad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$$

$$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$$

Irrational **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

Definitions:

$$\frac{1}{2} - AE = \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$$

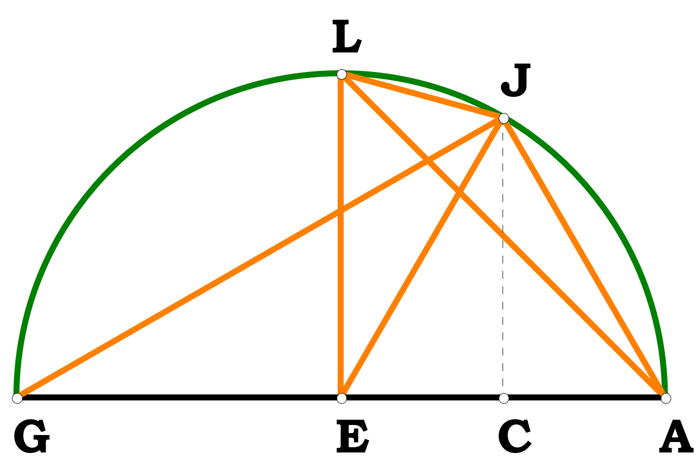
$$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$$

$$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$$

$$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$$

$$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$$

Outtake Four: Some Names 0507014



$$\mathbf{AG} := 1 \quad \mathbf{AE} := \frac{\mathbf{AG}}{2} \quad \mathbf{AC} := \frac{\mathbf{AE}}{2}$$

$$\mathbf{CG} := \mathbf{AG} - \mathbf{AC} \quad \mathbf{CJ} := \sqrt{\mathbf{AC}\cdot\mathbf{CG}}$$

$$\mathbf{EL} := \mathbf{AE} \qquad \mathbf{CE} := \mathbf{AC}$$

$$\mathbf{JL} := \sqrt{\mathbf{EL}^2 - 2\cdot\mathbf{EL}\cdot\mathbf{CJ} + \mathbf{CJ}^2 + \mathbf{CE}^2}$$

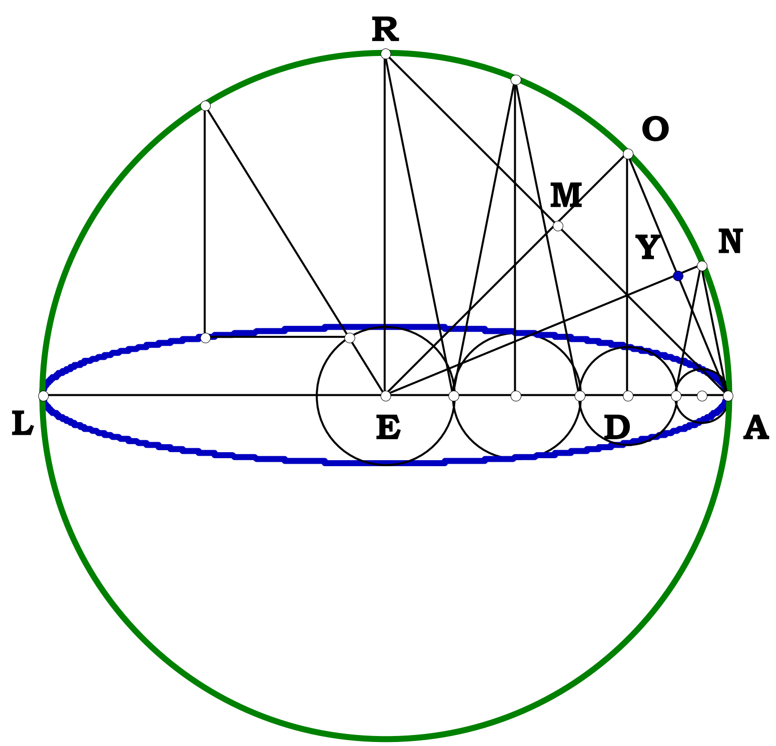
$$\mathbf{AJ} := \sqrt{\mathbf{AC}^2 + \mathbf{CJ}^2} \quad \mathbf{GJ} := \sqrt{\mathbf{CG}^2 + \mathbf{CJ}^2}$$

$$\mathbf{AL} := \sqrt{\mathbf{AE}^2 + \mathbf{EL}^2}$$

$$\mathbf{AE} - \frac{1}{2} = 0 \qquad \mathbf{AC} - \frac{1}{4} = 0 \qquad \mathbf{CG} - \left(1 - \frac{1}{4}\right) = 0 \qquad \mathbf{CJ} - \frac{1}{4}\cdot\sqrt{3} = 0 \qquad \mathbf{JL} - \left(\frac{1}{4}\cdot\sqrt{6} - \frac{1}{4}\cdot\sqrt{2}\right) = 0$$

$$\mathbf{AJ} - \frac{1}{2} = 0 \qquad \mathbf{GJ} - \frac{1}{2}\cdot\sqrt{3} = 0 \quad \mathbf{AL} - \frac{1}{2}\cdot\sqrt{2} = 0$$

Quadsection: 0507013



$$AL := 2.5 \quad AE := \frac{AL}{2} \quad AR := \sqrt{2 \cdot AE^2}$$

$$AM := \frac{AR}{2} \quad EO := AE \quad EM := AM$$

$$MO := EO - EM \quad AO := \sqrt{AM^2 + MO^2}$$

$$AY := \frac{AO}{2} \quad EY := \sqrt{AE^2 - AY^2}$$

$$EN := AE \quad NY := EN - EY \quad AN := \sqrt{AY^2 + NY^2}$$

$$AP := \frac{AN^2}{AL} \quad NP := \sqrt{AN^2 - AP^2} \quad ES := \frac{NP \cdot AE}{AL - AP}$$

Some Algebraic Names.

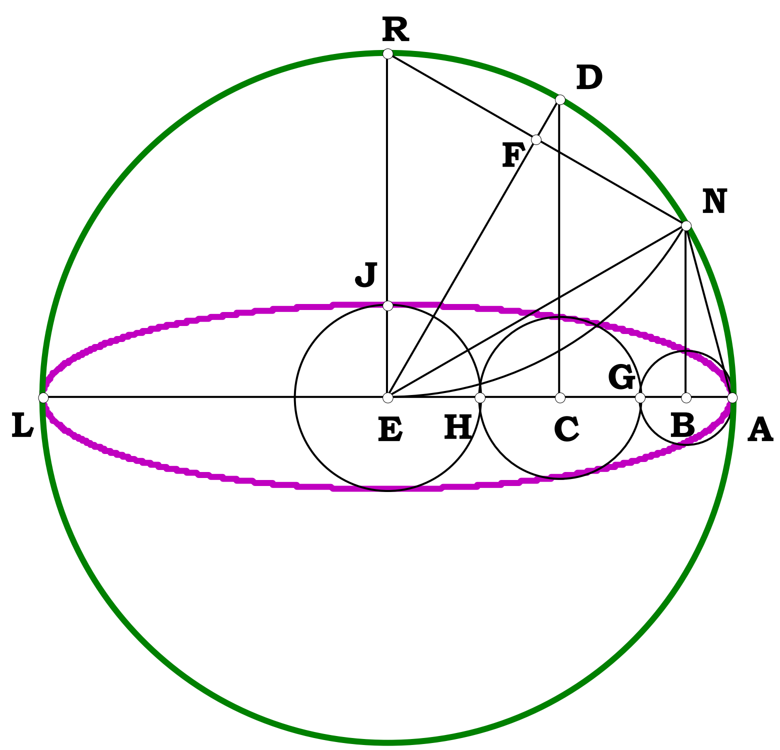
$$AR - \frac{AL}{2} \cdot \sqrt{2} = 0 \quad AM - \frac{AL}{4} \cdot \sqrt{2} = 0 \quad MO - \frac{AL}{4} \cdot \left(2 - \sqrt{2}\right) = 0 \quad AO - \frac{AL}{2} \cdot \sqrt{2 - \sqrt{2}} = 0$$

$$AY - \frac{AL}{4} \cdot \sqrt{2 - \sqrt{2}} = 0 \quad EY - \frac{AL}{4} \cdot \sqrt{2 + \sqrt{2}} = 0 \quad NY - \frac{AL}{4} \cdot \left(2 - \sqrt{2 + \sqrt{2}}\right) = 0$$

$$AN - \frac{AL}{2} \cdot \sqrt{2 - \sqrt{2 + \sqrt{2}}} = 0 \quad \frac{2 - \sqrt{2 + \sqrt{2}}}{4} \cdot AL - AP = 0 \quad \frac{AL}{4} \cdot \sqrt{2 - \sqrt{2}} - NP = 0$$

$$\frac{1}{2} \cdot AL \cdot \frac{\sqrt{2 - \sqrt{2}}}{2 + \sqrt{2 + \sqrt{2}}} - ES = 0$$

Trisection: 0507013



$N := 4$

$AL := N \quad AE := \dfrac{AL}{2} \quad ER := AE$

$NR := ER \quad FN := \dfrac{NR}{2} \quad EN := AE$

$EF := \sqrt{EN^2 - FN^2} \quad DE := AE$

$DF := DE - EF \quad DN := \sqrt{DF^2 + FN^2}$

$AN := DN \quad AB := \dfrac{AN^2}{AL} \quad EL := AE$

$BL := AL - AB \quad BN := \sqrt{AN^2 - AB^2}$

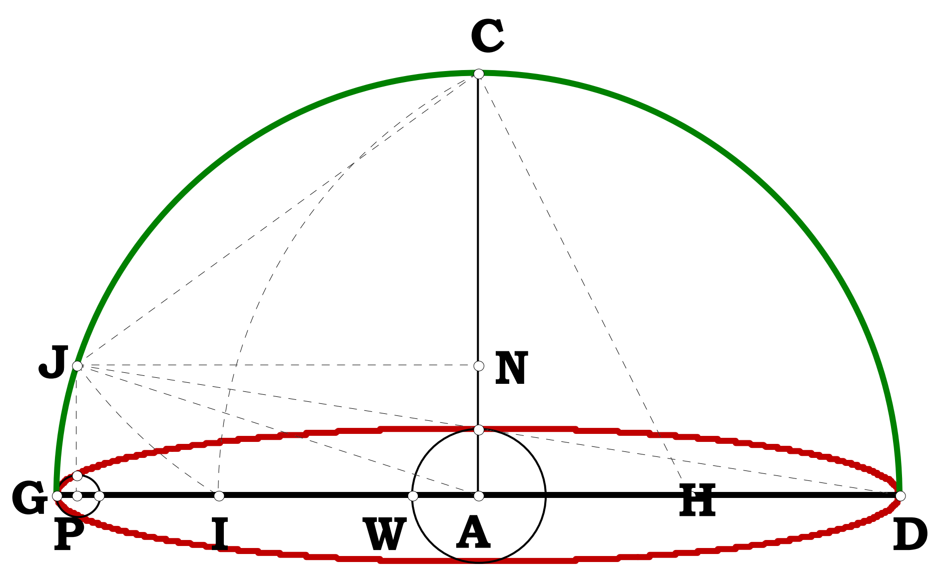
$EJ := \dfrac{BN \cdot EL}{BL} \quad GH := AE - (EJ + 2 \cdot AB)$

Some Algebraic Names.

$FN - \dfrac{N}{4} = 0 \qquad EF - \dfrac{N}{4} \cdot \sqrt{3} = 0 \qquad DF - \dfrac{N \cdot \left(2 - \sqrt{3}\right)}{4} = 0 \qquad AN - \dfrac{\sqrt{2} \cdot N \cdot \left(\sqrt{3} - 1\right)}{4} = 0 \qquad \dfrac{N \cdot \left(2 - \sqrt{3}\right)}{4} - AB = 0$

$\dfrac{N \cdot \left(\sqrt{3} + 2\right)}{4} - BL = 0 \qquad \dfrac{\sqrt{N^2}}{4} - BN = 0 \qquad \dfrac{\sqrt{N^2}}{2 \cdot \sqrt{3} + 4} - EJ = 0 \qquad \dfrac{N + \sqrt{3} \cdot N - \sqrt{N^2}}{2 \cdot \left(\sqrt{3} + 2\right)} - GH = 0$

Pentasection 050701



$N := 3.333 \quad DG := N$

$AD := \frac{DG}{2} \quad AC := AD$

$AG := AD \quad AH := \frac{AD}{2}$

$CH := \sqrt{AH^2 + AC^2}$

$HI := CH \quad AI := HI - AH \quad CI := \sqrt{AI^2 + AC^2} \quad CJ := CI \quad CN := \frac{CJ^2}{2 \cdot AC}$

$AN := AC - CN \quad JP := AN \quad JN := \sqrt{CJ^2 - CN^2} \quad AP := JN \quad AW := \frac{JP \cdot AD}{AD + AP}$

Definitions:

$AD := \frac{N}{2} \quad AC := \frac{N}{2} \quad AG := \frac{N}{2} \quad AH := \frac{N}{4} \quad \frac{\sqrt{5} \cdot N}{4} - CH = 0$

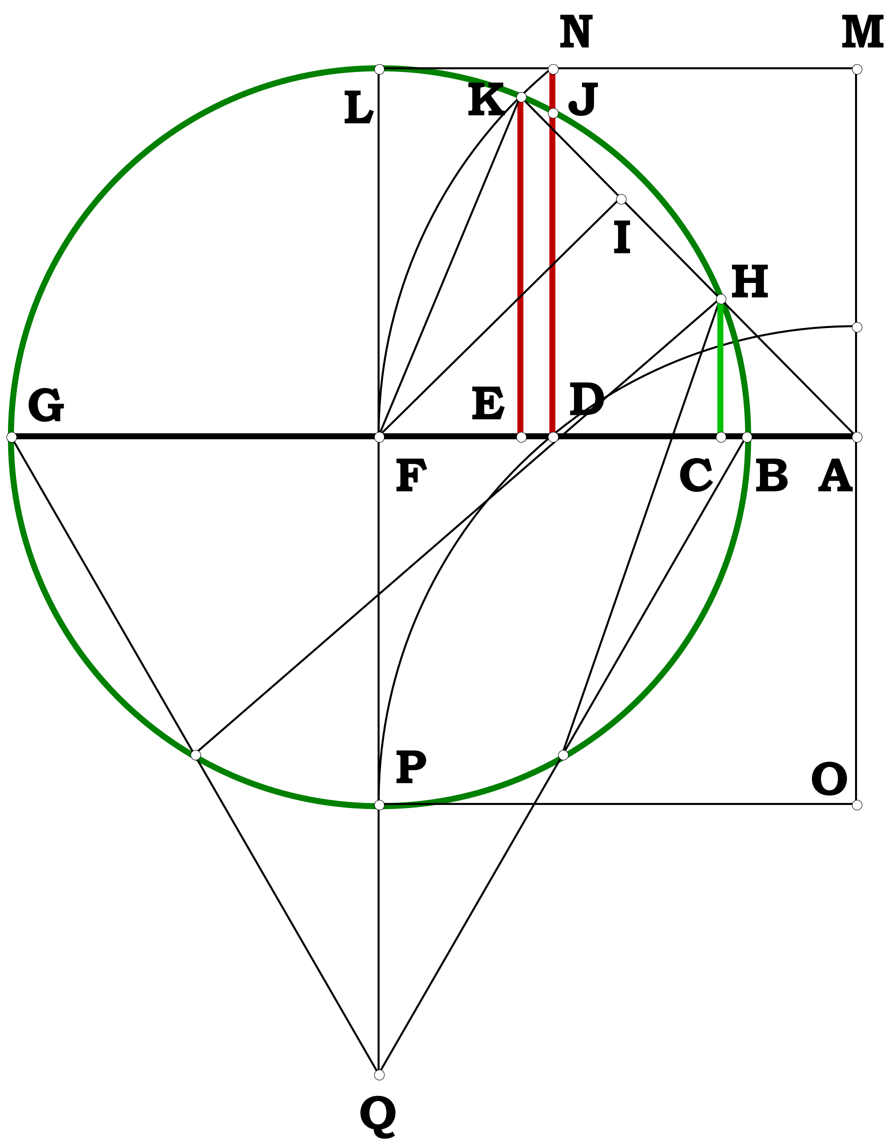
$\frac{\sqrt{5} \cdot N}{4} - HI = 0 \quad \frac{N \cdot \left(\sqrt{5} - 1\right)}{4} - AI = 0 \quad \frac{\sqrt{2} \cdot N \cdot \sqrt{5 - \sqrt{5}}}{4} - CI = 0$

$\frac{N \cdot \left(5 - \sqrt{5}\right)}{8} - CN = 0 \quad \frac{N \cdot \left(\sqrt{5} - 1\right)}{8} - AN = 0$

$\frac{\sqrt{N^2 \cdot \sqrt{2 \cdot \sqrt{5} + 10}}}{8} - JN = 0 \quad \frac{N \cdot \left(\sqrt{5} - 1\right)}{2 \cdot \sqrt{2 \cdot \sqrt{5} + 10} + 8} - AW = 0$

On Trisection 051301

For any given trisection what is the Algebraic names of BC and BE taking BG as unit?



$$N := 9 \quad BG := 1 \quad AG := BG \cdot N$$

$$AB := AG - BG \quad BF := \frac{BG}{2} \quad AF := AB + BF \quad AN := AF \quad AK := AN$$

$$FK := BF \quad S_1 := AF \quad S_2 := AK \quad S_3 := FK$$

$$AE := \frac{{S_2}^2 + {S_1}^2 - {S_3}^2}{2 \cdot S_1} \quad AI := AE \quad IK := AK - AI \quad HI := IK \quad AH := AK - (HI + IK)$$

$$AC := \frac{AE \cdot AH}{AK} \quad BC := AC - AB \quad BE := AE - AB$$

Some Algebraic Names:

$$N - 1 - AB = 0 \quad \frac{1}{2} - BF = 0 \quad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0 \quad \frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0$$

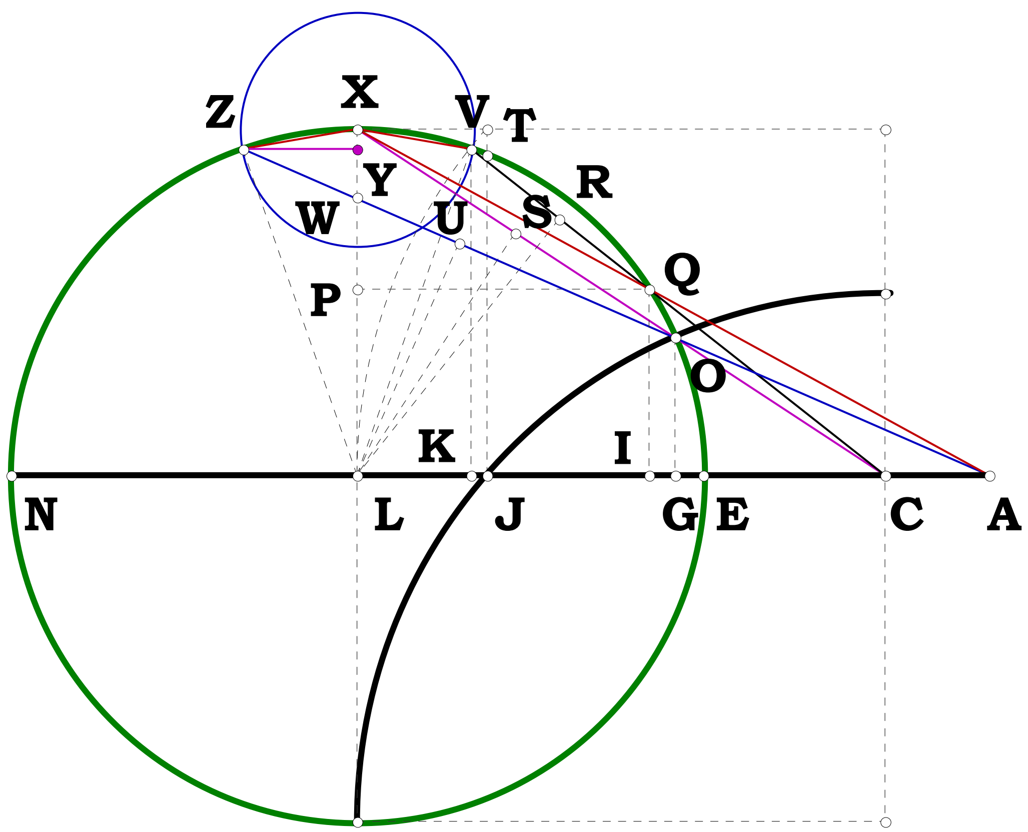
$$\frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0 \quad 2 \cdot N \cdot \frac{(N-1)}{(2 \cdot N - 1)} - AH = 0 \quad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$$

$$(N-1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \quad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$$

$$\frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$$

On Trisection 051401.MCD

For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?



$\mathbf{N := 5} \quad \mathbf{CE := 1} \quad \mathbf{CN := CE \cdot N}$

$\mathbf{EN := CN - CE} \quad \mathbf{EL := \dfrac{EN}{2}} \quad \mathbf{LV := EL}$

$\mathbf{CL := CE + EL}$

$\mathbf{CV := CL} \quad \mathbf{S_1 := CL} \quad \mathbf{S_2 := CV}$

$\mathbf{S_3 := LV} \quad \mathbf{CK := \dfrac{{S_2}^2 + {S_1}^2 - {S_3}^2}{2 \cdot S_1}} \quad \mathbf{CR := CK} \quad \mathbf{RV := CV - CR}$

$\mathbf{QR := RV} \quad \mathbf{CQ := CV - (QR + RV)} \quad \mathbf{CI := \dfrac{CK \cdot CQ}{CV}} \quad \mathbf{EI := CI - CE}$

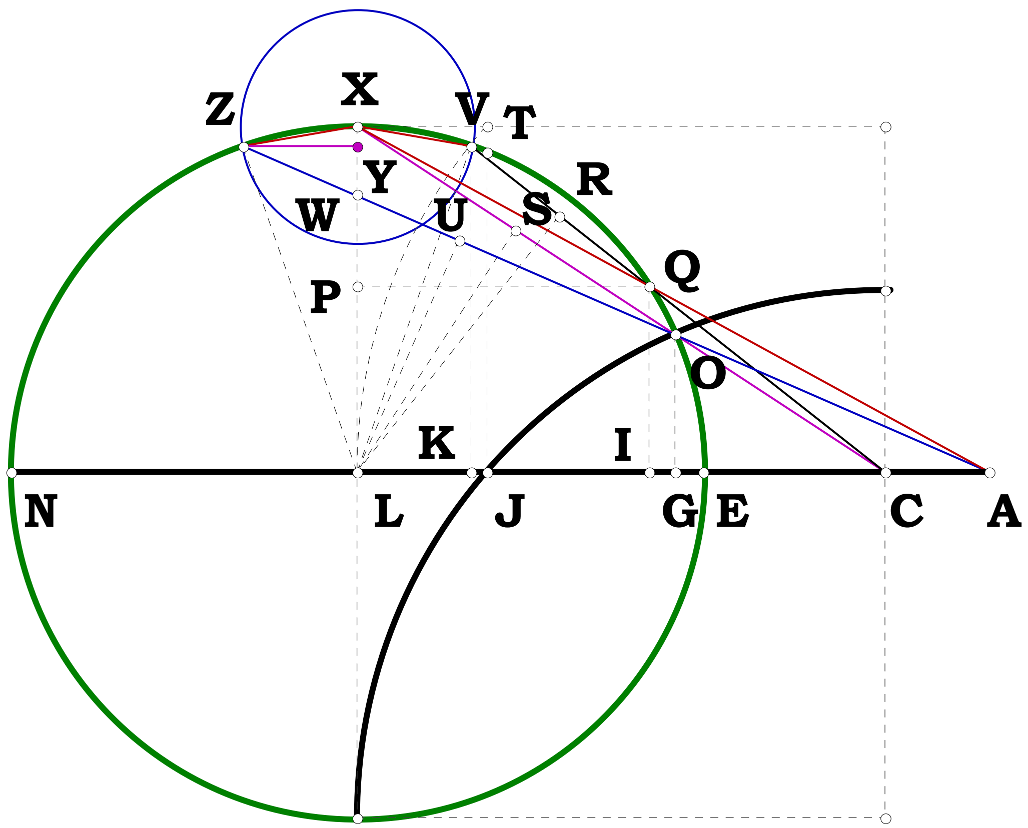
$\mathbf{IN := EN - EI} \quad \mathbf{IQ := \sqrt{EI}} \mathbf{LP := IQ} \quad \mathbf{LX := E} \mathbf{IL := EL - EI} \quad \mathbf{PX := LX - LP}$

$\mathbf{AL := \dfrac{IL \cdot LX}{PX}} \quad \mathbf{AE := AL - EL} \quad \mathbf{AC := AE - CE} \quad \mathbf{AI := AC + CI} \quad \mathbf{AX := \sqrt{AL^2 + LX^2}}$

$\mathbf{AQ := \dfrac{AX \cdot AI}{AL}} \quad \mathbf{QX := AX - AQ} \quad \mathbf{CX := \sqrt{CL^2 + LX^2}} \quad \mathbf{CS := \dfrac{CL^2}{CX}} \quad \mathbf{SX := CX - CS}$

$\mathbf{OS := SX} \quad \mathbf{CO := CX - (SX + OS)} \quad \mathbf{GO := \dfrac{LX \cdot CO}{CX}} \quad \mathbf{CG := \dfrac{CL \cdot CO}{CX}} \quad \mathbf{EG := CG - CE}$

$\mathbf{AG := AE + EG} \quad \mathbf{AO := \sqrt{AG^2 + GO^2}} \quad \mathbf{AU := \dfrac{AG \cdot AL}{AO}} \quad \mathbf{OU := AU - AO} \quad \mathbf{UZ := OU} \quad \mathbf{AZ := AO + (OU + UZ)}$



$$LW := \frac{GO \cdot AL}{AG} \quad AW := \frac{AO \cdot AL}{AG} \quad WZ := AZ - AW \quad WY := \frac{GO \cdot WZ}{AO}$$

$$YZ := \frac{AG \cdot WZ}{AO} \quad YX := LX - (LW + WY) \quad XZ := \sqrt{YX^2 + YZ^2}$$

Some Algebraic Names:

$$\frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - CK = 0$$

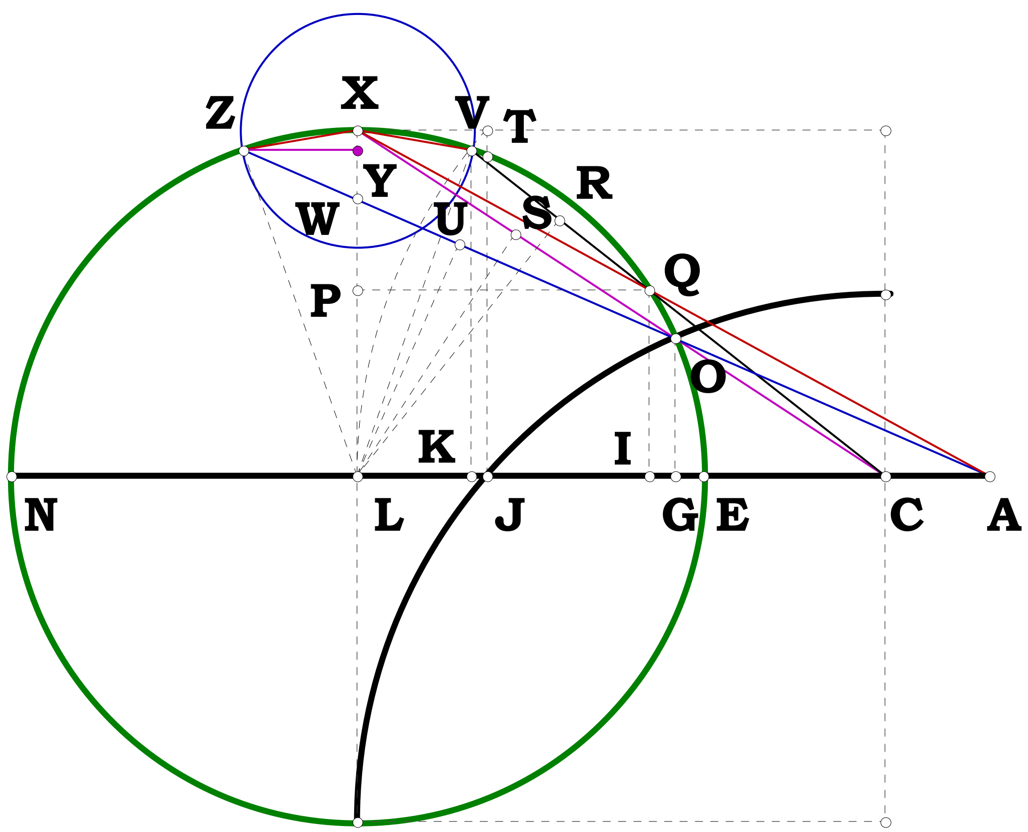
$$\frac{1}{4} \cdot CE \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{(1 + N)} - RV = 0 \qquad 2 \cdot CE \cdot \frac{N}{(1 + N)} - CQ = 0$$

$$CE \cdot \frac{N^3 + 6N^2 + N}{(N + 1)^3} - CE - EI = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - CI = 0$$

$$CE \cdot N^2 \cdot \frac{\left(-3 + 2 \cdot N + N^2\right)}{(1 + N)^3} - IN = 0 \qquad (N - 1) \cdot N \cdot CE \cdot \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(1 + N)^3} - IQ = 0$$

$$\frac{1}{2} \cdot CE \cdot \frac{\left(2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4 + 1\right)}{(1 + N)^3} - IL = 0$$

$$\left[\frac{1}{2} \cdot CE \cdot (N - 1) \cdot \frac{\left[N^3 + 3 \cdot N^2 - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}{(1 + N)^3}\right] - PX = 0$$



$$\frac{1}{2}\cdot \mathbf{CE}\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-\mathbf{AL}=0$$

$$\mathbf{CE}\cdot(N-1)\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N-1\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-\mathbf{AE}=0$$

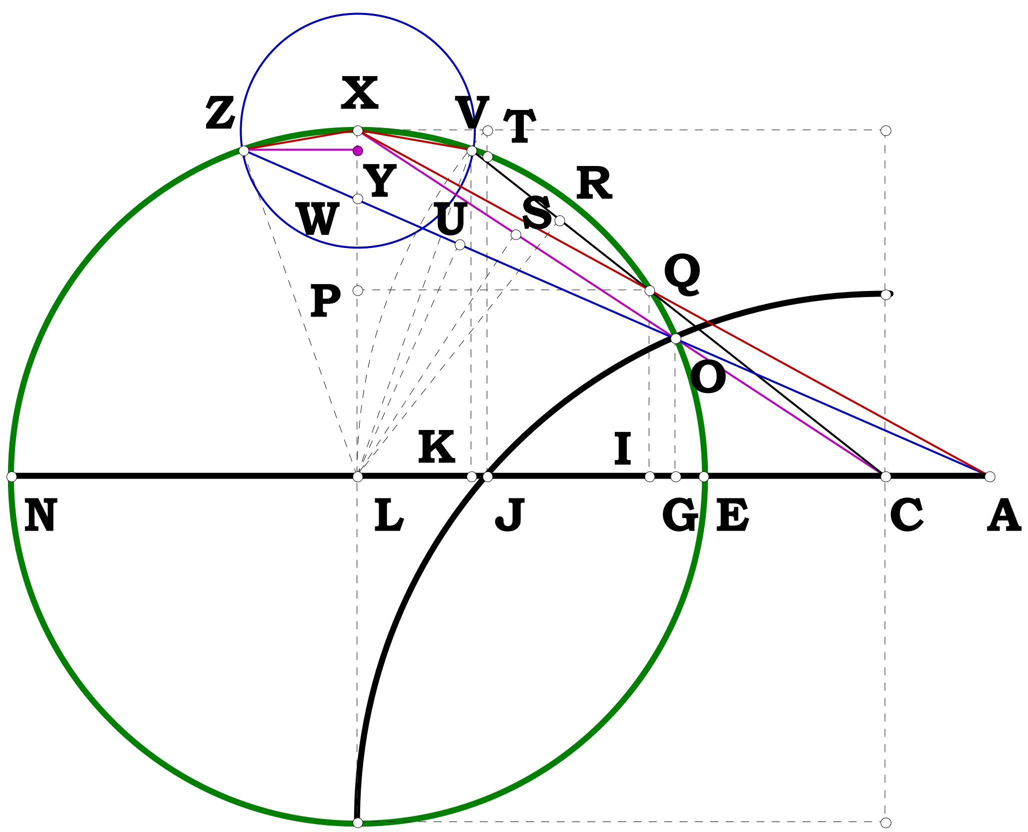
$$-\mathbf{CE}\cdot N\cdot\frac{\left[-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-\mathbf{AC}=0$$

$$\frac{\mathbf{CE}\cdot(N-1)^2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+4\cdot N+1\right)}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot(1+N)^3}-\mathbf{AI}=0$$

$$\frac{1}{2}\cdot\mathbf{CE}\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}-\mathbf{AX}=0$$

$$\mathbf{CE}\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}\cdot N\cdot\frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{(1+N)^3}-\mathbf{AQ}=0$$

$$\sqrt{\frac{\left[2\cdot(1+N)^3\right]}{\left[\begin{array}{l}N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\ +\ 3\cdot N+1\end{array}\right]}}\cdot\mathbf{CE}\cdot(N-1)\cdot\frac{\left[\begin{array}{l}\left(N^3+3\cdot N^2\right)-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\ +\ 3\cdot N+1\end{array}\right]}{2\cdot(1+N)^3}\ \ldots\ =0$$
$$+\ -\mathbf{QX}$$



$$\frac{CE}{2}\cdot\sqrt{2}\cdot\sqrt{\left(1+N^2\right)} - CX = 0 \qquad \frac{1}{4}\cdot CE\cdot(1+N)^2\cdot\frac{\sqrt{2}}{\sqrt{1+N^2}} - CS = 0$$

$$\frac{1}{4}\cdot\sqrt{2}\cdot CE\cdot\frac{(N-1)^2}{\sqrt{1+N^2}} - SX = 0 \qquad \frac{\sqrt{2}}{\sqrt{1+N^2}}\cdot CE\cdot N - CO = 0 \qquad CE\cdot N\cdot\frac{(N-1)}{\left(1+N^2\right)} - GO = 0$$

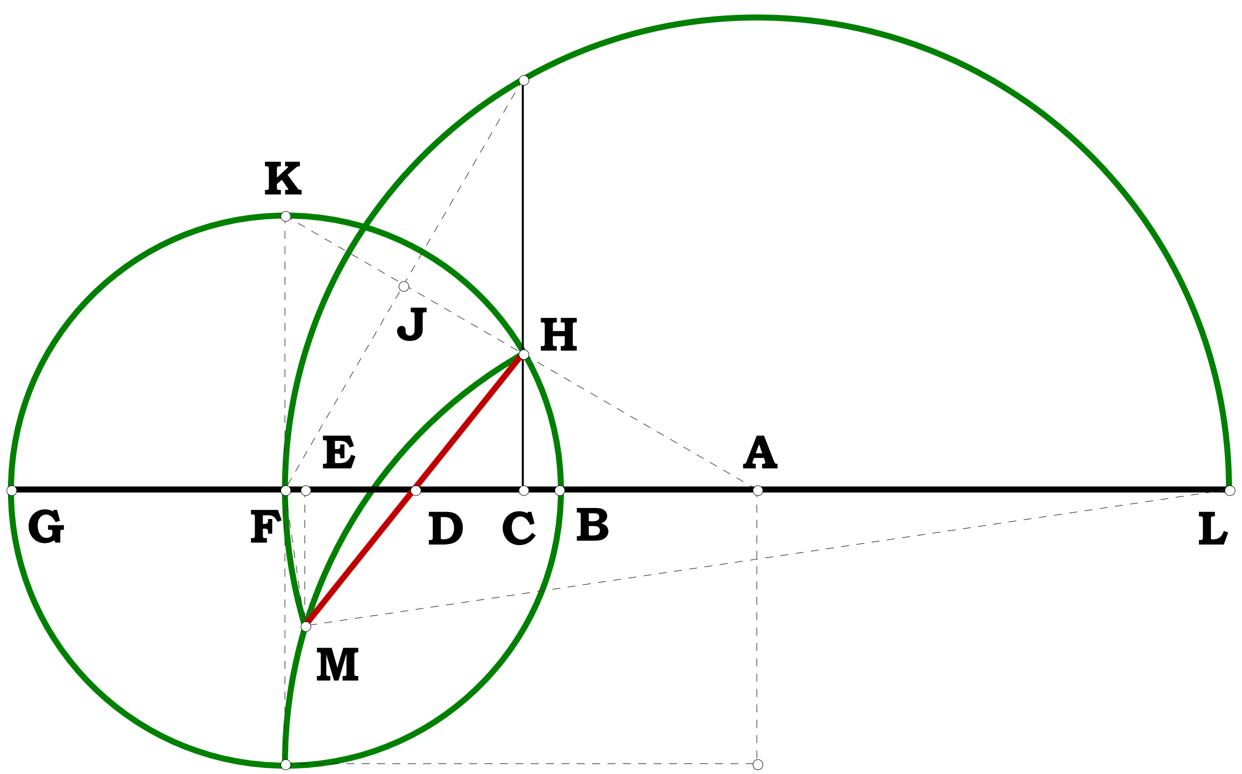
$$CE\cdot\frac{N^2+N}{\left(1+N^2\right)} - CG = 0 \qquad CE\cdot\frac{(N-1)}{\left(1+N^2\right)} - EG = 0$$

$$CE\cdot(N-1)^2\cdot N\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} - 2\cdot N + \sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot\left(1+N^2\right)\right]} - AG = 0$$

$$\frac{CE\cdot(N-1)}{2}\cdot\sqrt{\frac{\left[\left[2\cdot N^4+14\cdot N^3-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+32\cdot N^2\right]-7\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots \atop + -7\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+14\cdot N+2-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[(1+N)\cdot\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\right]}} - XZ = 0$$

Segment DF And HM 052201

Given AB and AG, what is HM and DF?



$$N := 7.111 \qquad AB := .375$$

$$AG := AB \cdot N \qquad BG := AG - AB \qquad BF := \frac{BG}{2} \qquad FK := BF \qquad AF := AB + BF$$

$$AK := \sqrt{AF^2 + FK^2} \qquad AJ := \frac{AF^2}{AK} \qquad JK := AK - AJ \qquad HJ := JK$$

$$AH := AK - (JK + HJ) \qquad AC := \frac{AF \cdot AH}{AK} \qquad EM := \frac{BF}{2} \qquad FL := 2 \cdot AF$$

$$EF := \frac{FL}{2} - \frac{\sqrt{-4EM^2 + FL^2}}{2} \qquad AE := AF - EF \qquad CE := AE - AC \qquad CH := \frac{FK \cdot AH}{AK}$$

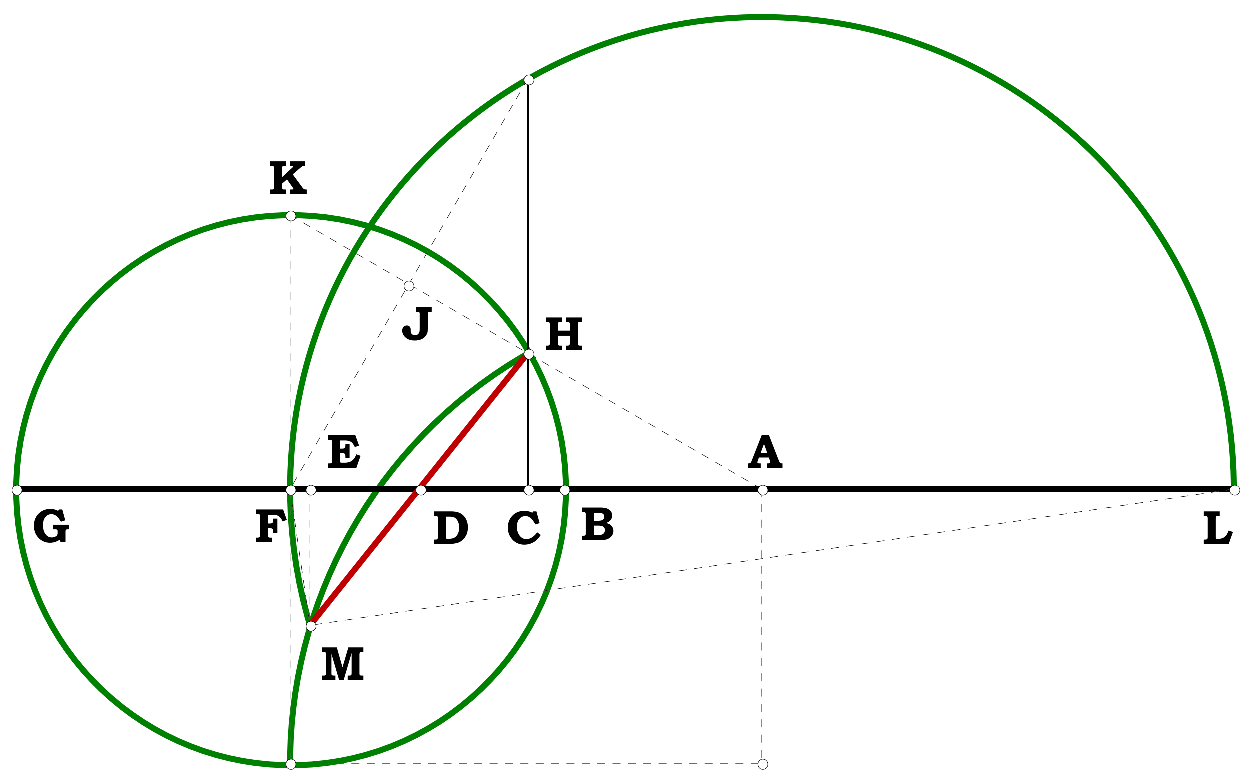
$$HM := \sqrt{(EM + CH)^2 + CE^2} \qquad DE := \frac{CE \cdot EM}{EM + CH} \qquad DF := DE + EF$$

Some Algebraic Names:

$$AB \cdot N - AG = 0 \qquad AB \cdot N - AB - BG = 0 \qquad \frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0 \qquad \frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - FK = 0$$

$$\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0 \qquad \frac{1}{2} \cdot AB \cdot \sqrt{2} \cdot \sqrt{1 + N^2} - AK = 0 \qquad \frac{1}{4} \cdot AB \cdot (1 + N)^2 \cdot \frac{\sqrt{2}}{\sqrt{1 + N^2}} - AJ = 0$$

$$\frac{1}{4} \cdot \sqrt{2} \cdot AB \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{\sqrt{1 + N^2}} - JK = 0 \qquad AB \cdot \sqrt{2} \cdot \frac{N}{\sqrt{1 + N^2}} - AH = 0 \qquad AB \cdot (1 + N) \cdot \frac{N}{1 + N^2} - AC = 0$$



$$\frac{1}{4}\cdot AB\cdot N-\frac{1}{4}\cdot AB-EM=0 \qquad AB+AB\cdot N-FL=0$$

$$\left(\frac{1}{2}\cdot AB+\frac{1}{2}\cdot AB\cdot N\right)-\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-EF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-AE=0$$

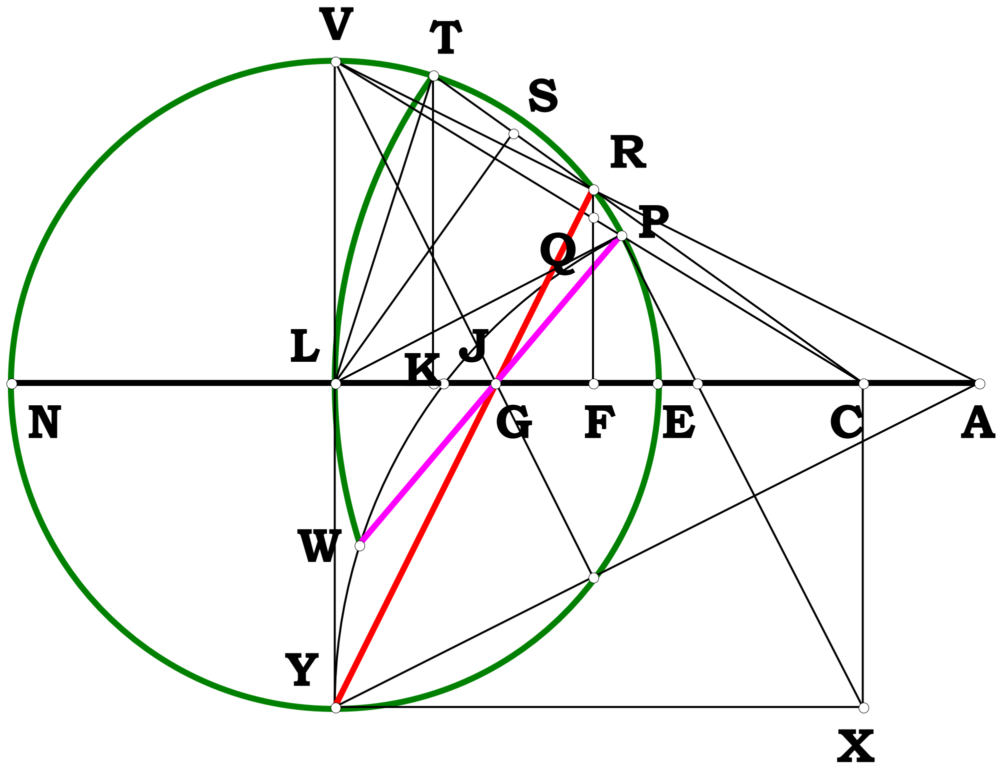
$$\frac{1}{4}\cdot AB\cdot\frac{\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)}{\left(1+N^2\right)}-CE=0$$

$$(N-1)\cdot AB\cdot\frac{N}{\left(1+N^2\right)}-CH=0$$

$$\frac{1}{2}\cdot AB\cdot\sqrt{(1+N)\cdot\frac{\left[N^3+3\cdot N^2-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+3\cdot N+1\right]}{\left(1+N^2\right)}}-HM=0$$

$$\frac{1}{4}\cdot\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)\cdot\frac{AB}{\left(N^2+4\cdot N+1\right)}-DE=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N+3\cdot N^2+1+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\right)}{\left(N^2+4\cdot N+1\right)}-DF=0$$



Point of Intersection 052701

Do RY and PW intersect at G?

$N := 5 \quad CE := 1 \quad CN := CE \cdot N \quad EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$LT := EL \quad LY := EL \quad CL := CE + EL \quad CT := CL \quad S_1 := CL \quad S_2 := CT \quad S_3 := LT$

$KL := \frac{S_3{}^2 + S_1{}^2 - S_2{}^2}{2 \cdot S_1} \quad CK := CL - KL \quad CS := CK \quad ST := CT - CS \quad RS := ST$

$CR := CT - (ST + RS) \quad CF := \frac{CK \cdot CR}{CT} \quad FR := \sqrt{CR^2 - CF^2} \quad FQ := \frac{LV \cdot CF}{CL} \quad FL := CL - CF$

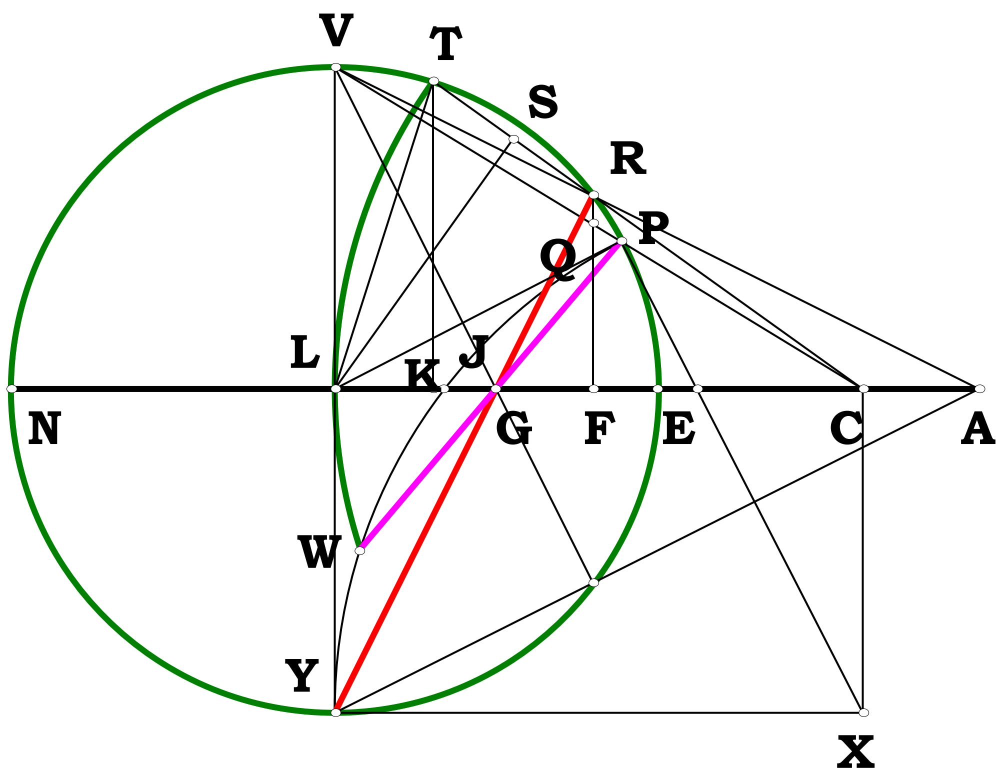
$GL := \frac{FL \cdot LY}{(LY + FR)}$

Some Algebraic Names:

$CE \cdot N - CN = 0 \quad CE \cdot N - CE - EN = 0 \quad \frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \quad \frac{1}{2} \cdot CE + \frac{1}{2} \cdot CE \cdot N - CL = 0 \quad \frac{1}{4} \cdot CE \cdot \frac{(N-1)^2}{(1+N)} - KL = 0$

$\frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)} - CK = 0 \quad \frac{1}{4} \cdot CE \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1+N)} - ST = 0 \quad 2 \cdot CE \cdot \frac{N}{(1+N)} - CR = 0 \quad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)^3} \cdot N - CF = 0$

$CE \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \frac{(N-1)}{(1+N)^3} - FR = 0 \quad CE \cdot (N-1) \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1+N)^4} - FQ = 0 \quad \frac{1}{2} \cdot CE \cdot \frac{\left(1 + 2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4\right)}{(1+N)^3} - FL = 0$



$$GL := \frac{1}{2}\cdot\frac{CE\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{\left[N^3+3\cdot N^2+2N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}$$

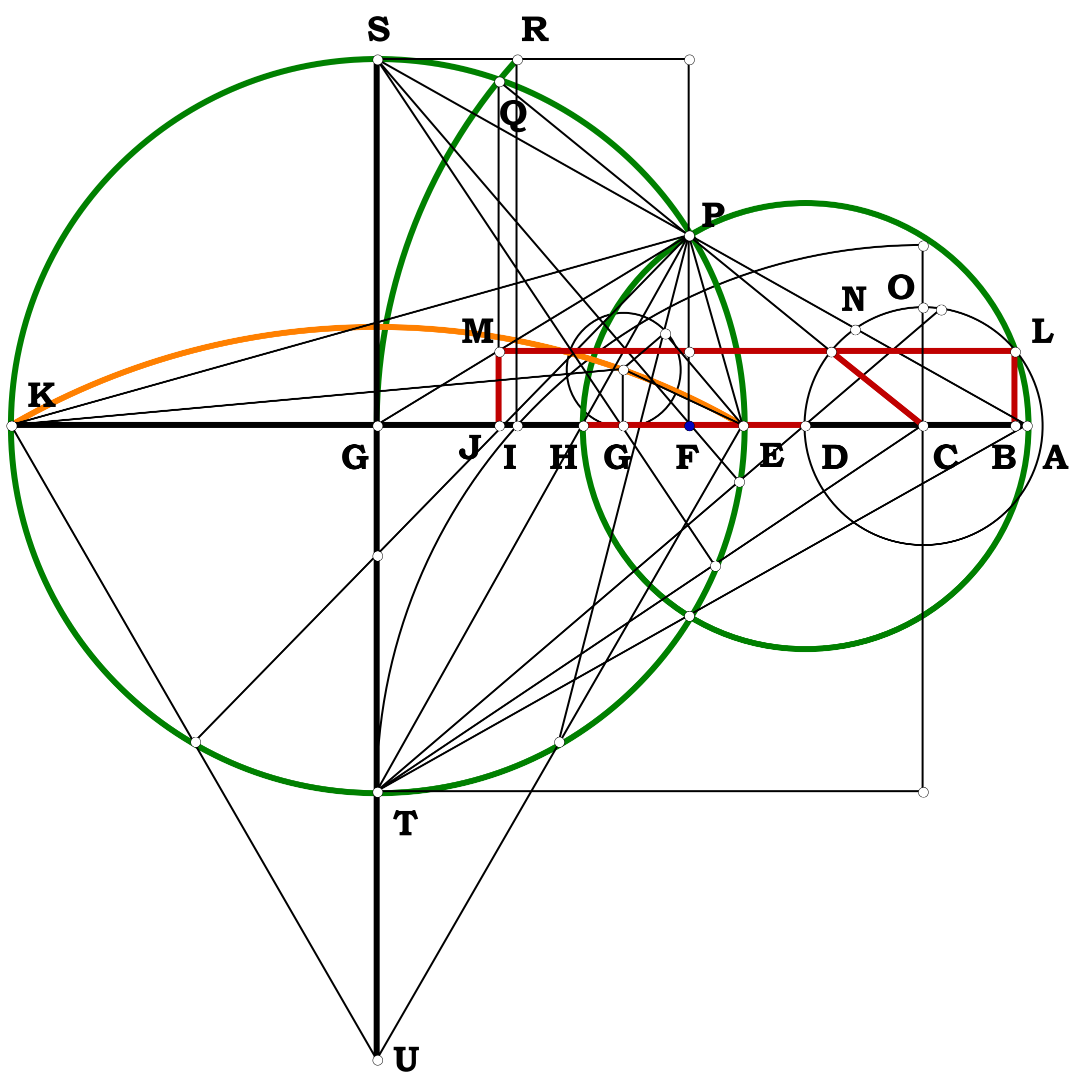
From Segment DF And HM 052201:

$$\frac{1}{2}\cdot\frac{CE\cdot\left(N^3+3\cdot N^2+3\cdot N-2N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{\left(N^2+4\cdot N+1\right)}-GL=0$$

Which does reduce to,

$$\frac{\frac{1}{2}\cdot\frac{CE\cdot\left(N^3+3\cdot N^2+3\cdot N-2N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{\left(N^2+4\cdot N+1\right)}}{\frac{1}{2}\cdot\frac{CE\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{\left[N^3+3\cdot N^2+2N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}=1 \qquad 1=1$$

0530011



What is CD and BL?

$$N_1 := 1 \quad N_2 := 5$$

$$CE := N_1 \quad EK := N_2 \quad EG := \frac{EK}{2} \quad CG := CE + EG$$

$$CQ := CG \quad PQ := \frac{EG^2}{CQ} \quad CP := CQ - PQ \quad GT := EG \quad CF := \frac{CP^2 + CG^2 - EG^2}{2 \cdot CG}$$

$$PF := \sqrt{CP^2 - CF^2} \quad GS := EG \quad FG := CG - CF \quad FS := \sqrt{FG^2 + GS^2}$$

$$DG := \frac{GS \cdot GT}{FG} \quad CD := CG - DG \quad BL := \frac{PF \cdot CD}{CP}$$

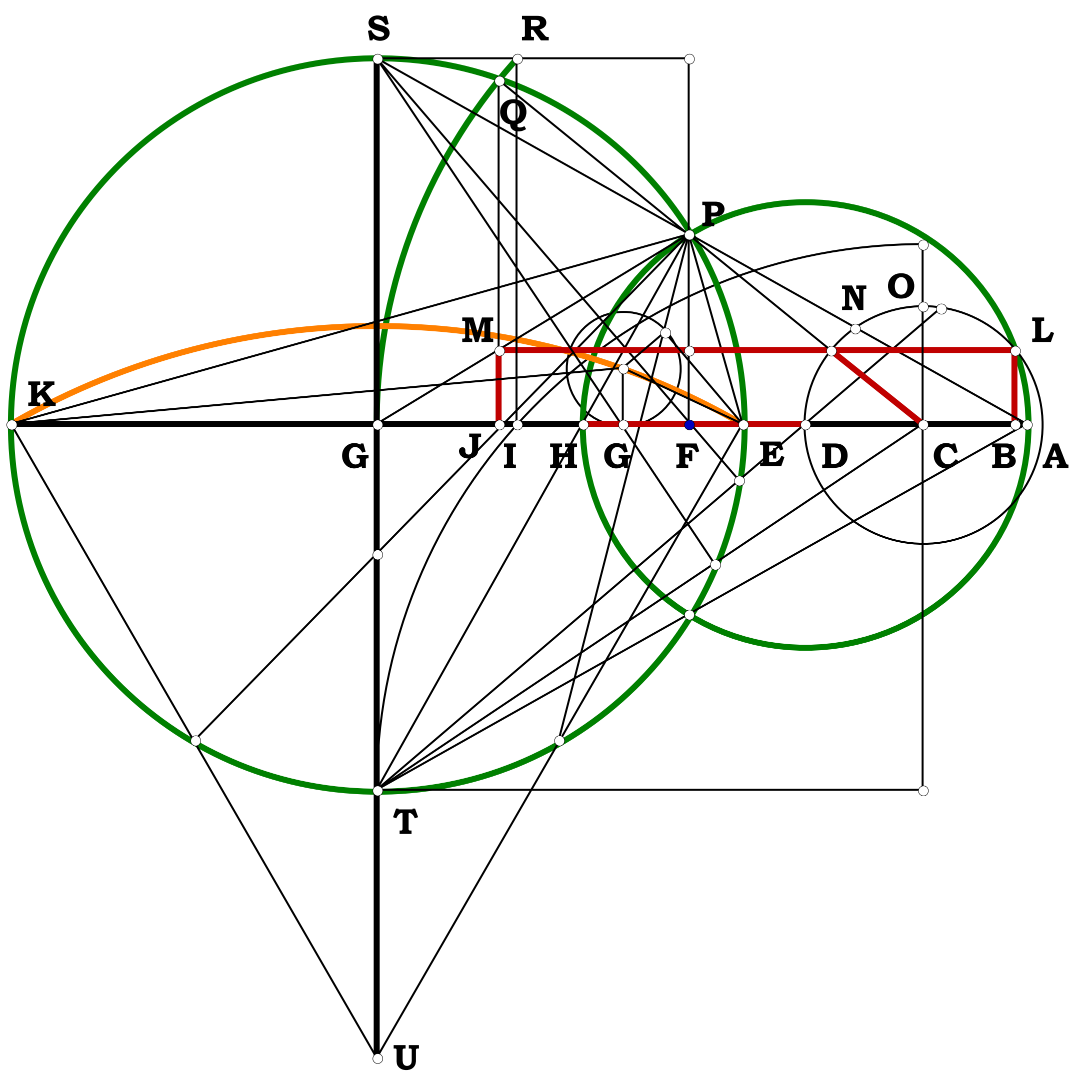
Some Algebraic Names;

$$CE := N_1 \quad EK := N_2 \quad EG - \frac{N_2}{2} = 0 \quad CG - \left(N_1 + \frac{N_2}{2}\right) = 0$$

$$PQ - \frac{1}{2} \cdot \frac{N_2^{\ 2}}{(2 \cdot N_1 + N_2)} = 0 \quad CP - 2 \cdot N_1 \cdot \frac{(N_1 + N_2)}{(2 \cdot N_1 + N_2)} = 0$$

$$CF := N_1 \cdot (N_1 + N_2) \cdot \frac{\left(8 \cdot N_1^{\ 2} + 8 \cdot N_1 \cdot N_2 + N_2^{\ 2}\right)}{(2 \cdot N_1 + N_2)^3}$$

$$PF - (N_1 + N_2) \cdot N_1 \cdot \sqrt{(N_2 + 4 \cdot N_1) \cdot (3 \cdot N_2 + 4 \cdot N_1)} \cdot \frac{N_2}{(2 \cdot N_1 + N_2)^3} = 0$$



$$FG - \frac{1}{2}\cdot N_2{}^2\cdot\frac{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3} = 0$$

$$FS - \sqrt{\left[\frac{1}{2}\cdot N_2{}^2\cdot\frac{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3}\right]^2 + \mathrm{OR}\left(\frac{N_2}{2}\right)^2} = 0$$

$$FS - \left(\frac{1}{2}\right)\cdot N_2\cdot\sqrt{2}\cdot\frac{\sqrt{\left(138\cdot N_2{}^2\cdot N_1{}^4+116\cdot N_2{}^3\cdot N_1{}^3+54\cdot N_2{}^4\cdot N_1{}^2+12\cdot N_2{}^5\cdot N_1\right)\ \ldots\ + N_2{}^6+32\cdot N_1{}^6+96\cdot N_1{}^5\cdot N_2}}{\left(2\cdot N_1+N_2\right)^3} = 0$$

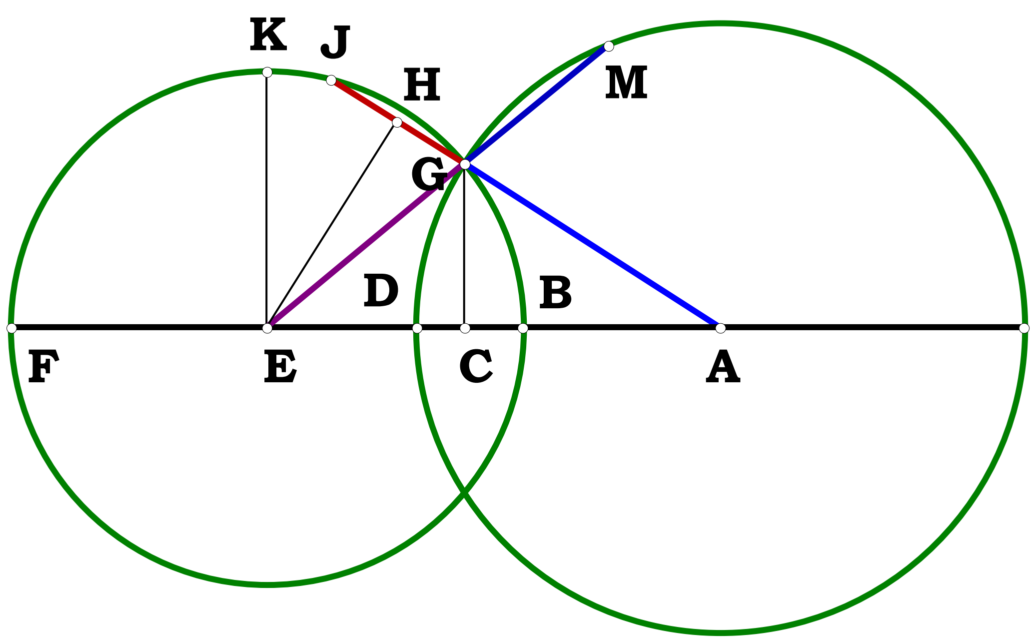
$$DG - \frac{\left(2\cdot N_1+N_2\right)^3}{\left[2\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)\right]} = 0$$

$$CD - N_1\cdot\left(N_1+N_2\right)\cdot\frac{\left(2\cdot N_1+N_2\right)}{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)} = 0$$

$$BL - \frac{\left(N_1+N_2\right)\cdot N_1\cdot\sqrt{\left(N_2+4\cdot N_1\right)\cdot\left(3\cdot N_2+4\cdot N_1\right)}\cdot N_2}{2\cdot\left(2\cdot N_1+N_2\right)\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)} = 0$$

A Small Extrapolation 060101

Given AE, AG, and EG, what is the Algebraic name of the segment GJ?



$S_1 := 6.00604 \qquad S_2 := 4.02167 \qquad S_3 := 3.38667$

$AE := S_1 \qquad AG := S_2 \qquad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2AE}$$

$$AH := \frac{AC \cdot AE}{AG} \qquad GH := AH - AG$$

$$HJ := GH \qquad GJ := GH + HJ$$

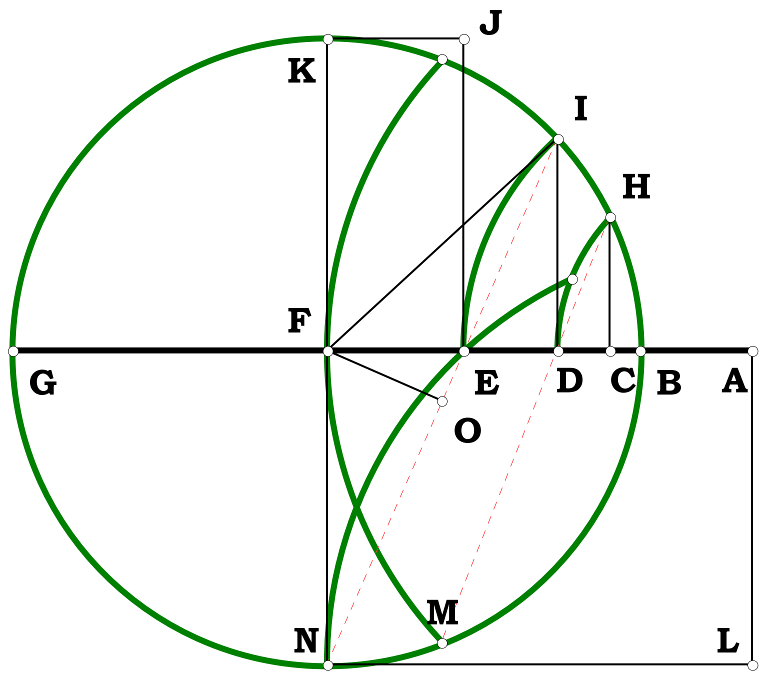
Some Algebraic Names:

$$\frac{S_2{}^2 + S_1{}^2 - S_3{}^2}{2S_1} - AC = 0 \qquad \frac{S_1{}^2 + S_2{}^2 - S_3{}^2}{2S_2} - AH = 0 \qquad \frac{S_1{}^2 - S_2{}^2 - S_3{}^2}{2S_2} - GH = 0$$

$$\frac{S_1{}^2 - S_2{}^2 - S_3{}^2}{S_2} - GJ = 0 \qquad \frac{S_1{}^2 - S_2{}^2 - S_3{}^2}{S_3} - \color{red}{GM} = \quad ?$$

Units From Both Sides 060201

Start with AB as unit and find. . . . then start with . .
. . as unit and find AB.



$$N := 5.727 \quad AB := .573 \quad AG := AB \cdot N$$

$$BG := AG - AB \quad BF := \frac{BG}{2} \quad AF := AB + BF$$

$$AE := \sqrt{AB \cdot AG} \quad BE := AE - AB$$

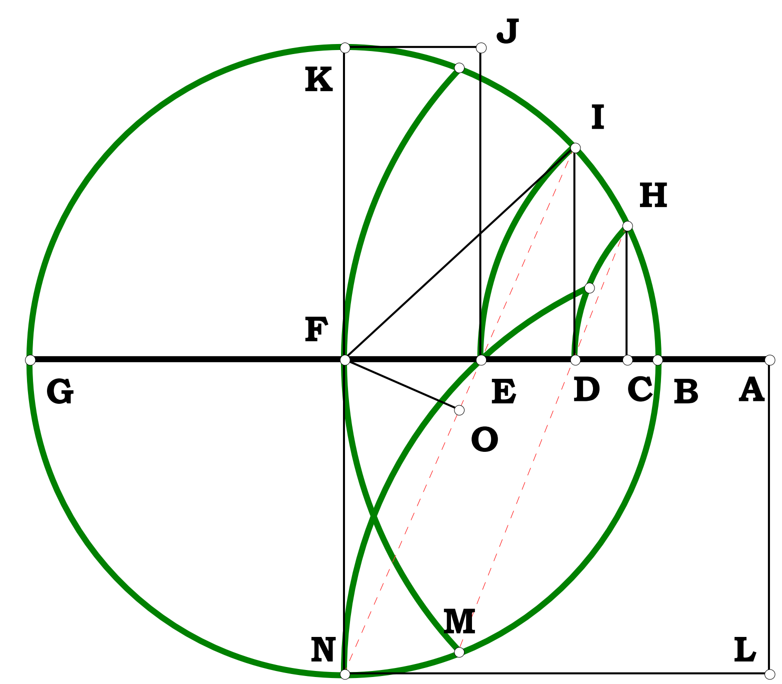
$$FN := BF \quad EF := BF - BE \quad EN := \sqrt{FN^2 + EF^2}$$

$$NO := \frac{FN^2}{EN} \quad NI := 2 \cdot NO \quad EI := NI - EN \quad \frac{1}{2} \cdot AB \cdot (N-1) - BF = 0$$

$$DE := \frac{EF \cdot EI}{EN} \quad AB \cdot (N-1) - BG = 0$$

$$AB\sqrt{N} - AE = 0 \quad AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$$

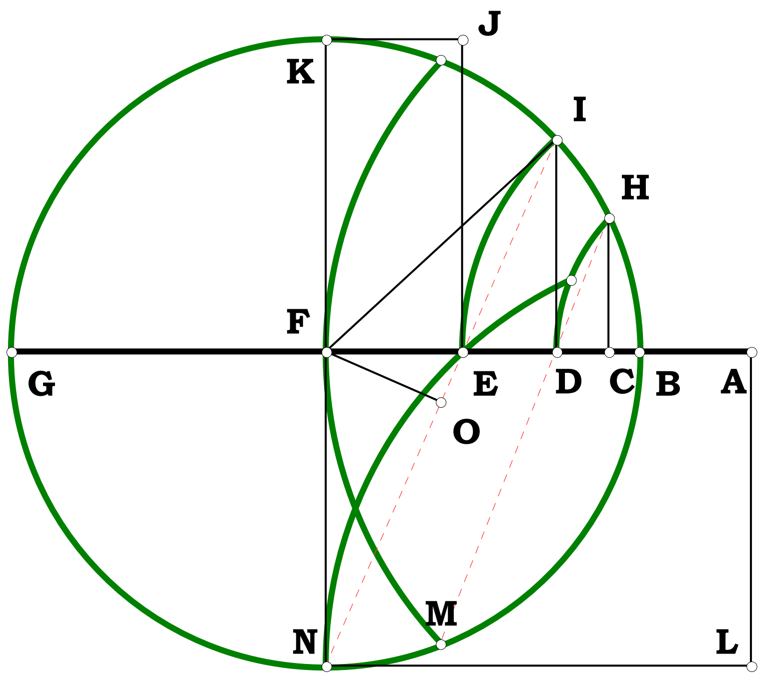
$$\frac{1}{2} \cdot AB \cdot (1 + N) - AF = 0$$



$$N_2 := 2 \quad BG_2 := 1 \quad BF_2 := \frac{BG_2}{2} \quad BE_2 := \frac{BF_2}{N_2} \quad EF_2 := BF_2 - BE_2$$

$$FN_2 := BF_2 \quad EN_2 := \sqrt{EF_2^{\,2} + FN_2^{\,2}} \quad NP_2 := \frac{EN_2}{2} \quad LN_2 := \frac{EN_2 \cdot NP_2}{EF_2}$$

$$AF_2 := LN_2 \quad AB_2 := AF_2 - BF_2$$



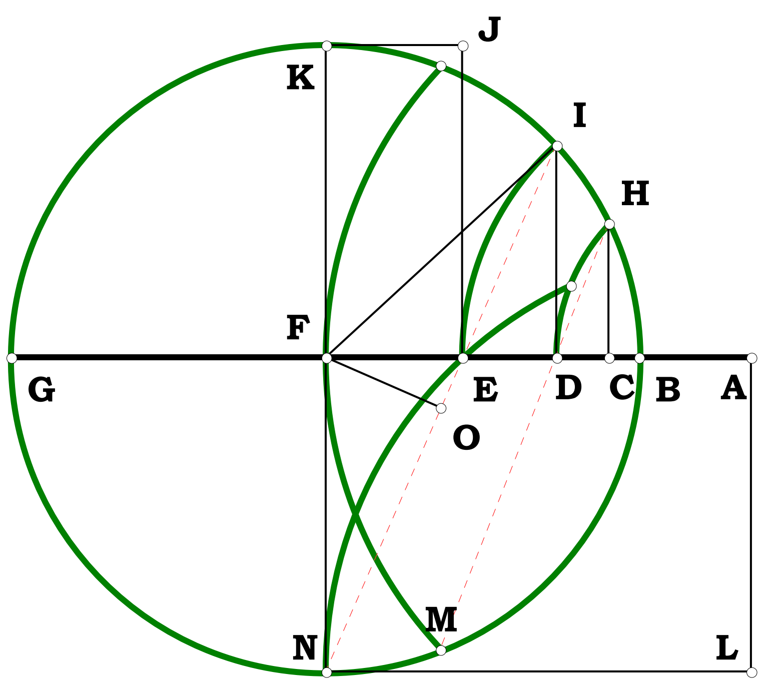
$$\frac{BG_2}{2} - BF_2 = 0 \qquad \frac{BG_2}{(2 \cdot N_2)} - BE_2 = 0 \qquad \frac{BG_2}{2} \cdot \frac{(N_2 - 1)}{N_2} - EF_2 = 0$$

$$\frac{BG_2}{2} \cdot \frac{\sqrt{2 \cdot N_2{}^2 - 2 \cdot N_2 + 1}}{N_2} - EN_2 = 0 \qquad \frac{BG_2}{4} \cdot \frac{\sqrt{2 \cdot N_2{}^2 - 2 \cdot N_2 + 1}}{N_2} - NP_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{\left(2 \cdot N_2{}^2 - 2 \cdot N_2 + 1\right)}{\left[N_2 \cdot \left(N_2 - 1\right)\right]} - LN_2 = 0 \qquad \frac{BG_2}{4 \cdot N_2 \cdot \left(N_2 - 1\right)} - AB_2 = 0$$

$$AI := AE \qquad FI := BF \qquad AD := \frac{AI^2 + AF^2 - FI^2}{2AF} \qquad BD := AD - AB$$

$$AD - 2 \cdot AB \cdot \frac{N}{(N+1)} = 0 \qquad BD - AB \cdot \frac{(N-1)}{(1+N)} = 0$$



$$N_3 := 3.364 \quad BG_3 := 2.708 \quad BF_3 := \frac{BG_3}{2} \quad BD_3 := \frac{BF_3}{N_3} \quad DG_3 := BG_3 - BD_3$$

$$DI_3 := \sqrt{BD_3 \cdot DG_3} \quad DF_3 := BF_3 - BD_3 \quad FN_3 := BF_3 \quad EF_3 := \frac{DF_3 \cdot FN_3}{FN_3 + DI_3}$$

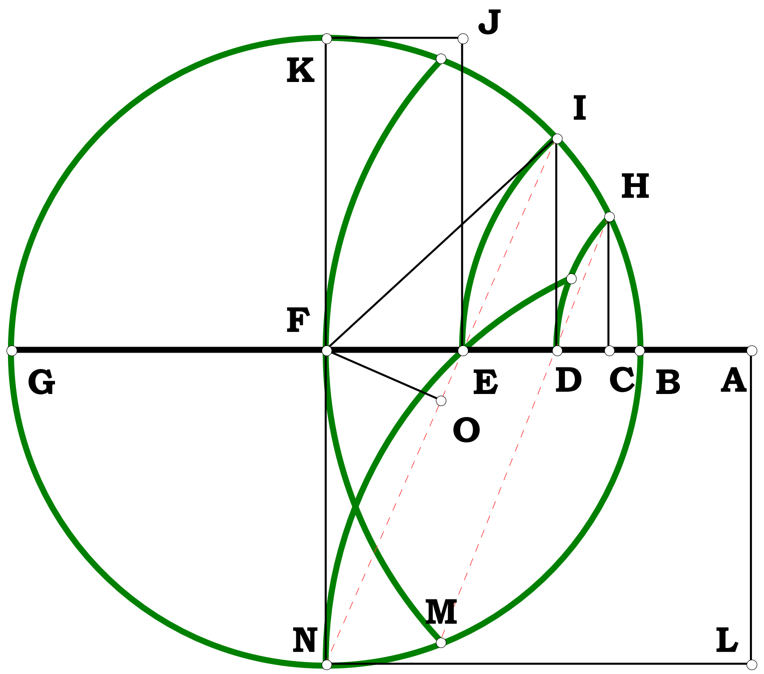
$$EN_3 := \sqrt{EF_3^{\,2} + FN_3^{\,2}} \quad NP_3 := \frac{EN_3}{2} \quad LN_3 := \frac{EN_3 \cdot NP_3}{EF_3} \quad AF_3 := LN_3$$

$$AB_3 := AF_3 - BF_3$$

$$BD_3 - \frac{1}{2} \cdot \frac{BG_3}{N_3} = 0 \quad DG_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{(2 \cdot N_3 - 1)}{N_3} = 0 \quad DI_3 - \frac{1}{(2 \cdot N_3)} \cdot BG_3 \cdot \sqrt{2 \cdot N_3 - 1} = 0 \quad DF_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{N_3} = 0$$

$$EF_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{\left(N_3 + \sqrt{2 \cdot N_3 - 1}\right)} = 0 \quad NP_3 - \frac{1}{4} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{\left(N_3 + \sqrt{2 \cdot N_3 - 1}\right)}} = 0 \quad EN_3 - \frac{1}{2} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{\left(N_3 + \sqrt{2 \cdot N_3 - 1}\right)}} = 0$$

$$LN_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{N_3}{(N_3 - 1)} = 0 \quad AB_3 - \frac{1}{2} \cdot \frac{BG_3}{(N_3 - 1)} = 0$$



$$AH := AD$$

$$FH := BF \qquad AC := \frac{AH^2 + AF^2 - FH^2}{2 \cdot AF}$$

$$BC := AC - AB$$

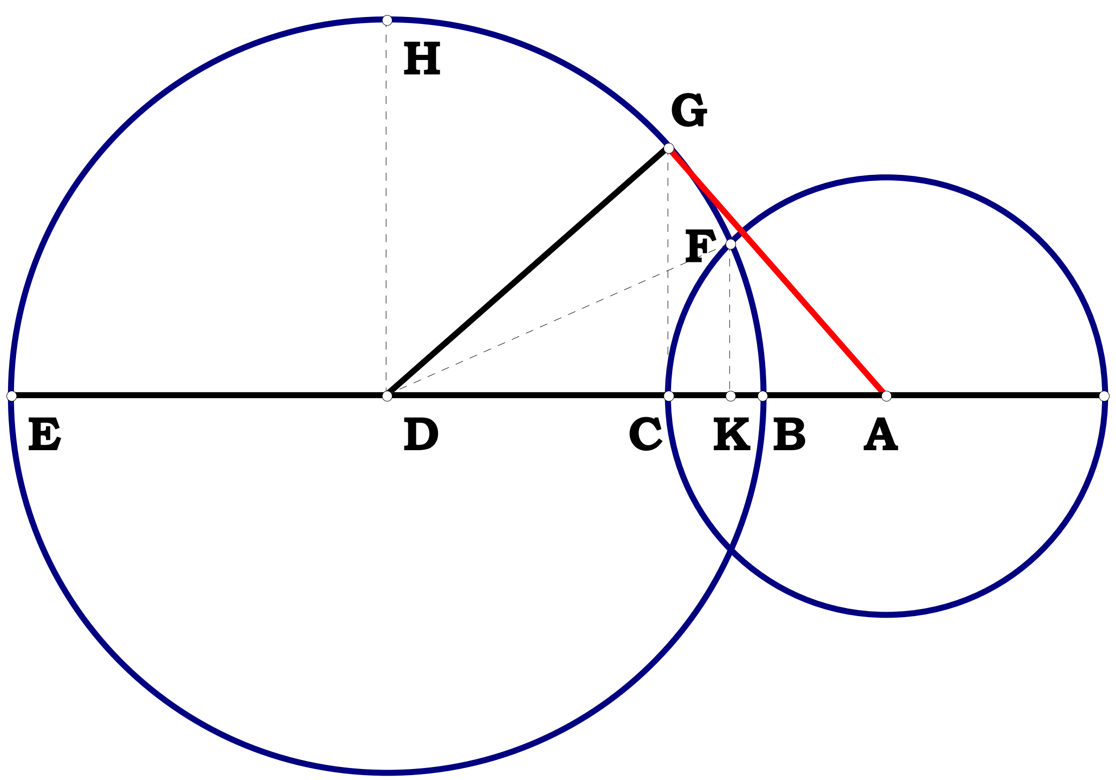
$$AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$$

$$AC - AB \cdot N \cdot \frac{\left(6 \cdot N + 1 + N^2\right)}{(N + 1)^3} = 0$$

$$BC - AB \cdot (3 \cdot N + 1) \cdot \frac{(N - 1)}{(N + 1)^3} = 0$$

Isolating A Problem 060301

If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?



$$N := 4 \qquad BE := 1$$

$$BD := \frac{BE}{2} \qquad BC := \frac{BE}{N} \qquad CE := BE - BC$$

$$CG := \sqrt{BC \cdot CE} \qquad CD := BD - BC \qquad AC := \frac{CG^2}{CD}$$

$$AF := AC \qquad AD := AC + CD \qquad DF := BD$$

$$DK := \frac{DF^2 + AD^2 - AF^2}{2AD} \qquad BK := BD - DK$$

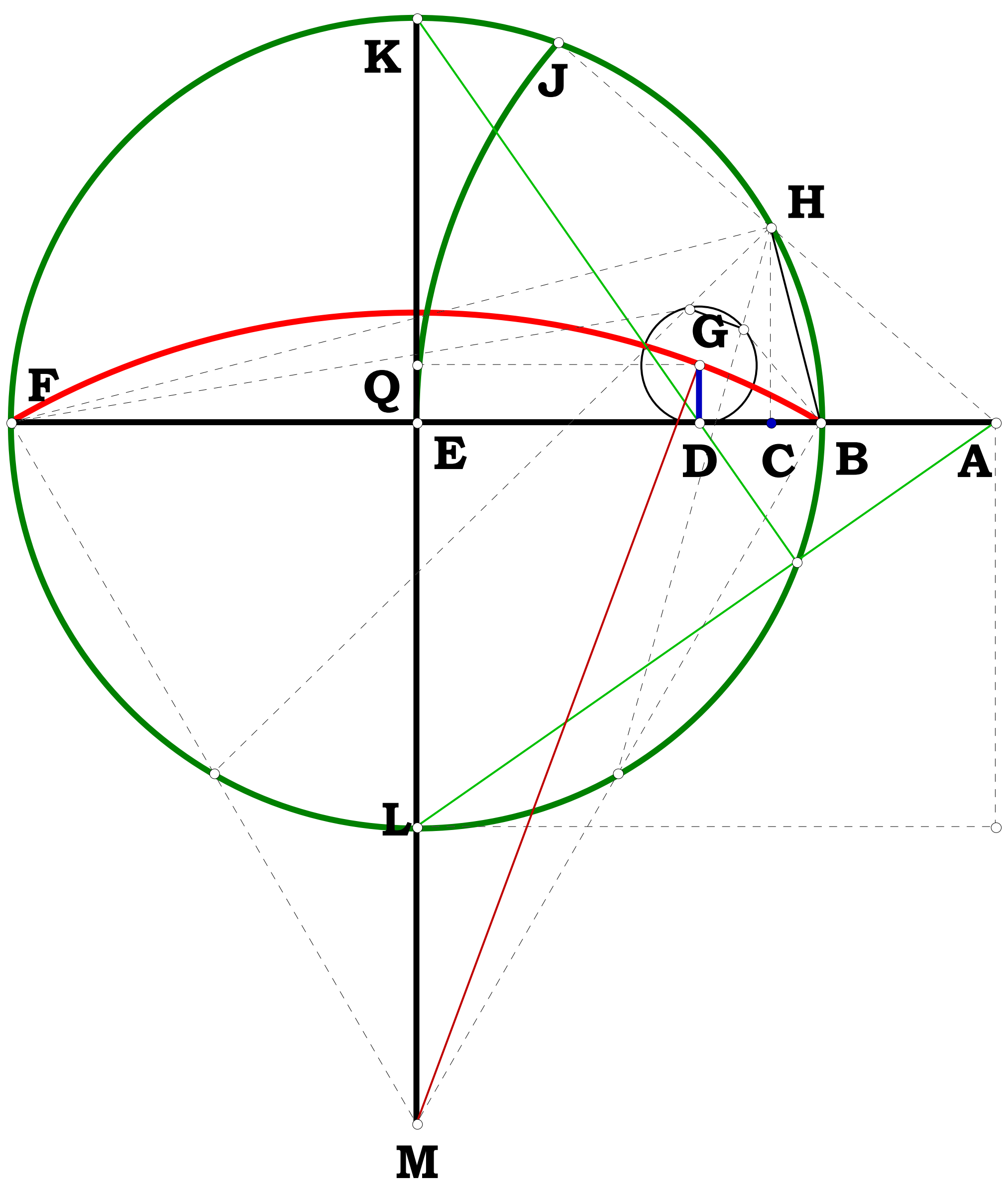
$$CK := BC - BK$$

$$BE \cdot \frac{(N-1)}{N} - CE = 0 \qquad BE \cdot \frac{\sqrt{(N-1)}}{N} - CG = 0 \qquad BE \cdot \frac{(N-2)}{2 \cdot N} - CD = 0$$

$$BE \cdot \frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0 \qquad BE \cdot \frac{N}{2 \cdot (N-2)} - AD = 0 \qquad BE \cdot \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$$

$$BE \cdot \frac{(3 \cdot N - 2)}{N^3} - BK = 0 \qquad BE \cdot \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0 \qquad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$$

For Any AB 061001



For any AB, AF what is DG?

$N := 4.39 \quad AB := .615 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad EK := BE$

$AE := AB + BE \quad DE := \frac{EK^2}{AE} \quad EF := BE$

$FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM \quad DG := EQ$

Some Algebraic Names:

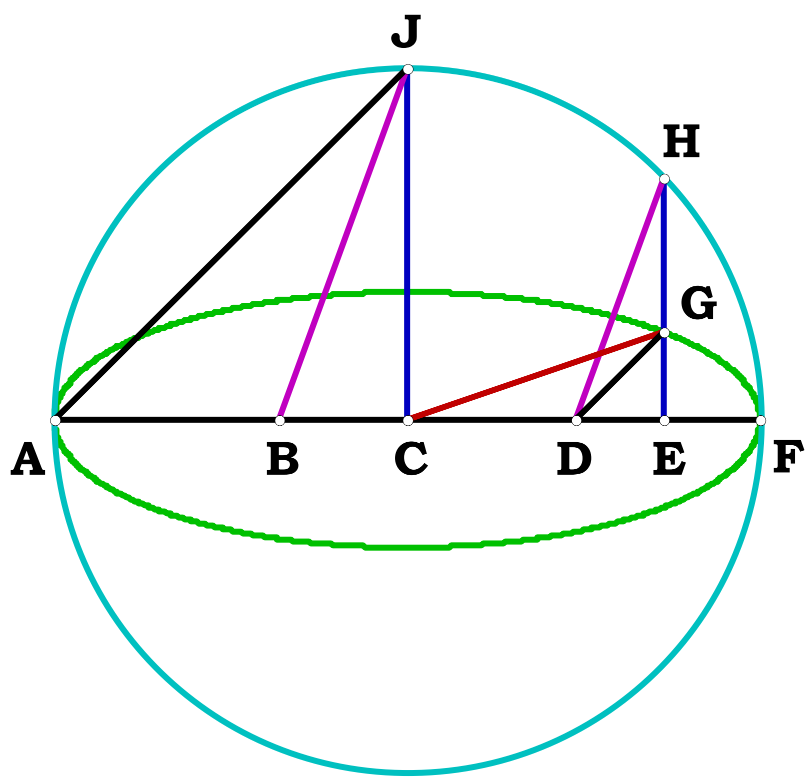
$AB \cdot (N-1) - BF = 0 \qquad \frac{AB \cdot (N-1)}{2} - BE = 0 \qquad \frac{1}{2} \cdot AB \cdot (N+1) - AE = 0$

$\frac{1}{2} \cdot AB \cdot \frac{(N-1)^2}{(N+1)} - DE = 0 \qquad \frac{1}{2} \cdot \sqrt{3} \cdot AB \cdot (N-1) - EM = 0$

$$\frac{AB \cdot \left[\sqrt{(N+3) \cdot (3 \cdot N+1)} \cdot (N-1)\right]}{2 \cdot (N+1)} - MQ = 0$$

$$\frac{AB \cdot (N-1) \cdot \left[\sqrt{(N+3) \cdot (3 \cdot N+1)} - \sqrt{3} - \sqrt{3} \cdot N\right]}{2 \cdot (N+1)} - DG = 0$$

Elipse By Parallels 082601



$N_1 := 4 \quad N_2 := 1$

$AF := 1 \quad AC := \dfrac{AF}{2} \quad CJ := AC$

$BC := \dfrac{AC}{N_1} \quad AE := \dfrac{AF}{N_2}$

$EF := AF - AE \quad EH := \sqrt{AE \cdot EF} \quad EG := \dfrac{BC \cdot EH}{CJ}$

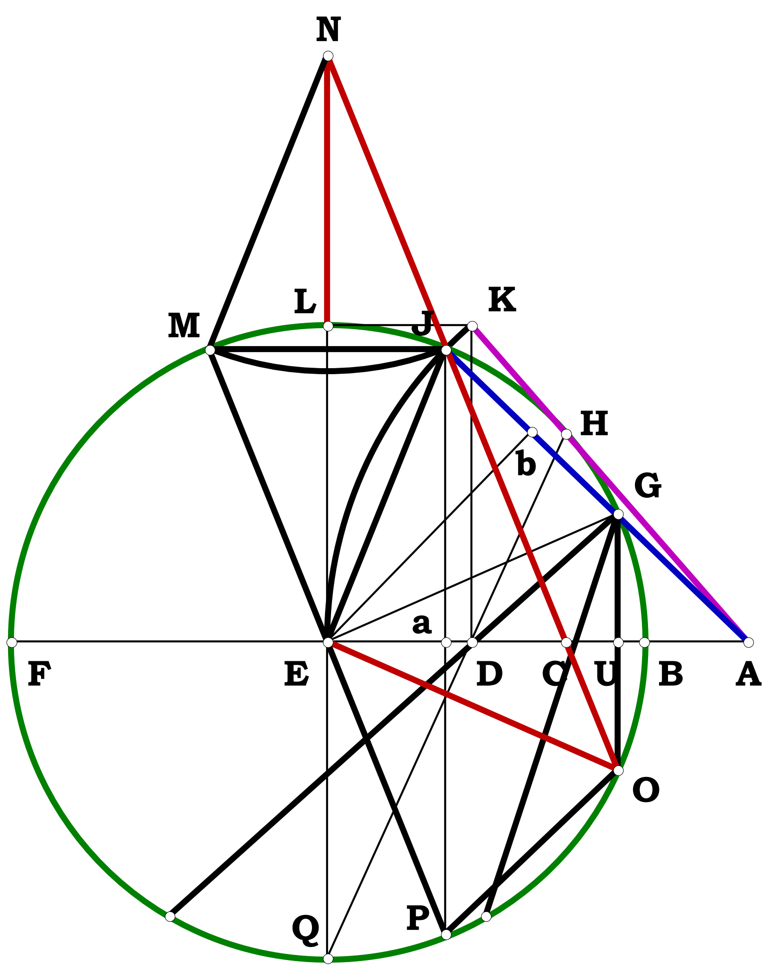
$CE := AE - AC \quad CG := \sqrt{CE^2 + EG^2}$

Some Algebraic Names:

$BC - \dfrac{1}{(2 \cdot N_1)} = 0 \quad AE - \dfrac{1}{N_2} = 0 \quad EF - \left(1 - \dfrac{1}{N_2}\right) = 0 \quad EH - \dfrac{\sqrt{N_2 - 1}}{N_2} = 0 \quad EG - \dfrac{\sqrt{N_2 - 1}}{N_1 \cdot N_2} = 0$

$CE - \left(\dfrac{1}{N_2} - \dfrac{1}{2}\right) = 0 \qquad CG - \dfrac{1}{2} \cdot \dfrac{\sqrt{4 \cdot N_1^2 - 4 \cdot N_1^2 \cdot N_2 + N_1^2 \cdot N_2^2 + 4 \cdot N_2 - 4}}{(N_1 \cdot N_2)} = 0$

Just Another Proof Of Paper 010202



$$N := 7 \quad AB := 1 \quad AF := AB \cdot N$$

$$BF := AF - AB \quad BE := \frac{BF}{2} \quad AE := BE + AB$$

$$AJ := AE \quad EJ := BE \quad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$$

$$Gb := Ea \quad GJ := 2 \cdot Gb \quad AG := AJ - GJ$$

$$Aa := AE - Ea \quad AU := \frac{Aa \cdot AG}{AJ}$$

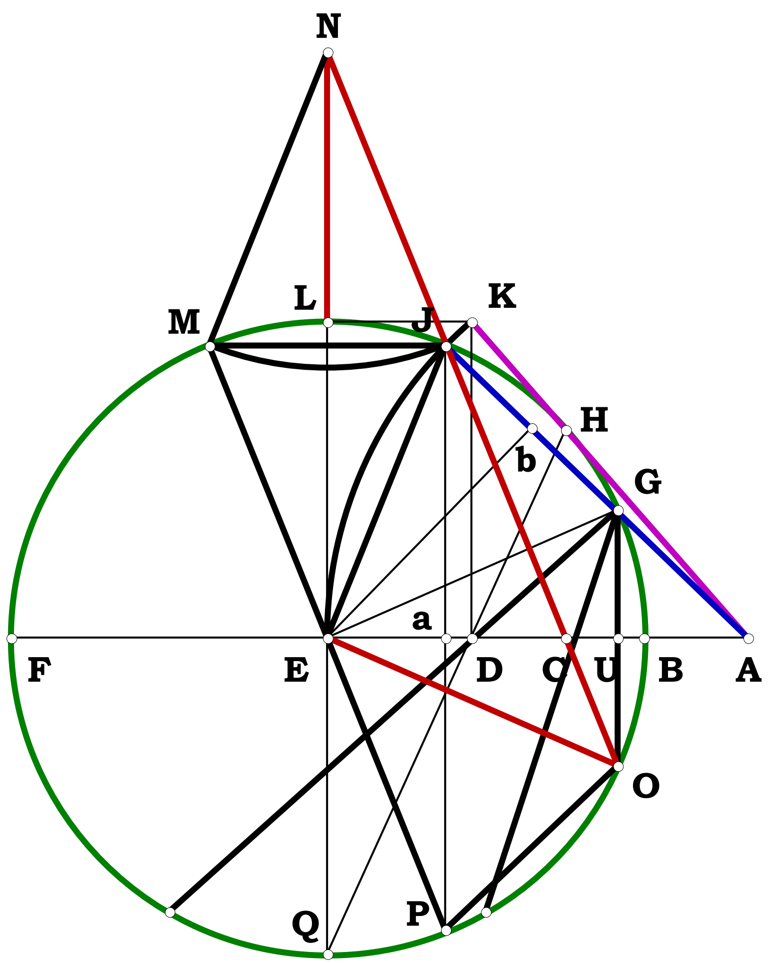
$$Ja := \sqrt{AJ^2 - Aa^2} \quad GU := \frac{Ja \cdot AG}{AJ}$$

$$Ua := Aa - AU \quad JO := \sqrt{Ua^2 + (GU + Ja)^2}$$

$$JN := \frac{JO \cdot Ea}{Ua} \quad JN - BE = 0$$

From 4/29/94 $\quad OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$$OP - 2 \cdot Ea = 0 \quad NO := JO + JN \quad EU := Ua + Ea \quad EN := \sqrt{NO^2 - EU^2} \quad EL := BE \quad LN := EN - EL$$



Some Algebraic Names;

$$N-1-BF=0 \qquad \frac{N-1}{2}-BE=0 \qquad \frac{N+1}{2}-AE=0 \qquad \frac{(N-1)^2}{4\cdot(N+1)}-Ea=0$$

$$\frac{(N-1)^2}{2\cdot(N+1)}-GJ=0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)}-Aa=0 \qquad \frac{2\cdot N}{N+1}-AG=0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3}-AU=0$$

$$\frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)}-Ja=0 \qquad \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}-GU=0$$

$$\frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3}-Ua=0 \qquad \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2}-JO=0 \qquad \frac{N-1}{2}-JN=0$$

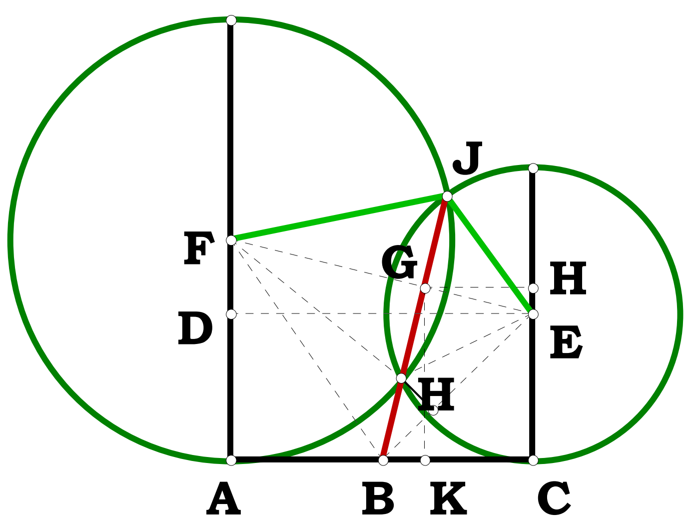
$$\frac{(N-1)^2}{2\cdot(N+1)}-OP=0 \qquad \frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2}-NO=0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3}-EU=0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3}-EN=0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3}-\frac{N-1}{2}-LN=0$$

Easy Power-Line 06_20_02

For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.



$AC := 5 \quad AF := 4$

$CE := 3 \quad FH := AF \quad EH := CE$

$AD := CE \quad DF := AF - AD$

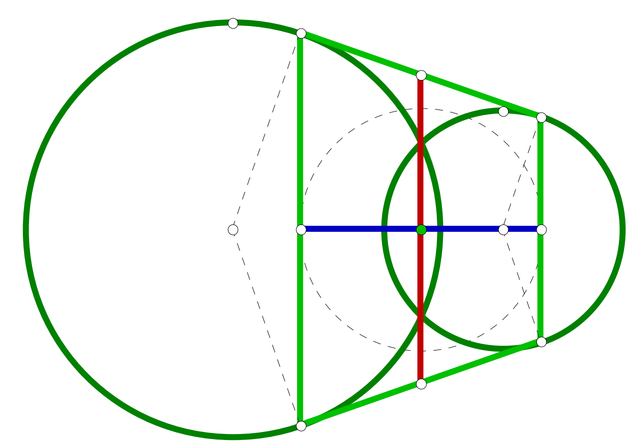
$EJ := CE \quad FJ := AF$

$DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

$$EG := \frac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF}$$

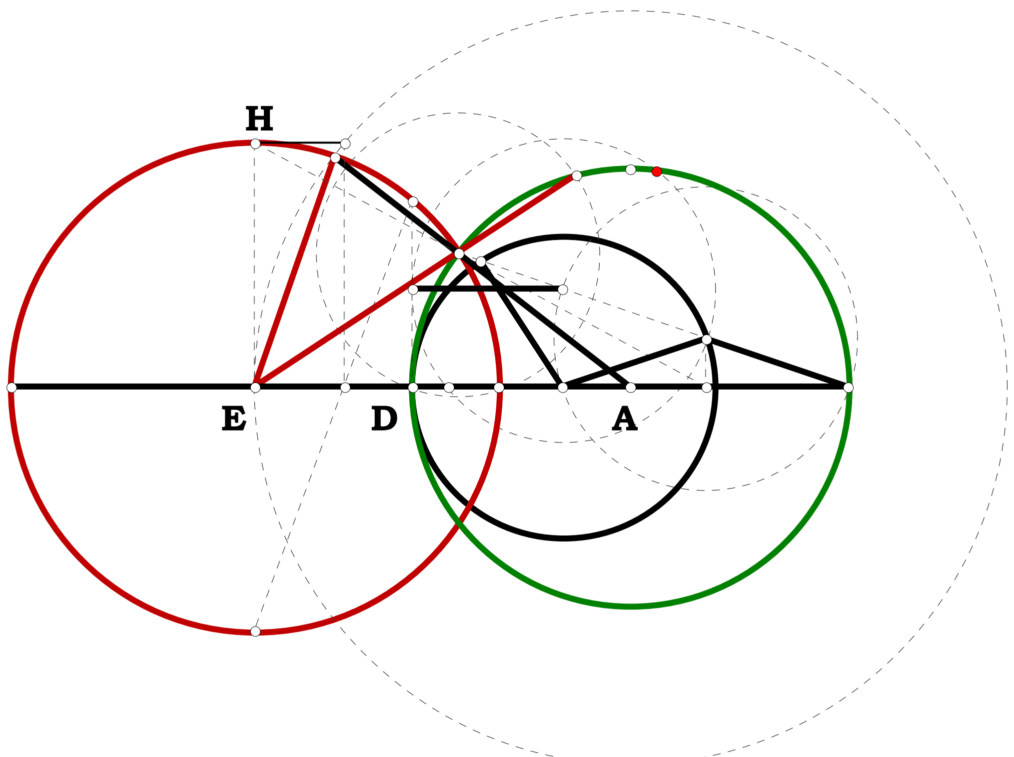
$$EH := \frac{DF \cdot EG}{EF} \quad CH := CE + EH \quad GH := \frac{DE \cdot EG}{EF} \quad CK := GH \quad KG := CH$$

$$BK := \frac{DF \cdot KG}{DE} \quad BC := CK + BK \quad BC - \frac{AC}{2} = 0$$



021603

From AD project a trisection to EH.



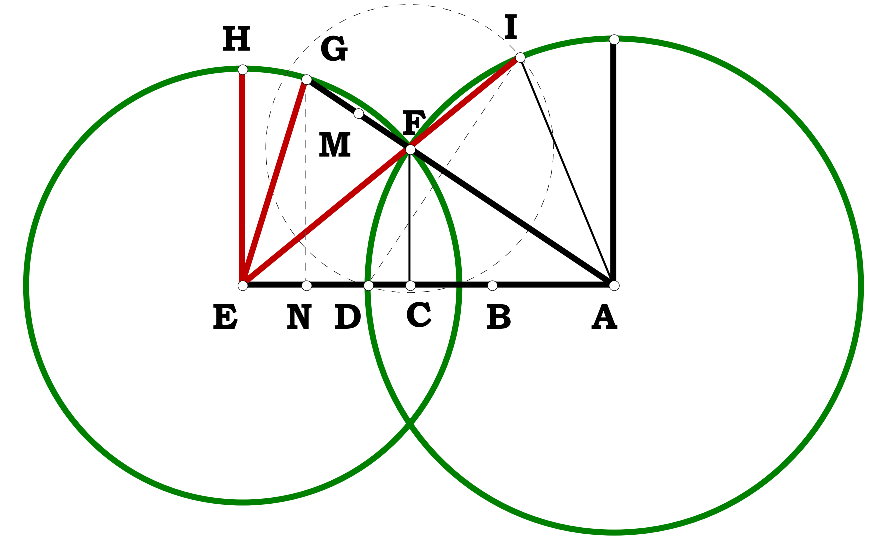
$AD := .969 \quad AB := \dfrac{AD}{2} \quad N := 1.51$

$BD := AD - AB \quad BC := \dfrac{BD}{N}$

$CD := BD - BC \quad AC := AB + BC$

$AF := AD \quad CF := \sqrt{AF^2 - AC^2}$

$FI := \sqrt{CD^2 + CF^2}$

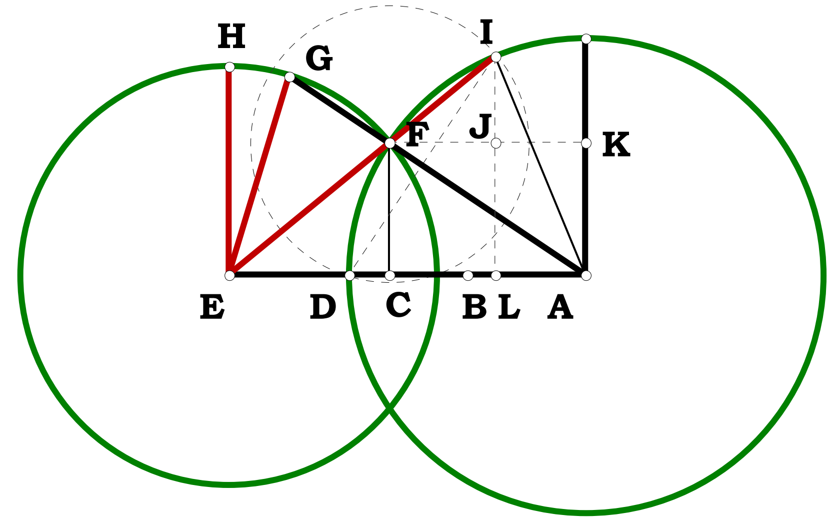


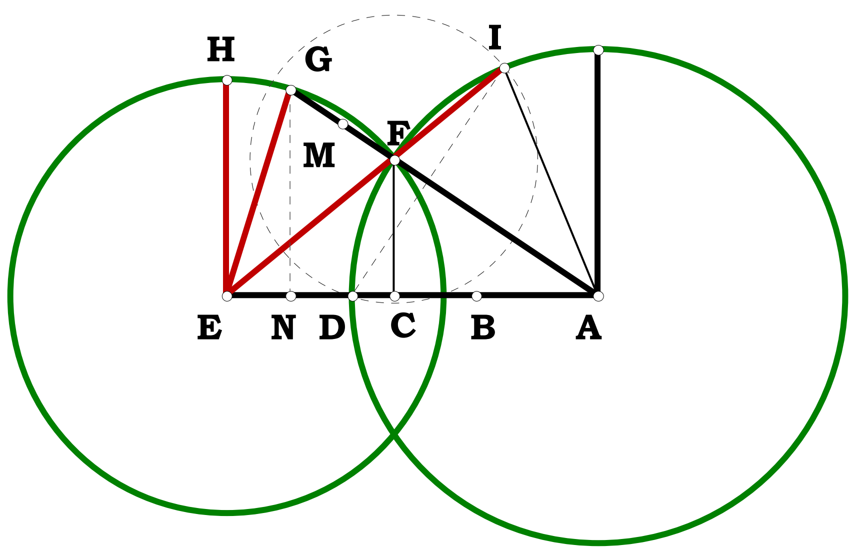
$DI := 2 \cdot CF \quad AI := AD \quad AL := \dfrac{\left(2AD^2 - DI^2\right)}{2 \cdot AD}$

$FJ := AC - AL \quad IL := \sqrt{AI^2 - AL^2} \quad IJ := IL - CF$

$EL := \dfrac{FJ \cdot IL}{IJ} \quad AE := AL + EL \quad EI := \dfrac{FI \cdot IL}{IJ}$

$EF := EI - FI$





$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF} \qquad AG := AF + FG \qquad AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \qquad FM := \frac{FG}{2} \qquad FM - EN = 0$$

Some Algebraic Names;

$$BC - \frac{AD - AB}{N} = CD - (N-1)\cdot\frac{(AD - AB)}{N} AC - \frac{(AB\cdot N + AD - AB)}{N} = 0$$

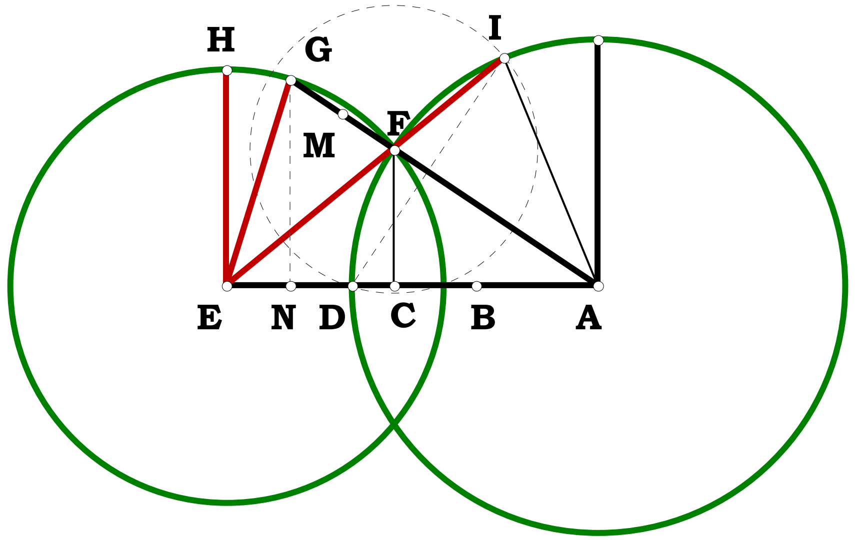
$$CF - \frac{\sqrt{(N-1)\cdot(AD-AB)\cdot(AD\cdot N + AD - AB + AB\cdot N)}}{N} = 0 \qquad FI - \sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}} = 0$$

$$DI - 2\cdot\frac{\sqrt{(N-1)\cdot(AD-AB)\cdot(AD\cdot N + AD - AB + AB\cdot N)}}{N} = 0$$

$$AL - \frac{-\left(AD^2\cdot N^2 - 4\cdot AD\cdot N\cdot AB + 4\cdot AB^2\cdot N - 2\cdot N^2\cdot AB^2 - 2\cdot AD^2 + 4\cdot AD\cdot AB - 2\cdot AB^2\right)}{\left(AD\cdot N^2\right)} = 0$$

$$FJ - (N-1)\cdot(AD-AB)\cdot\frac{(AD\cdot N + 2\cdot AD - 2\cdot AB + 2\cdot AB\cdot N)}{\left(AD\cdot N^2\right)} = 0$$

$$IL - \frac{(AB\cdot N + AD - AB)}{\left(AD\cdot N^2\right)}\cdot\sqrt{4}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot(AD\cdot N + AD - AB + AB\cdot N)} = 0$$



$$IJ - -\sqrt{(N-1)\cdot(AD-AB)\cdot(AD\cdot N+AD-AB+AB\cdot N)}\cdot\frac{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}{\left(AD\cdot N^{2}\right)} = 0$$

$$EL - \frac{-2\cdot(N-1)\cdot(AD-AB)\cdot(AD\cdot N+2\cdot AD-2\cdot AB+2\cdot AB\cdot N)\cdot(AB\cdot N+AD-AB)}{AD\cdot\left[N^{2}\cdot(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)\right]} = 0$$

$$AE - -AD^{2}\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$EI - 2\cdot\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot\frac{(AB\cdot N+AD-AB)}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)} = 0$$

$$EF - -\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot AD\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$FG - -2\cdot AD\cdot(AD-AB)\cdot\frac{(N-1)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$AG - -AD^{2}\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

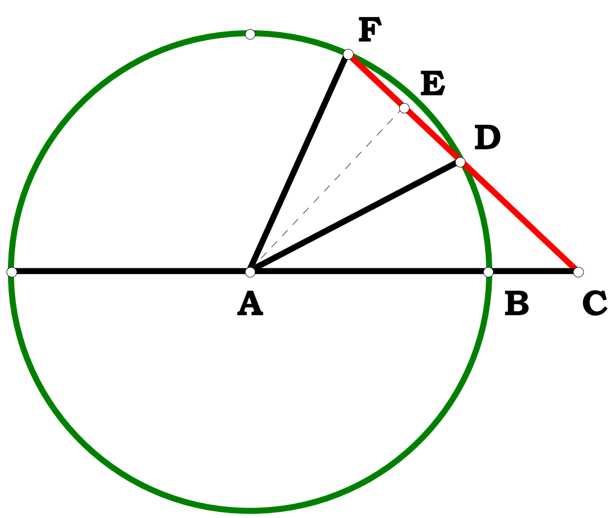
$$AN - (AB\cdot N+AD-AB)\cdot\frac{AD}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)} = 0$$

$$EN - -AD\cdot(N-1)\cdot\frac{(AD-AB)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$FM - EN = 0$$

Given AC, AB and either point of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C?

032304



$N_1 := 1.708 \quad N_2 := 1.649$

$N_3 := 1.24$

$AC := N_1 \quad CF := N_2$

$AF := N_3$

$$DF_1 := \frac{N_3^{\,2} + N_2^{\,2} - N_1^{\,2}}{N_2}$$

$CD := CF - D\,N_2 := CD$

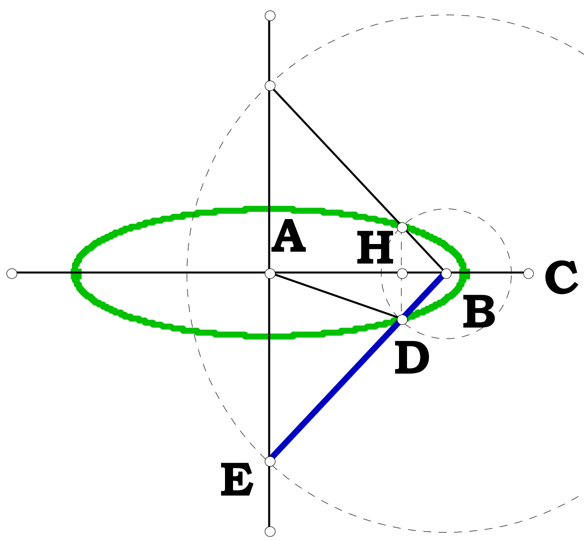
$$DF_2 := \frac{N_3^{\,2} + N_2^{\,2} - N_1^{\,2}}{N_2}$$

$DF_1 = 0.812333 \quad DF_2 = -0.812333 \quad DF_1 + DF_2 = 0$

041904

Straight Line Ellipse

Cardinal



$U := 1 \quad N_1 := 3 \quad N_2 := 2$

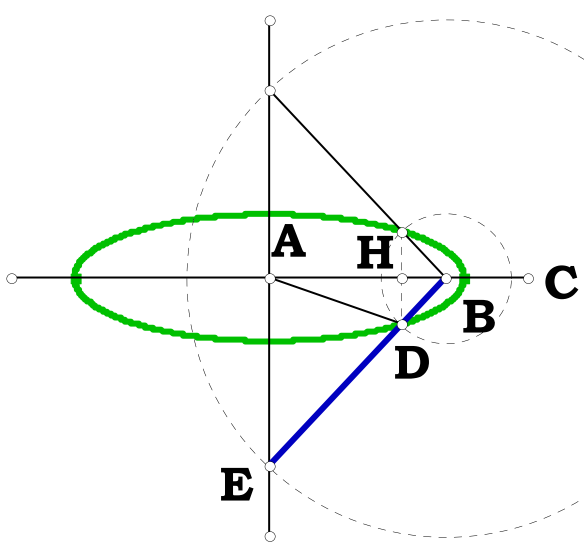
$AC := U \quad BE := AC \quad BD := \dfrac{BE}{N_1}$

$AB := \dfrac{AC}{N_2} \quad AE := \sqrt{BE^2 - AB^2} \quad BH := \dfrac{AB \cdot BD}{BE} \quad DH := \dfrac{AE \cdot BD}{BE}$

$AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$

$$\frac{U}{(N_1 \cdot N_2)} \cdot \sqrt{(N_1)^2 - 2 \cdot N_1 + (N_2)^2} - AD = 0 \qquad \frac{\sqrt{(N_1)^2 - 2 \cdot N_1 + (N_2)^2}}{N_1 \cdot N_2} - \frac{AD}{U} = 0$$

Ordinal



$N_1 := 1.344 \quad N_2 := .3 \quad N_3 := .5$

$AC := N_1 \quad BD := N_2 \quad AB := N_3$

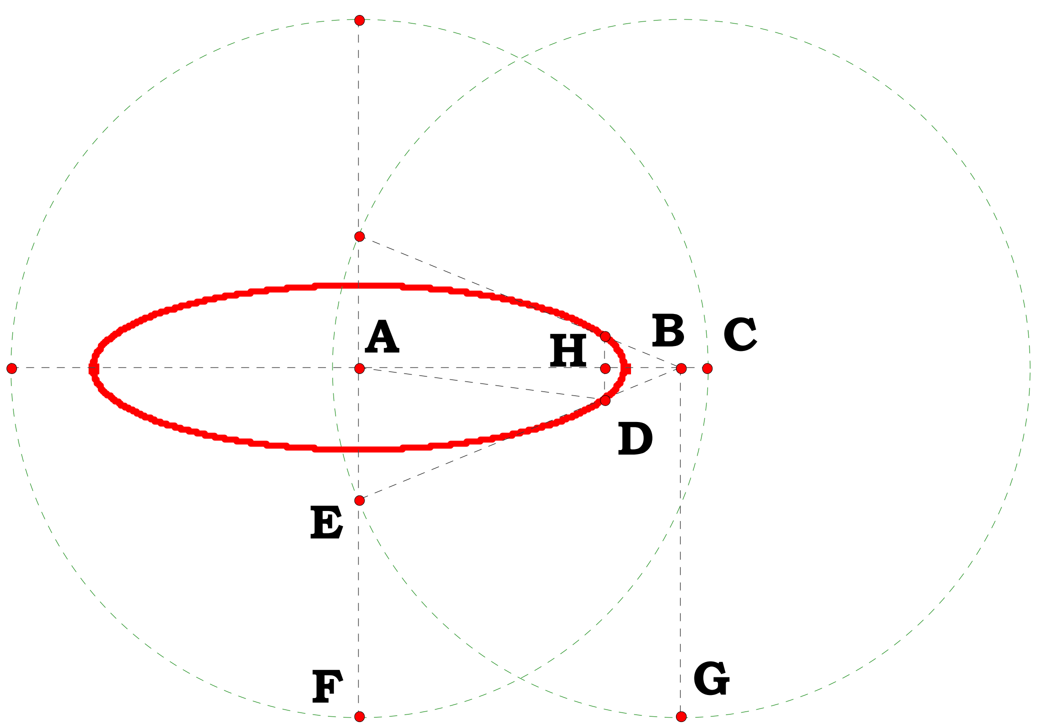
$AF := AC \quad BE := AC \quad AE := \sqrt{BE^2 - AB^2}$

$BH := \dfrac{AB \cdot BD}{BE} \quad DH := \dfrac{AE \cdot BD}{BE} \quad AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$

$$\sqrt{\frac{1}{N_1} \cdot \left[(N_3)^2 \cdot N_1 - 2 \cdot (N_3)^2 \cdot N_2 + (N_2)^2 \cdot N_1\right]} - AD = 0$$

Straight Line Ellipse

041904B



$$N_1 := 1.344 \quad N_2 := .415 \quad N_3 := 1.102$$

$$AC := N_1 \quad BD := N_2 \quad AB := N_3 \quad AF := AC$$

$$BE := AC \quad AE := \sqrt{BE^2 - AB^2}$$

$$BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE}$$

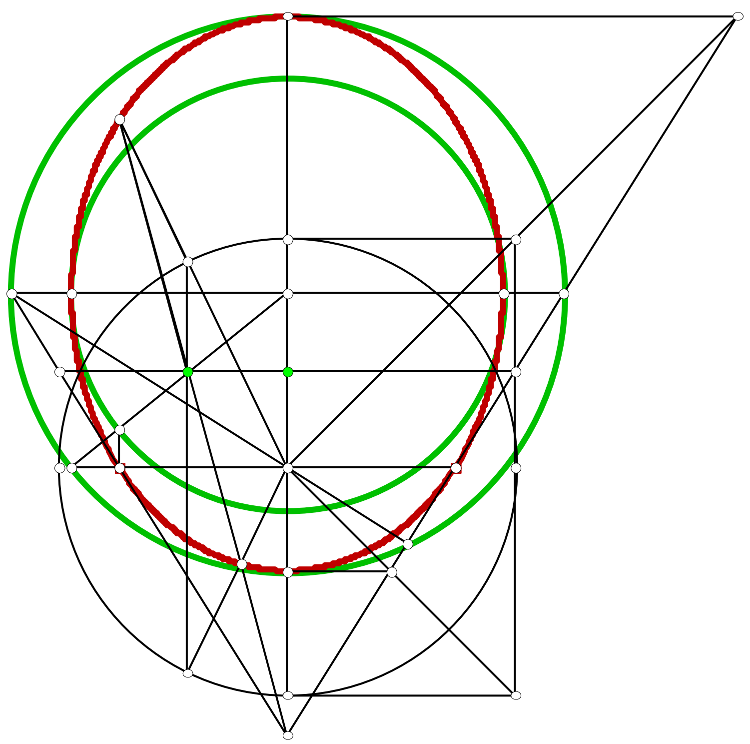
$$AH := AB - BH$$

$$AD := \sqrt{AH^2 + DH^2}$$

$$AD := \sqrt{\frac{1}{N_1} \cdot \left[\left(N_3\right)^2 \cdot N_1 - 2 \cdot \left(N_3\right)^2 \cdot N_2 + \left(N_2\right)^2 \cdot N_1 \right]}$$

Another Ellipse 031405a

The locus formed by N and I as determined by L provides an ellipse.
Privide an Algebraic name for the Major and Minor Axis.



$N_1 := 1.25 \quad N_2 := 3$

$AC := N_1 \quad AD := N_2$

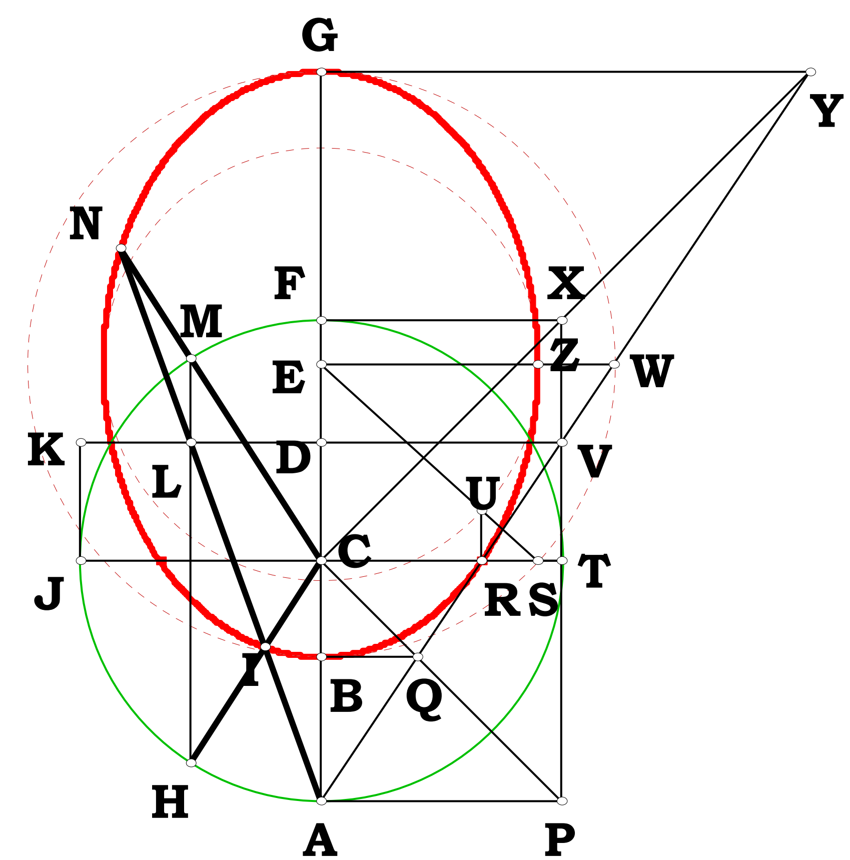
$DV := AC \quad AV := \sqrt{AD^2 + DV^2} \quad AF := 2 \cdot AC$

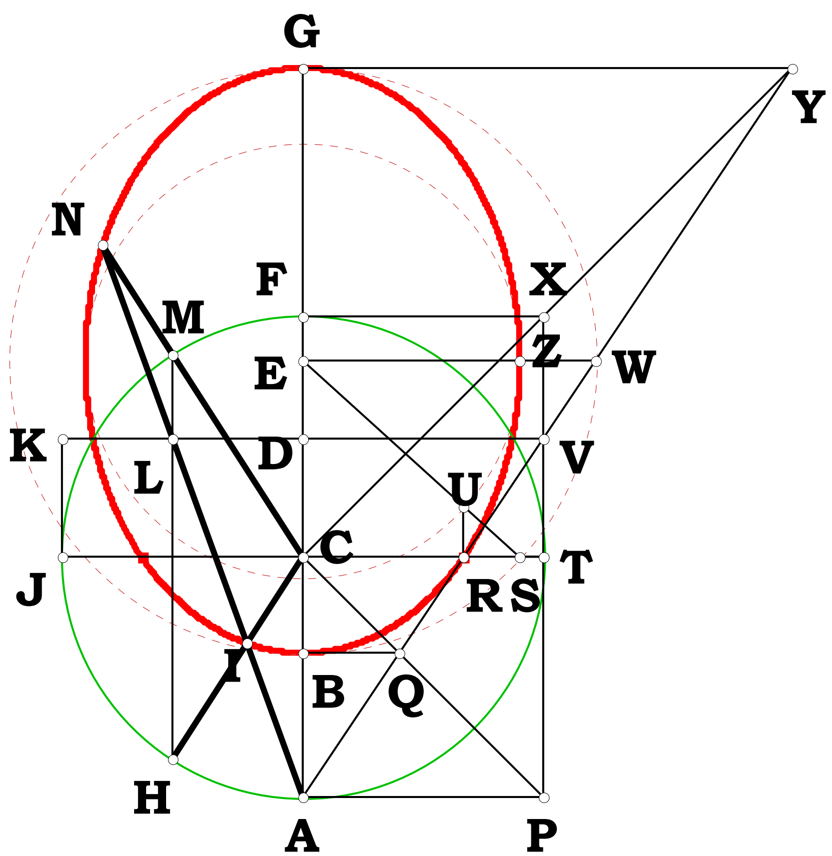
$VX := AF - AD \quad AY := \dfrac{AV \cdot AC}{AC - VX} \quad AG := \dfrac{AD \cdot AY}{AV}$

$BC := \dfrac{AC \cdot AC}{AC + AD} \quad CG := AG - AC \quad BG := BC + CG$

$BE := \dfrac{BG}{2} \quad CE := BE - BC \quad ES := BE$

$CS := \sqrt{ES^2 - CE^2} \quad CR := \dfrac{DV \cdot AC}{AD}$





$$EU := \frac{ES \cdot CR}{CS}$$

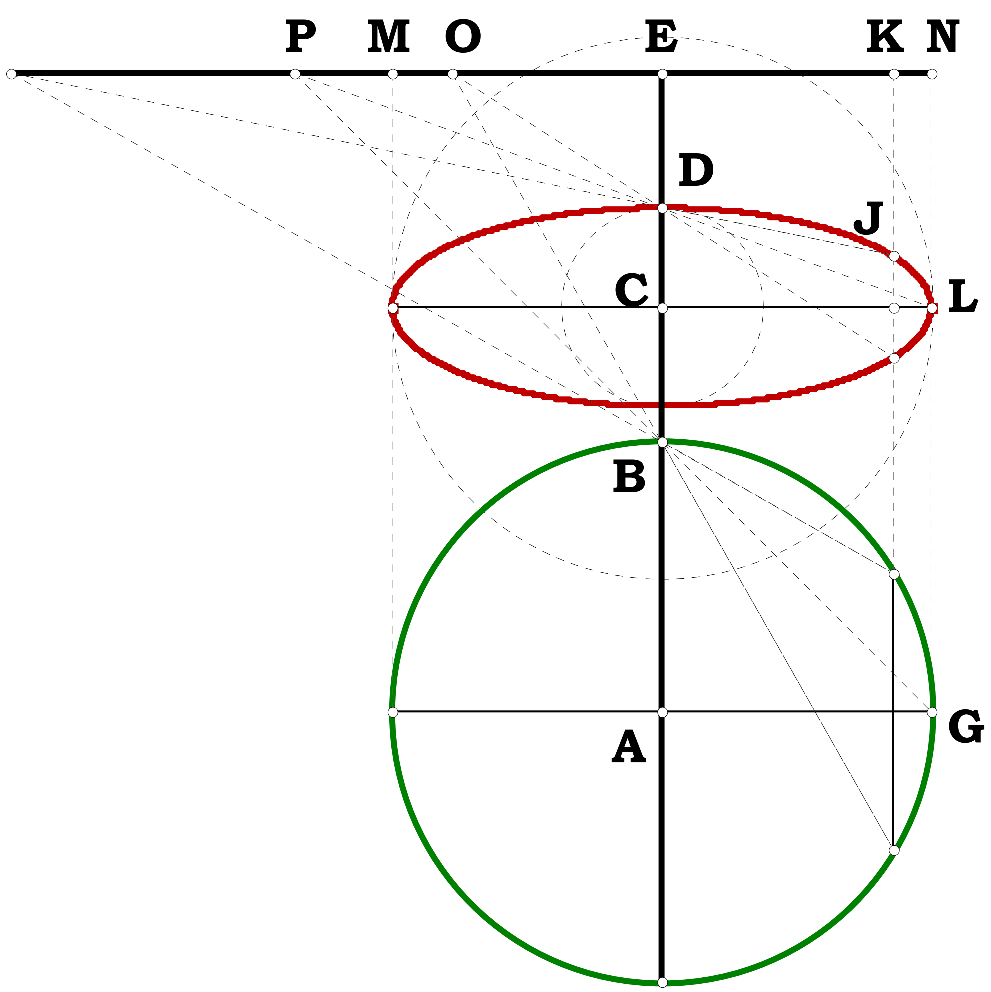
$$EZ := EU$$

$$EZ - \frac{N_1^{\,2} \cdot \sqrt{N_2^{\,2} - N_1^{\,2}}}{(N_1 + N_2) \cdot (N_2 - N_1)} = 0$$

$$BG - \frac{2 \cdot N_1^{\,2} \cdot N_2}{(N_1 + N_2) \cdot (N_2 - N_1)} = 0$$

032305a

An Ellipse



$N_1 := 1 \quad N_2 := 2 \quad N_3 := .5$

$AB := N_1 \quad MN := 2 \cdot AB$

$BE := N_2 \quad BD := N_3$

$DE := BE - BD \quad AG := AB$

$EP := \frac{AG \cdot BE}{AB} \quad EN := AB$

$NL := \frac{DE \cdot (EP + EN)}{EP}$

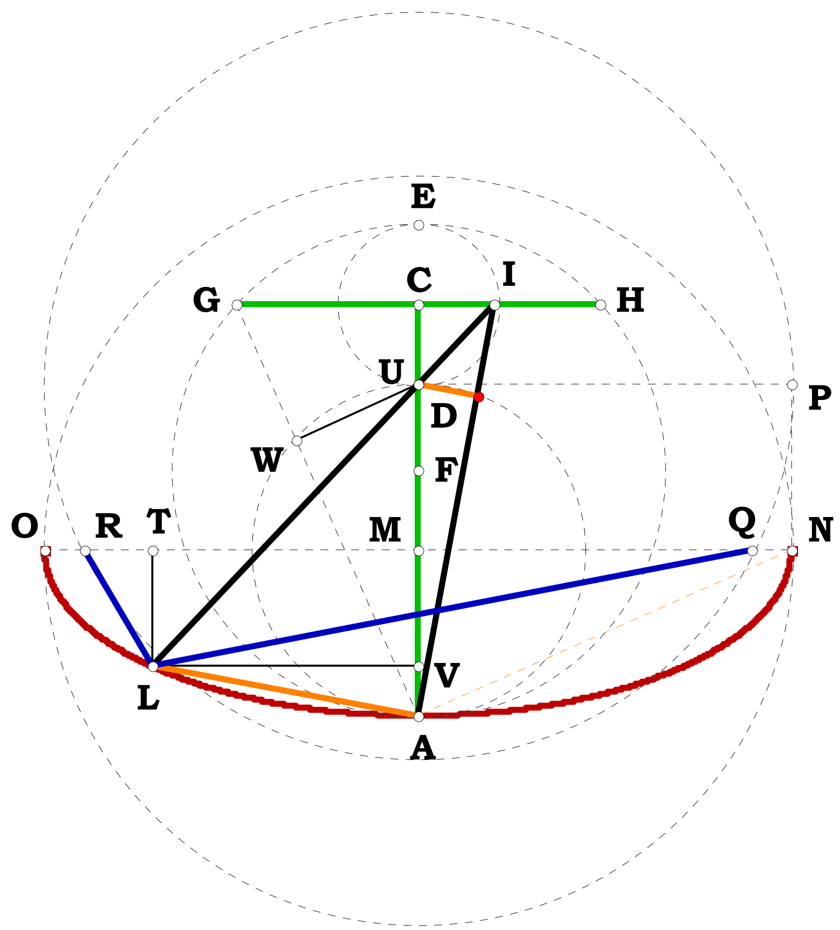
$CE := NL \quad CD := CE - DE$

$N_1 \cdot \frac{(N_2 - N_3)}{N_2} - CD = 0$

032905

Elipse Projected From a Perpendicular.

Let AC be some perpendicular on some line GH.



$N_1 := 1.9167 \qquad N_2 := .3244 \qquad N_3 := .437$

$AC := N_1 \quad CD := N_2 \quad CE := CD \quad CG := \sqrt{CE \cdot AC} \quad GH := 2 \cdot CG$

$GI := N_3 \quad CI := CG - GI \quad AD := AC - CD \quad AI := \sqrt{AC^2 + CI^2} \quad AM := \dfrac{AD}{2}$

$DM := AM \quad DU := \dfrac{CI \cdot AD}{AI} \quad AU := \dfrac{AC \cdot AD}{AI} \quad IU := AI - AU \quad AL := \dfrac{DU \cdot AI}{IU}$

$IL := \sqrt{AI^2 + AL^2} \quad DI := \sqrt{DU^2 + IU^2} \quad DL := IL - DI \quad DV := \dfrac{CD \cdot DL}{DI}$

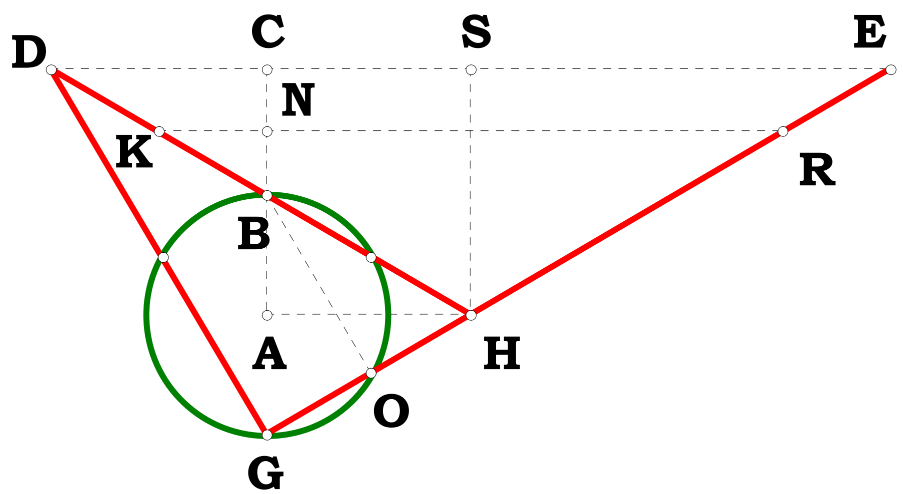
$MV := DV - DM \quad AV := AM - MV \quad LV := \sqrt{AL^2 - AV^2} \quad MT := LV$

$AG := \sqrt{CG^2 + AC^2} \quad DW := \dfrac{CG \cdot AD}{AG} \quad AW := \dfrac{AC \cdot AD}{AG} \quad GW := AG - AW$

$AN := \dfrac{DW \cdot AG}{GW} \quad MN := \sqrt{AN^2 - AM^2} \quad ON := 2 \cdot MN \quad ON - \sqrt{(N_2 - N_1)^2 \cdot \dfrac{N_1}{N_2}} = 0$

033005c

N^3 and More



$N_1 := 1 \quad N_2 := 2 \quad N_3 := 4 \qquad A := N_2 \qquad N := N_3$

AB is the unit AH/AB is the constant one. BC/BN is constant two.

$$AB := N_1 \quad AH := N_2 \quad BH := \sqrt{AB^2 + AH^2} \quad GO := \frac{AB \cdot 2 \cdot AB}{BH} \quad HO := BH - GO$$

$$DH := \frac{BH \cdot BH}{HO} \quad AC := \frac{AB \cdot DH}{BH} \quad BO := \sqrt{BH^2 - HO^2} \quad DG := \frac{BO \cdot BH}{HO}$$

$$DE := \frac{BH \cdot DG}{AB} \quad CD := \frac{DG^2}{DE} \quad BC := AC - AB \quad CE := DE - CD$$

$$\frac{CE}{BC} - \left(\frac{AH}{AB}\right)^3 = 0 \qquad \frac{N_2{}^3}{N_1{}^3} - \frac{CE}{BC} = 0$$

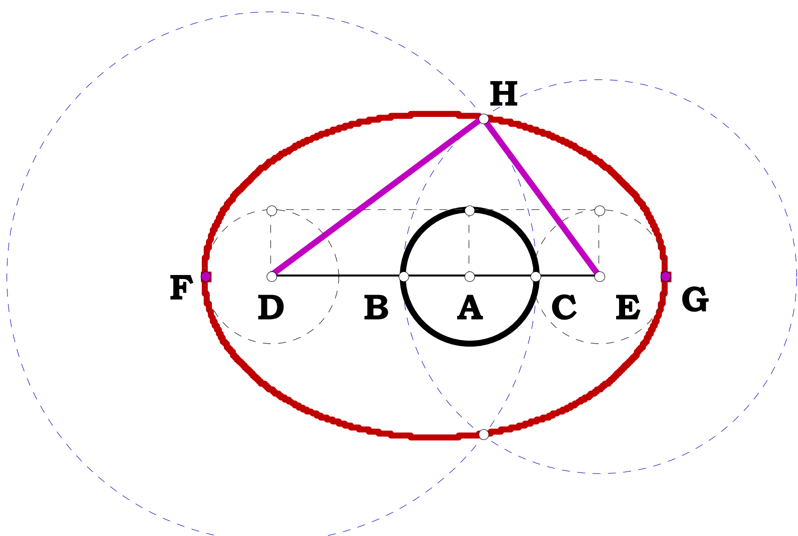
More $\left(A^3 \cdot N\right) - (N - 1) \cdot A$ Let N_3 be N

$$BN := \frac{BC}{N_3} \quad KN := \frac{CD}{N_3} \quad CG := AC + AB \quad GN := 2 \cdot AB + \frac{BC}{N_3} \quad NR := \frac{CE \cdot GN}{CG}$$

$$\left[\frac{N_2{}^3}{N_1{}^3} \cdot N_3 - \frac{N_2 \cdot \left(N_3 - 1\right)}{N_1}\right] - \frac{NR}{BN} = 0 \qquad A^3 \cdot N - (N - 1) \cdot A = 26 \qquad \frac{NR}{BN} = 26$$

033105b

Just Another Ellipse



Given the difference between the foci and difference between the proportional radii, etc., etc.

$N_1 := 1.708 \quad N_2 := .693 \quad N_3 := 1.032$

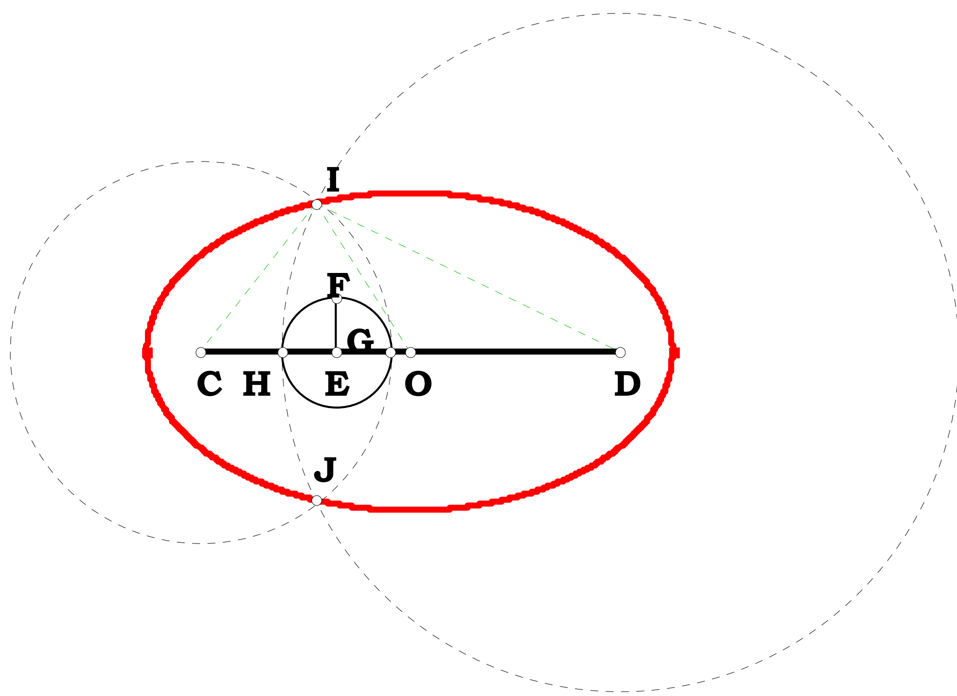
$DE := N_1 \quad BC := N_2 \quad FG := DE + BC \quad AB := \dfrac{BC}{2} \quad AD := N_3$

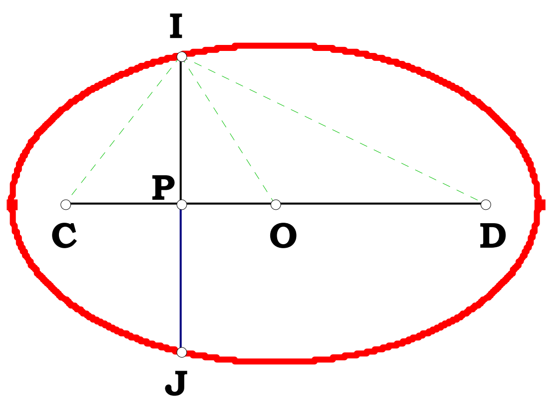
$CD := AD + AB \quad BE := DE - AD + AB \quad EH := BE \quad DH := CD$

$$DJ := \frac{DH^2 + DE^2 - EH^2}{2 \cdot DE} \qquad HJ := \sqrt{DH^2 - DJ^2}$$

$$\frac{\sqrt{-N_3 \cdot (N_3 - N_1) \cdot N_2 \cdot (N_2 + 2 \cdot N_1)}}{N_1} - HJ = 0$$

041205





$N_1 := 2.188 \quad N_2 := .278 \quad N_3 := 3.095$

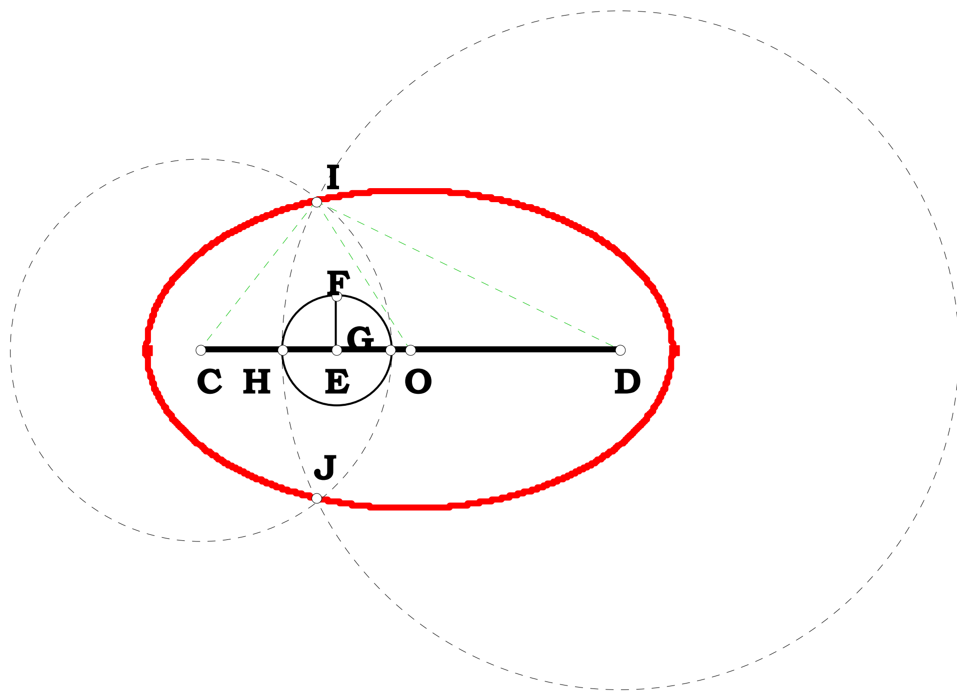
$CD := N_1 \quad CO := \dfrac{CD}{2} \quad EF := N_2 \quad CE := \dfrac{CD}{N_3}$

$CG := CE + EF \quad DH := CD - CG + 2 \cdot EF$

$CI := CG \quad DI := DH \qquad IO := \dfrac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$

$$2 \cdot \frac{\sqrt{N_2 \cdot (N_1 + N_2) \cdot (N_3 - 1)}}{N_3} - IP = 0$$

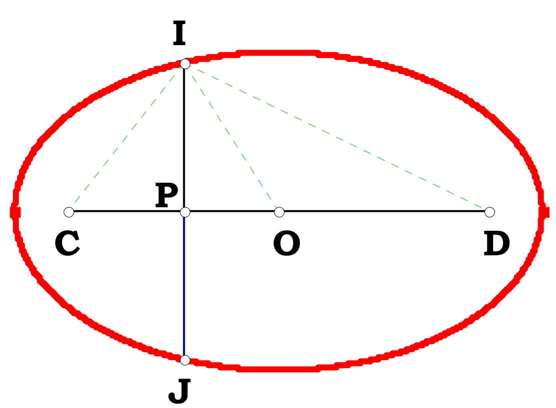


$$N_1 := 2.188 \quad N_2 := .278 \quad N_3 := .707$$

$$CD := N_1 \qquad CO := \frac{CD}{2} \qquad EF := N_2$$

$$CE := N_3 \quad CG := CE + EF \quad DH := CD - CG + 2 \cdot EF$$

$$CI := CG \quad DI := DH \qquad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$



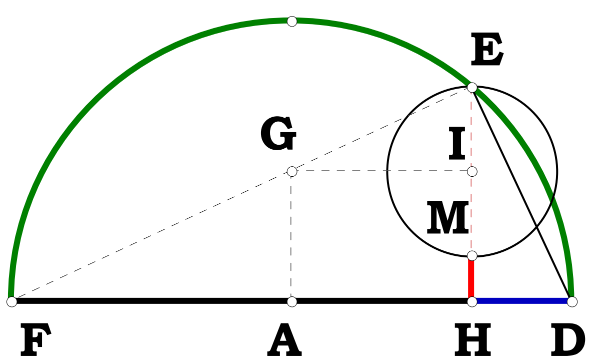
$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$

$$2 \cdot \frac{\sqrt{-CE \cdot EF \cdot (CD + EF) \cdot (-CD + CE)}}{CD} - IP = 0$$

$$2 \cdot \frac{\sqrt{-N_3 \cdot N_2 \cdot \left(N_1 + N_2\right) \cdot \left(-N_1 + N_3\right)}}{N_1} - IP = 0$$

$$IO := \frac{1}{2} \cdot \sqrt{4 \cdot CE^2 + 4 \cdot EF^2 + CD^2 - 4 \cdot CD \cdot CE + 4 \cdot CD \cdot EF}$$

041305b



N^3 **N Cubed**

$N_1 := 1 \quad N_2 := 36$

$DH := N_1 \quad FH := N_2 \quad DF := DH + FH \quad AD := \dfrac{DF}{2}$

$EH := \sqrt{DH \cdot FH} \quad HI := \dfrac{EH \cdot AD}{FH} \quad HM := EH - 2 \cdot (EH - HI)$

$$\frac{FH}{HM} - \left(\frac{DH}{HM}\right)^3 = 0$$

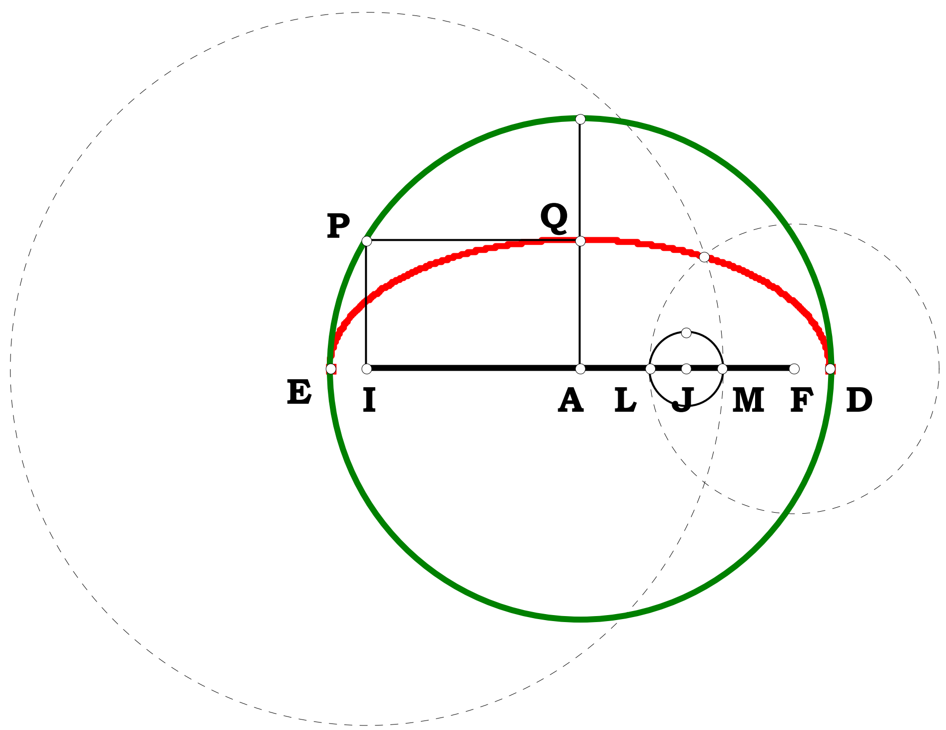
$DF := \sqrt{DH^2 + EH} \quad EF := \sqrt{FH^2 + EH^2} \quad \dfrac{FH}{HM} - \left(\dfrac{EF}{DF}\right)^3 = 0$

$\dfrac{FH}{HM} = 216 \qquad \left(\dfrac{FH}{DH}\right)^{1.5} = 216$

042205B

Given the major axis and the difference between the two foci, whatis the minor axis?

$N_1 := 2.604 \quad N_2 := 2.234$



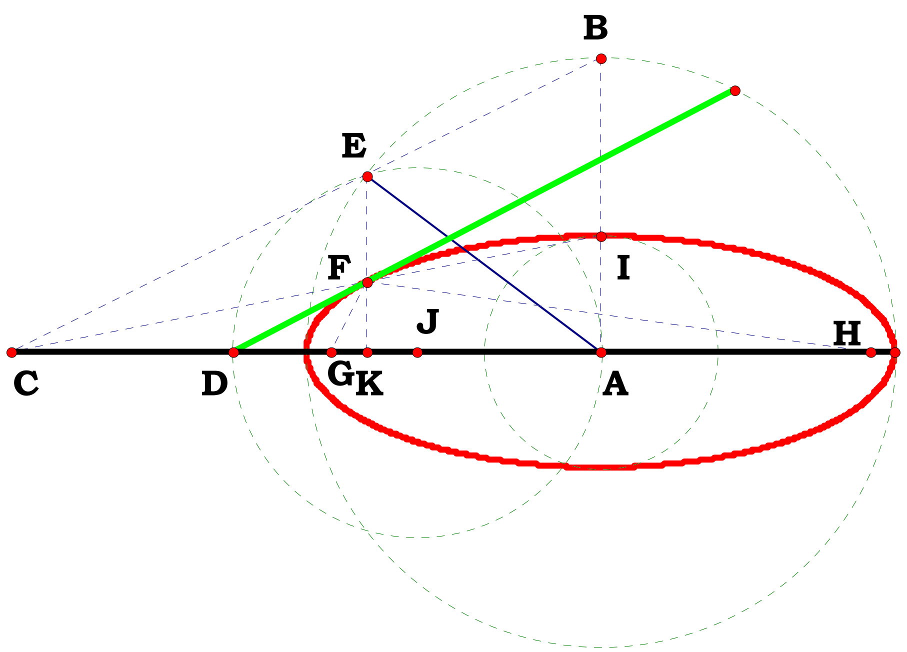
$DE := N_1 \quad FI := N_2 \quad EI := \dfrac{DE - FI}{2}$

$DI := DE - EI \qquad EP := \sqrt{EI \cdot DI}$

$AQ := EP$

$AQ - \dfrac{1}{2} \cdot \sqrt{(N_1 + N_2) \cdot (N_1 - N_2)} = 0 \qquad N_2 := AQ$

$FI - \sqrt{N_1^{\,2} - 4 \cdot N_2^{\,2}} = 0$



062007 C.mcd

Tangent from Major Axis

$$N_1 := 3.17500 \qquad N_2 := 1.24993 \qquad N_3 := 4.23333$$

$$AB := N_1 \qquad AI := N_2 \qquad AD := N_3$$

$$AJ := \frac{AD}{2} \qquad \text{From 080193} \quad EK := \frac{AB \cdot \sqrt{(2 \cdot AJ - AB) \cdot (2 \cdot AJ + AB)}}{2 \cdot AJ}$$

$$AK := \sqrt{AB^2 - EK^2} \qquad DK := AD - AK \qquad FK := \frac{AI \cdot EK}{AB} \qquad DF := \sqrt{DK^2 + FK^2}$$

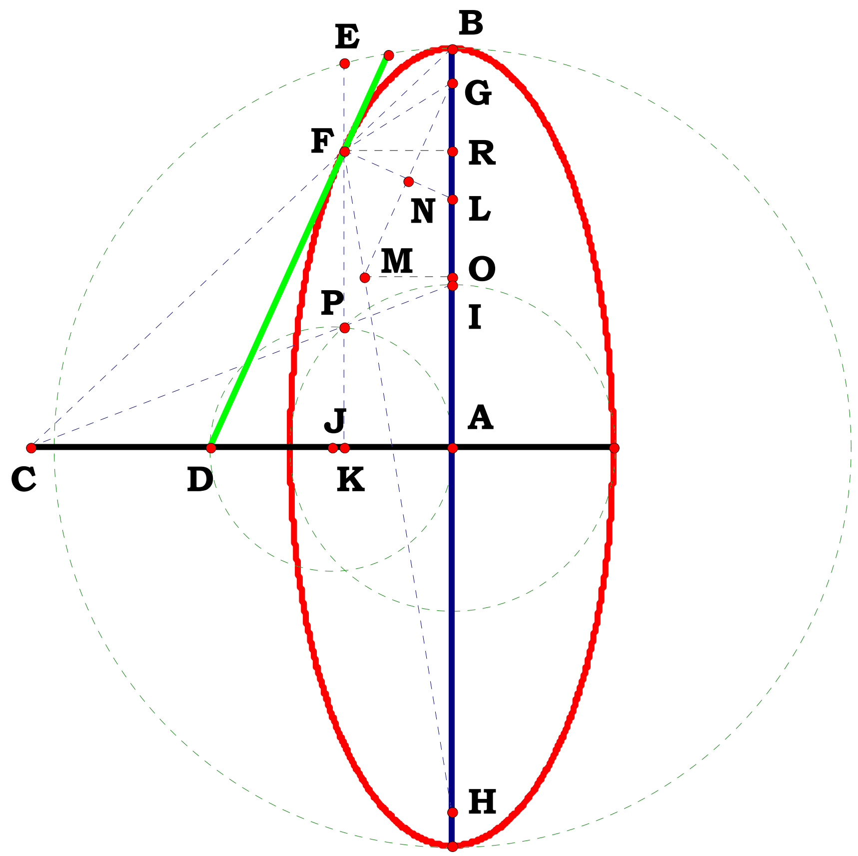
$$AG := \sqrt{AB^2 - AI^2} \qquad AH := AG \qquad HK := AH + AK \qquad GK := AG - AK$$

$$FG := \sqrt{FK^2 + GK^2} \qquad FM := FG \qquad FH := \sqrt{HK^2 + FK^2} \qquad HM := FH - FM$$

$$HO := \frac{HK \cdot HM}{FH} \qquad GH := 2 \cdot AH \qquad GO := GH - HO$$

$$MO := \frac{FK \cdot HM}{FH} \qquad \frac{GO}{MO} - \frac{DK}{FK} = 0 \qquad N_2 \cdot \frac{\sqrt{(N_3 - N_1) \cdot (N_3 + N_1)}}{N_3} - FK = 0$$

062007 D.mcd



Tangent from Minor Axis

$N_1 := 3.78354 \quad N_2 := 1.53747 \quad N_3 := 2.43417$

$AB := N_1 \quad AI := N_2 \quad AD := N_3$

$$AJ := \frac{AD}{2} \quad PK := \frac{AI\cdot\sqrt{(2\cdot AJ - AI)\cdot(2\cdot AJ + AI)}}{2\cdot AJ} \quad AK := \sqrt{AI^2 - PK^2}$$

$$DK := AD - AK \quad FK := \frac{PK\cdot AB}{AI} \quad DF := \sqrt{DK^2 + FK^2} \quad AG := \sqrt{AB^2 - AI^2}$$

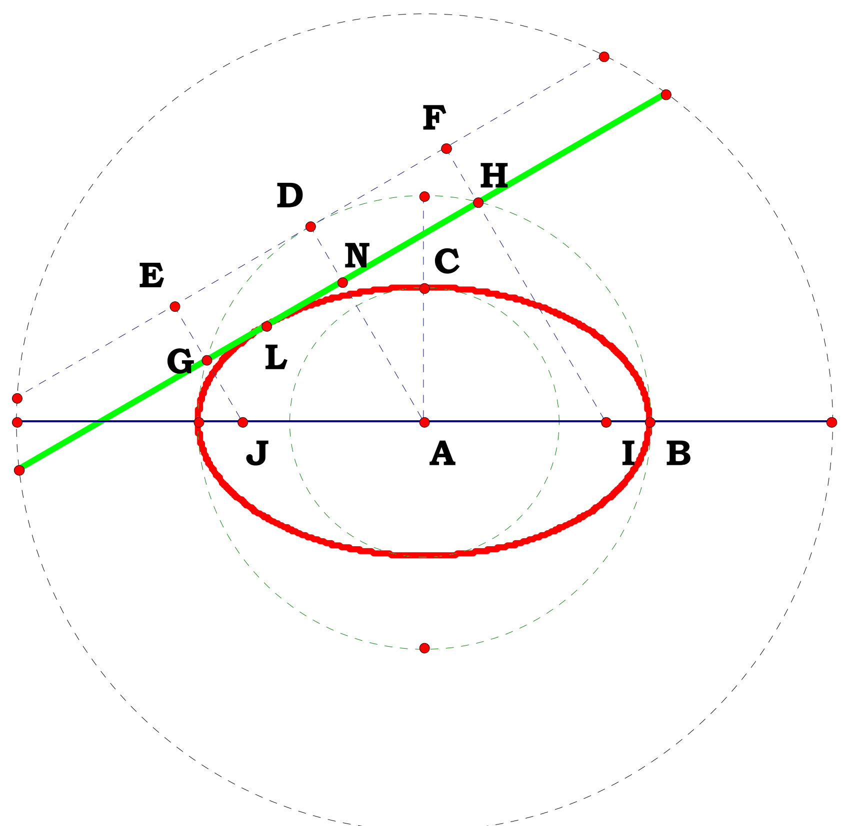
$$AH := AG \quad HR := AH + FK \quad GR := AG - FK \quad FG := \sqrt{AK^2 + GR^2} \quad FM := FG$$

$$\frac{\sqrt{(N_3 - N_2)\cdot(N_3 + N_2)}}{N_3}\cdot N_1 - FK = 0 \qquad Major := N_2\cdot\frac{\sqrt{(N_3 - N_1)\cdot(N_3 + N_1)}}{N_3}$$

$$FH := \sqrt{HR^2 + AK^2} \quad HM := FH - FM \quad HO := \frac{HR\cdot HM}{FH} \quad GH := 2\cdot AH$$

$$GO := GH - HO \quad MO := \frac{AK\cdot HM}{FH} \quad \frac{GO}{MO} - \frac{FK}{DK} = 0$$

Found on the Internet 062407



Found the construction, now I explore it with Algebra.

$$N_1 := 2.98979 \quad N_2 := 1.77791 \quad N_3 := 1.83972$$

$$AB := N_1 \quad AC := N_2 \quad EJ := N_3$$

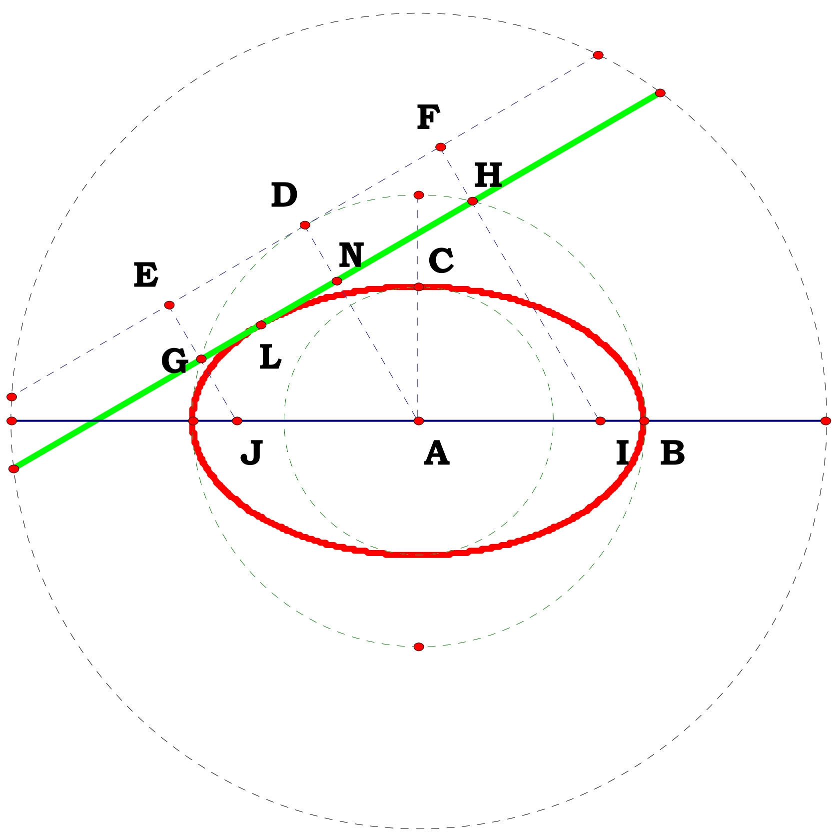
$$AJ := \sqrt{AB^2 - AC^2} \quad FI := (AB - EJ) + AB$$

$$DE := \sqrt{AJ^2 - (AB - EJ)^2} \quad AN := \frac{\sqrt{DE^2 \cdot (DE + AB) \cdot (-DE + AB)}}{DE}$$

$$DN := AB - AN \quad GJ := EJ - DN \quad HI := FI - DN \quad GL := \frac{GJ \cdot 2 \cdot DE}{GJ + HI}$$

$$HL := 2 \cdot DE - GL \quad JL := \sqrt{GL^2 + GJ^2} \quad IL := \sqrt{HI^2 + HL^2}$$

$$(JL + IL) - 2 \cdot AB = 0$$



Definitions:

$$AJ - \sqrt{N_1^2 - N_2^2} = 0 \qquad FI - \left(2 \cdot N_1 - N_3\right) = 0$$

$$DE - \sqrt{-N_2^2 + 2 \cdot N_1 \cdot N_3 - N_3^2} = 0 \qquad \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - AN = 0$$

$$N_1 - \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - DN = 0$$

$$N_3 - N_1 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - GJ = 0$$

$$\left(N_1 - N_3 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}\right) - HI = 0$$

$$\left(N_3 - N_1 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}\right) \cdot \frac{\sqrt{-N_2^2 + 2 \cdot N_1 \cdot N_3 - N_3^2}}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}} - GL = 0$$

$$\sqrt{-N_2^2 + 2 \cdot N_1 \cdot N_3 - N_3^2} \cdot \frac{\left(\sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - N_3 + N_1\right)}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}} - HL = 0$$

$$N_1 \cdot \frac{\sqrt{2 \cdot N_1^2 + N_2^2 + 2 \cdot N_3^2 + 2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2} \cdot \left(N_3 - N_1\right) - 4 \cdot N_1 \cdot N_3}}{\sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2}} - JL = 0$$

$$N_1 \cdot \frac{\sqrt{2 \cdot N_1^2 + N_2^2 + 2 \cdot N_3^2 + 2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2} \cdot \left(N_1 - N_3\right) - 4 \cdot N_1 \cdot N_3}}{\sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2}} - IL = 0$$

My Name is John.

Hello. My name is John and I am going to explain how to multiply and divide a line by a line in Geometry. Now, if you are going to ask me if I am a geometer, I have to reply by myth. Explanation by myth is one the ancient Greek's methods of teaching by discourse.

Once upon a time, God created man; They created him male and female, in the image of God. Or one can say, male and female created They him, which is rather awkward, but it does have that ancient New England flair to it. At any rate, once upon a time is not this time. It came to pass as men multiplied on the earth that men started to work for a living and not being god's themselves needed a way to designate each other and so individuals, which are not, by definition man started calling each other by their craft. That is where we got Mr. Smith and Mr. Clark, etc. A vestige of this remains today. Not being man, we tend to think of each other by an assigned craft. I work in a factory, but my name is Clark. The conflict of course is why I spend the entirety of my wages in therapy.

Now this works to my advantage. I have learned that individuals calling themselves geometers (I am personally hoping for the day I become part of man) cannot multiply and divide a line by a line. So, I guess one could say, that a geometer is someone who cannot do the math, which is really a sign for some serious expenditure on therapy—and, if those in mind—field knew what they were doing, the outlay would be advantageous. Too bad they cannot define a man. Now a non-Euclidean Geometer is someone who not only cannot do the math, they demand, as part of their initiation rights, that one will never be able to do the math. So, in due respect to non-Euclidean Geometers, please stop reading and go back to your scribbling—and contradicting yourself. Doing geometry inside of or on the outside of a tennis ball, or a Frisbee, makes me think that one has spent way to many days on the court, spiking one's tea, and certainly missing the ball.

Now, if your like me, a factory worker, and someone were to give you two lines and say,

Hey, you (He is hairy and has a club). Here are two lines, show me how to multiply one by the other, and after that, show me how to divide one by the other.

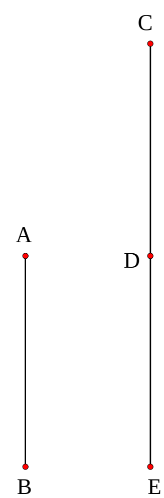
I would look at the man, think for a moment and draw a blank. What the heck does he mean? Then I would say, I am sorry, but I don't understand

what you mean. The man would leave off and I would go get another cup of coffee.

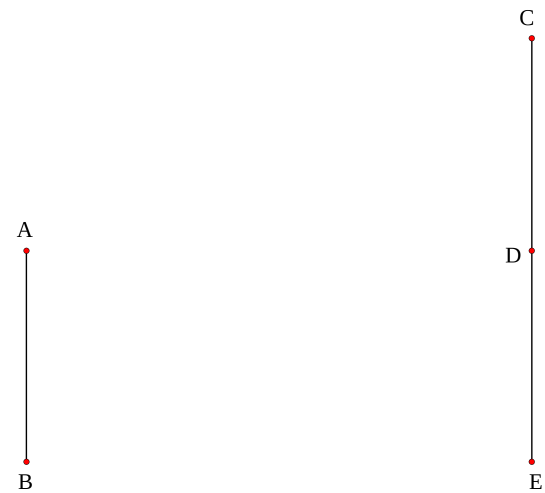
If I were a bit strange, I would consult Euclid's ***Elements*** and find to my dismay, the chap could do the math, but seems to have left this off for some reason, probably because it was too easy (so who don't lie for a friend?). Now, I happen to have in my possession a number of unpublished manuscripts which does have the answer in them and they are full of doing the math. I acquired them from the God's (and for those of you interested, the Delian Problem does have a solution—and it has something to do with Plato under extending himself). If it should be discovered that I am stealing a bit of fire, and giving it to man, please don't tell where you got it from. I have learned from first hand experience, you don't want to mess with Them—they be giants—really, really, big giants.

Now I am not going to explain this exactly as it was explained to me, as I have a poor memory. Please bear with me.

If I were given two lines, and asked to compare them, I would look at them and say;

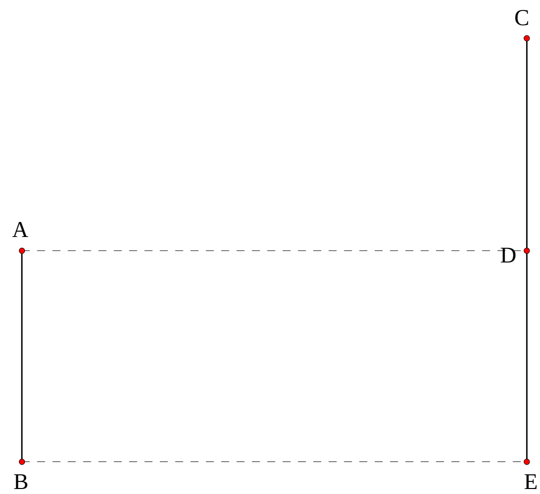


well, AB is shorter than CE. I mean, what can you do with two lines anyway. Reminds me of when I was a kid asking my mother what could I do with seven cents, realizing early on I was three cents short of a dime. If I were Euclid I would subtract one from the other and find that CE — AB = CD, or if you're a top down programmer, CE — AB = DE. If I move CE off a ways,

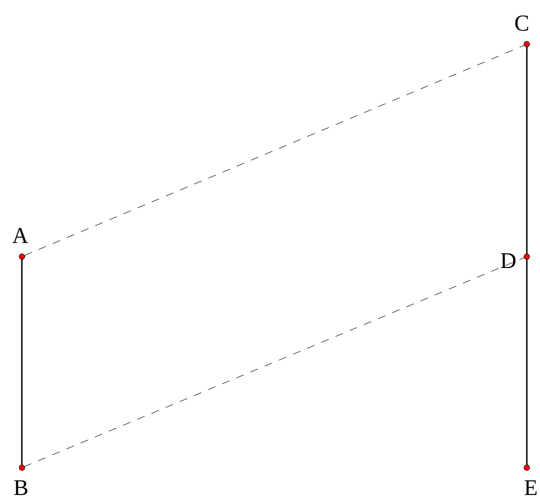


I would say that CE — AB = CD, or DE which ever you choose. Non-Euclidean Geometers, like Einstein, claim that this equality, this simultaneity, is not true and that at some point of moving AB and CE apart, as if it were part of the equation, does mysterious things to these segments. It amounts to a thief's logic—moving CE off sufficiently will make AB infinitely greater than CE 'cause we exact a kind of tribute on it and subtract that tribute as we go. It amounts to constructing a square say, of 25 square inches or so, and claiming if we repeat it enough, well, it just plain disappears—we wore it out. While on the other hand, there are those who claim that if I assert a point an infinite number of times, I can create a line. You know, like waving a knife in the air an infinite number of times an making a salad. This is the kind of mentality that makes credit card lenders rich. As I said, non-Euclidean Geometers are really crooked bankers in disguise—or really lousy cooks. A basic fact of abstraction, when you really know that a boundary is not the difference (a point is that which has not part), a form is in fact absolute, you know you can never attribute difference to that form, the form is applied as a boundary to any given difference—material. The cut is not the cutted! Wow, that was trashy!

Now if I had AB, and wanted to construct CE from it.



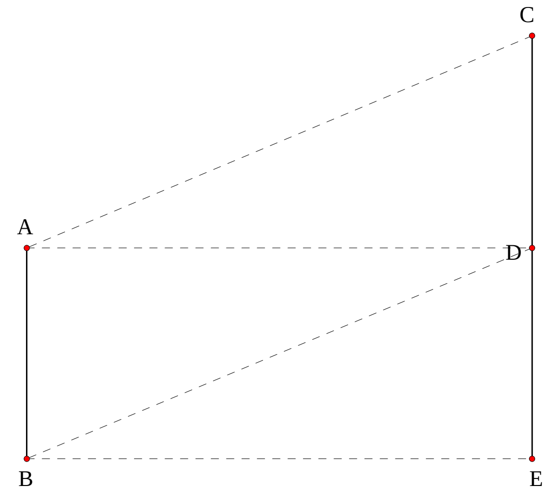
I could transfer one segment at a time



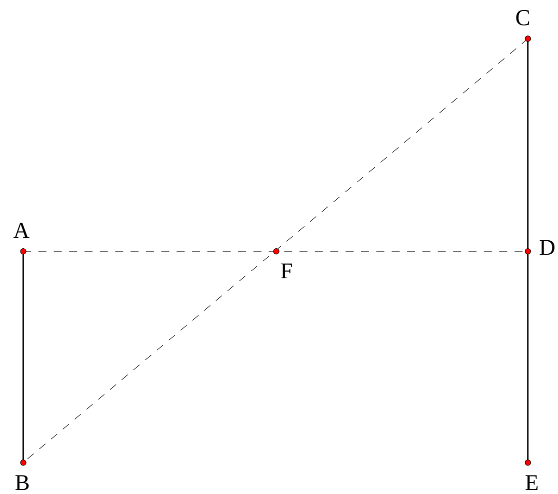
using parallel lines, but this is not multiplication, it is multiple processes, or simply addition. Parallel lines gives us the ability to do multiple additions, which is again not multiplication. One sign of that is that we have to assert each unit point in constructing CE. We have to assert each unit point just to do the parallels. Duh!

One of the things our ancient quibbling buddies, the Greeks, did tell us is that in order to multiply and divide, we have to have a unit. This is just part of plain simple Arithmetic. And they also said that when dealing with numbers in multiplication and division we were dealing with square and oblong (rectangular) numbers. Keep these ideas in mind. A square, an oblong, and a unit. Euclid drew a number of them. We will have need of them. For the moment let us learn what they did say about ratio,

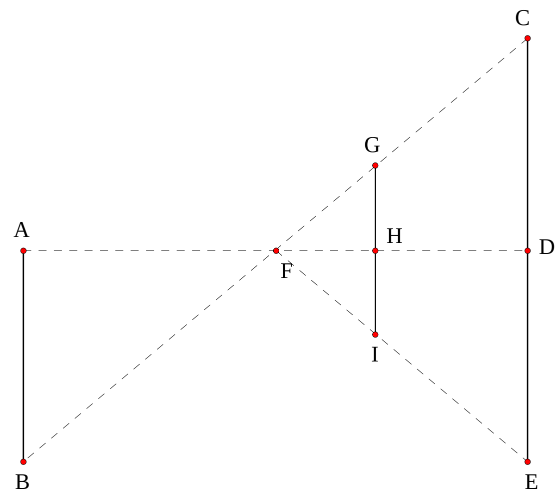
which we will also need. Now, if in constructing CE, we stayed up too late;—



and made a mistake in drawing—or were simply dyslexic;



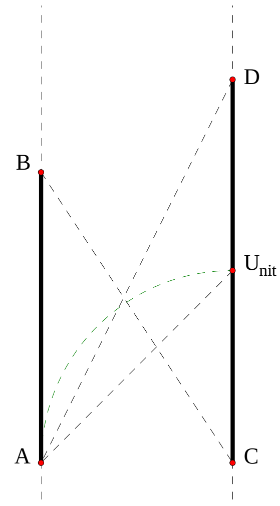
we would discover the ratio. As AB is to CD, so AF is to DF. And by George—(if you remember, he too was a hairy fellow and curious), One learns how to take any multiple and divide another segment of any length by the same multiple. From multiple addition, we have a kind of multiple division, but it is not division, it is still just a plain ratio, of anther segment.



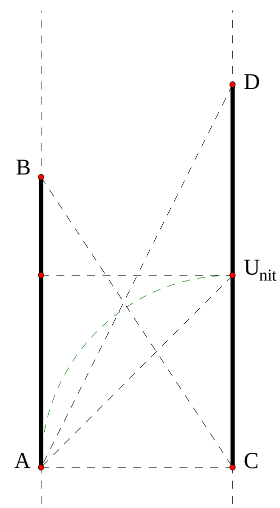
Now, as AB is to GH, so to DE is to HI, etc., etc. This is all fine and good, but, we still have not really learned how to multiply and divide. That is because these ratio's work regardless of the notion of unit, or square. Unless you are a crooked banker or a non-Euclidean Geometer, or a bad cook, this relationship is always true. There is one, and only one, difference between two points.

We are building our ideas up, one standard at a time. Intellectually, we fail, at the point we cannot abstract and use a standard—or what Plato called ***form*** because a boundary is not a difference and by definition (not a difference) always true. The divergence of language itself, starts with the inability to establish a standard even for a name. Many linguists call it the "growth" of language when meaning changes, but then they are non-Euclidean Geometers at heart also. What do they say of a government that has got its constitution saying exactly the opposite of what is written? If you want to reduce them to rubble, ask them outright, *Why can one word be or not be predicated of another*? Or again, if definition is conventional, and meaning can never be conventional, what in the heck does meaning have to do with definition? or even language? They will either get a funny look on their face mumbling to themselves, or start babbling non-sense to you. I have some books by the gods on that topic also. It is really simple, . . . but not here, not now.

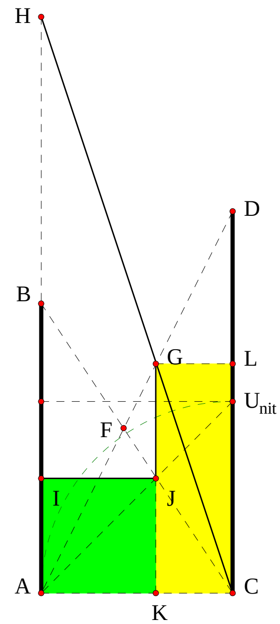
Multiplication and division rely on a standard in unit. So lets add that and see where we go.



At the outset the figure is very shy and unassuming. If you saw it laying in the street, you would hardly be pressed to pick it up. We have placed our segments the difference of our chosen unit apart, and we do have a square. No offence to Descartes who tried to find what I am doing, we don't have a number line, but a lined number. First time I ever seen a studious use of cross hairs actually miss the target.

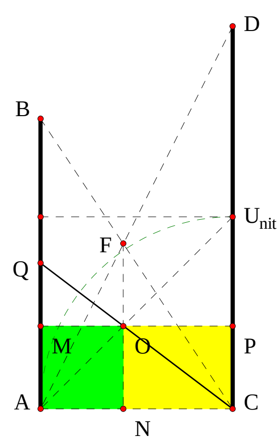


It don't look like much, but it can not only multiply and divide, one can use it to do much in the way of exponential manipulation as well. Let us take a closer look as to what the figure tells us.



$$\frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} - \frac{(\text{Area } IJKA)}{(\text{Area } KGLC)} = 0.00000$$

This is how we perform multiplication. Given AC as our unit, AB × CD = AH. In order to see this using the Arithmetic Grammar system, We divide AC by AC and get 1, our Unit. We then divide AB by AC which gives AB in terms of our unit. We then divide CE by AC and acquire that in units, and again for AH. We will find that by using the notion of Unit, Square and Oblong Numbers, which is incorporated in the idea of ratio, we can Multiply. And we can do what no binary calculator will ever do, we do it exactly. What about division?



$$\frac{(\text{Area } MANO)}{(\text{Area } ONCP)} - \frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} = 0.00000$$

Wouldn't you know it, there is a triplicate ratio in the figure! Right under our pencil. Didn't Euclid write that it was the hardest thing to do in geometry? Well, I have never taken geometry in school and set out to comprehend the triplicate ratio, guess I got somewhere. Going through our steps as before, we find that AB ÷ CD = AQ. Each of these steps is proven individually in Euclid. I suspect he was like Plato and wanted to see if his readers were smart enough to add and subtract ideas. And again, no binary computer will ever be up to Geometry, as Geometry is exact.

One can do a whole lot with this figure, through various projections. One can do a lot in the way of exponential manipulation. Try that with cross hairs! Some of the methods one will find in those unpublished books I was talking about. I don't know how long the gods will let me work on them, in fact, if it were not for Them, I would have been killed over thirty years ago. Imagine that, I am a walking contradiction, a living dead man. At any rate, I hope you have fun playing with the figure.

Now this is not the place to show the solution to the Delian Problem. My god, if one is just learning the simple four, by adding multiplication and division to our list of addition and subtraction, it may be too difficult realize a revolution in Euclidean Geometry based upon a standard long ago recognized but left unemployed—just like these. I will put the idea in the Geometer's Sketchpad file.

I hope I have made it clear that through multiple addition and subtraction, one leads into the understanding of ratio, just like Euclid did, but it is still a step away from multiplication and division. Those depend upon a respect for, and understanding of a standard in definition. We learn to add, and subtract. These teach us ratio—it is part of them. We learn about the units which is taught by them also. This then leads to multiplication and division and our primary four are thus established.

I do have some food for thought though. Using the facts of conventions in language, can you count the ways non-Euclidean geometries commit self-referential errors in simple logic? Apparently not, they are popular. Maybe it has something to do with linguist waving their knife in the air constructing sentences. What is prediction? Maybe I will read it to you sometime. The solution was once written on a Temple "Know Thyself." I will say this, as a sense system, the human mind is suppose to abstract form and create things with it. To deny form as the foundation for thought is simply a sign of dysfunction. I know, look at me.

Multiplication And Division of Lines

1. An unit is that by virtue of which each of the things that exist is called one.
Euclid's Elements

312

The Basic figures in this little thing are written up in my work Threee Pieces of Paper, or The Delian Quest. This is not a formal presentation, is a presentation of craft basics.

John Clark

Printed via import to MS Word.

Following the Yellow Brick Road

In Ancient Geometry, one calls the Green Square Numbers, and one calls the yellow Oblong Numbers. Today we just do swatches.

Multiplication

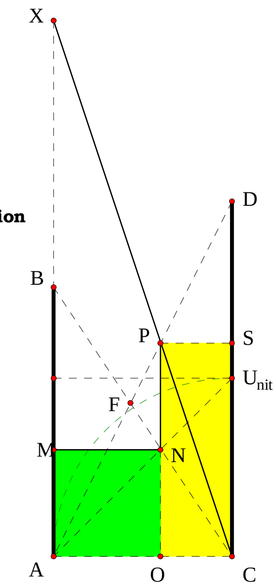
Unit = 1.00000

AB = 1.51095

CD = 1.99270

AX = 3.01087

AB·CD-AX = 0.00000



$$\frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} - \frac{(\text{Area MNOA})}{(\text{Area OPSC})} = 0.00000$$

Division

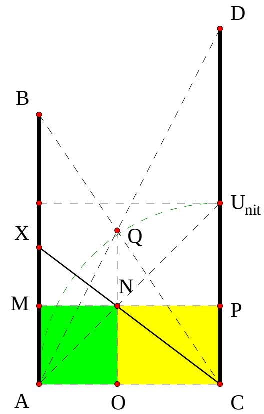
Unit = 1.00000

AB = 1.48921

CD = 1.96403

AX = 0.75824

$$\frac{AB}{CD} - AX = 0.00000$$



$$\frac{(\text{Area MAON})}{(\text{Area NOCP})} - \frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} = 0.00000$$

Introduction

Maybe I am too dogmatic, but I think one should have geometry teach one something of basic math. One should be able to add, subtract, multiply and divide with lines. These can provide proofs and constructible.

The figures can be modified in various ways to produce various results. I present a few here. The main figure is composed of the notion of common unit, and that multiplication and division works with square numbers, which is distinct from squaring a number. The square thus constructed provides the properties needed for multiplication and division.

I once read, in an Algebra book, that exponential notation had nothing to do with Geometry, that it was a pure mental abstract. What am I, then, to do with all the figures I have come up with that display the principles?

I would also like to see how the four basic operations of Math hold up in "non-Euclidean" Geometries. In fact, as part of their presentation, I think the four basic operations of mathematics should be a requirement. Perhaps by teaching the remaining two in geometry, something about reality and standards of thought will be learned.

The material in this little flyer is not new to me, it is part of four works I am currently engaged in, The Delian Quest, which is essentially completed, it needs some lipstick and a dress, Three Pieces of Paper, Eloi, and something with a puny Latin name.

Oh, and no, I have never studied geometry in an institution-I have never seen ideas survive in an institution. I have and probably will be again, be institutionalized at my own request.

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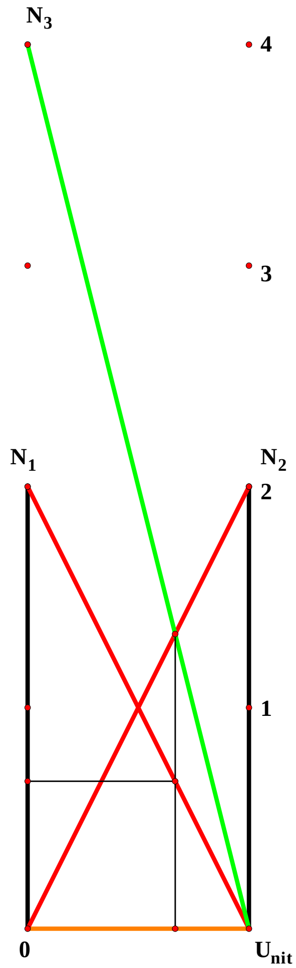


$N_1 = 2.00000$

$N_2 = 2.00000$

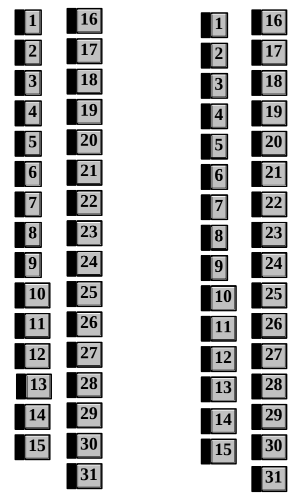
$N_1 \cdot N_2 = 4.00000$

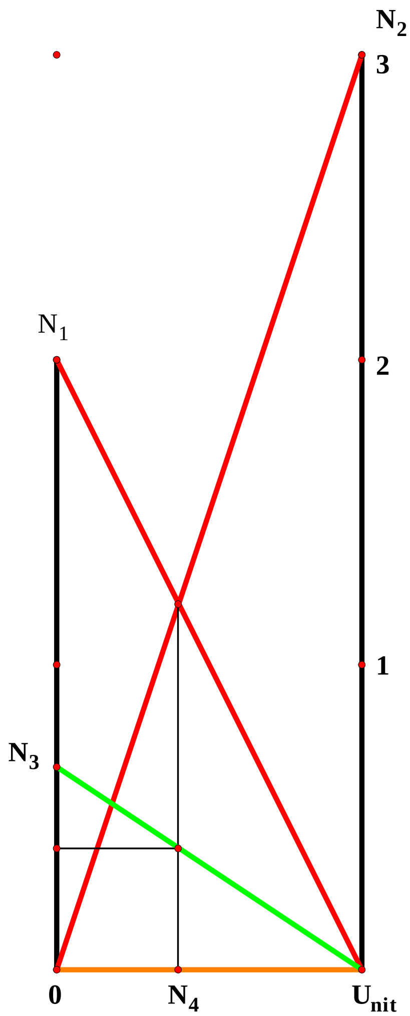
$(N_1 \cdot N_2) - N_3 = 0.00000$

$N_3 = 4.00000$

Multiply N1 by N2

N1 **N2**





$N_1 = 2.00000$

$N_2 = 3.00000$

$$\frac{N_1}{N_2} = 0.66667$$

$N_3 = 0.66667$

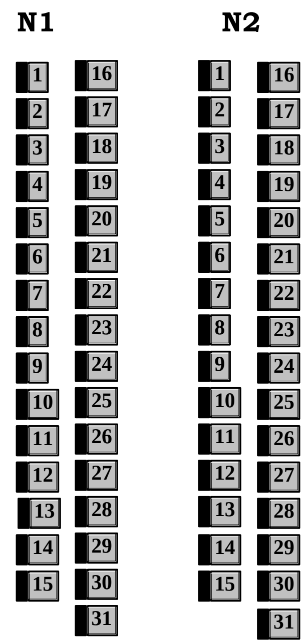
$$\frac{U_{nit}}{0N_4} = 2.50000$$

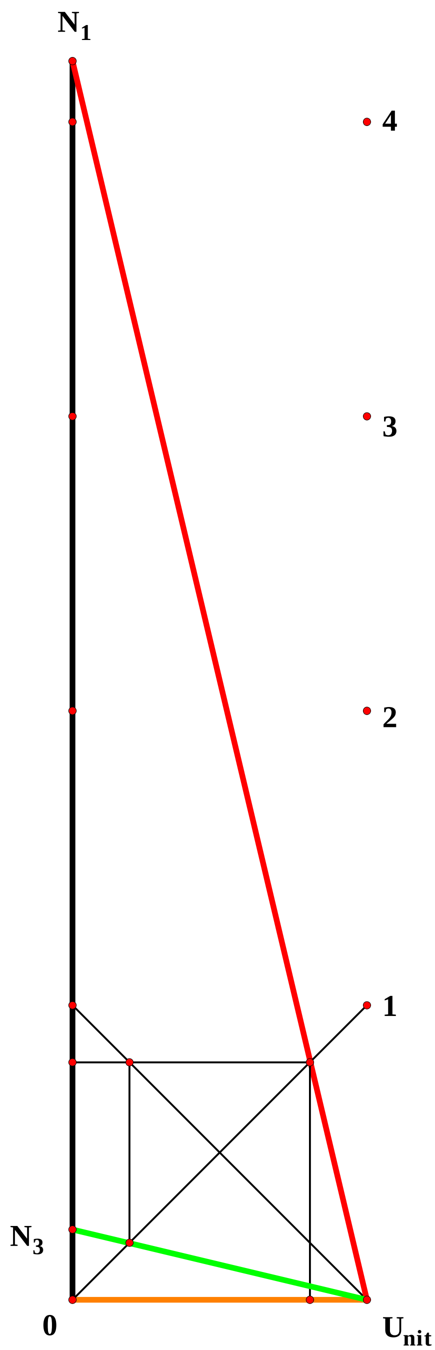
$$\frac{N_1+N_2}{N_1} = 2.50000$$

$$\frac{U_{nit}}{U_{nit}N_4} = 1.66667$$

$$\frac{N_1+N_2}{N_2} = 1.66667$$

Divide
N1 by N2





Find the Recprocal N1

$N_1 = 4.20930$

$\frac{1}{N_1} = 0.23757$

$N_3 = 0.23757$

$\frac{1}{N_1} - N_3 = 0.00000$

N1

1	16
2	17
3	18
4	19
5	20
6	21
7	22
8	23
9	24
10	25
11	26
12	27
13	28
14	29
15	30
	31

$N_1 = 3.00000$

$N_2 = 2.00000$

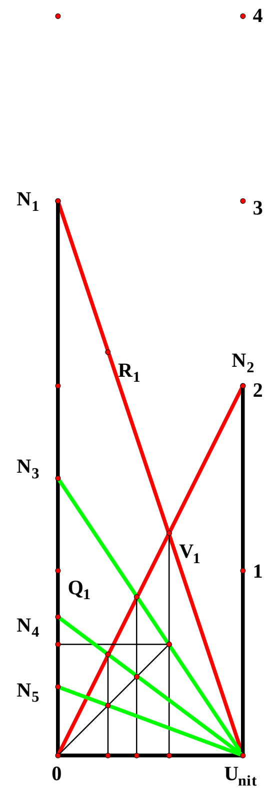
$\dfrac{N_1}{N_2} = 1.50000$

$N_3 = 1.50000$

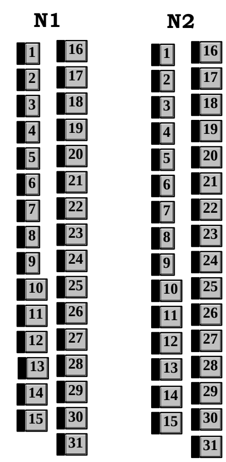
$N_4 = 0.75000$ $\dfrac{N_1}{N_2^2} - N_4 = 0.00000$

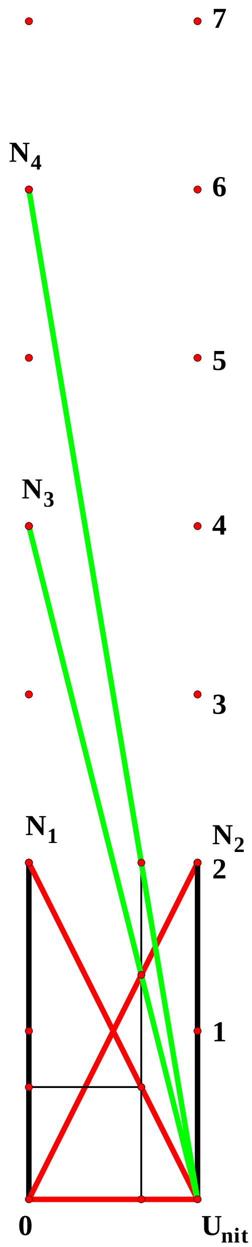
$N_5 = 0.37500$ $\dfrac{N_1}{N_3^3} - N_5 = 0.00000$

etc.



Divide N1 by a Power of N2





$N_1 = 2.00000$

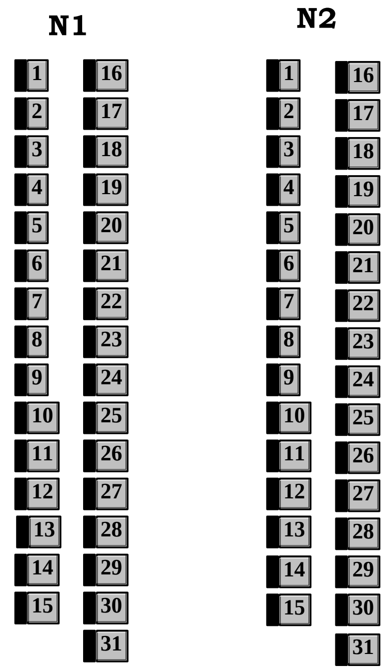
$N_2 = 2.00000$

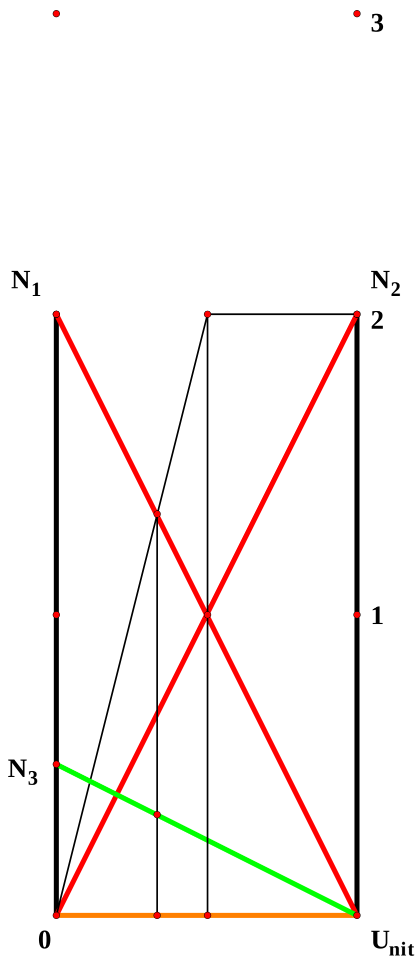
$N_1 \cdot N_2 = 4.00000$

$N_3 = 4.00000$

$N_4 = 6.00000$

$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$



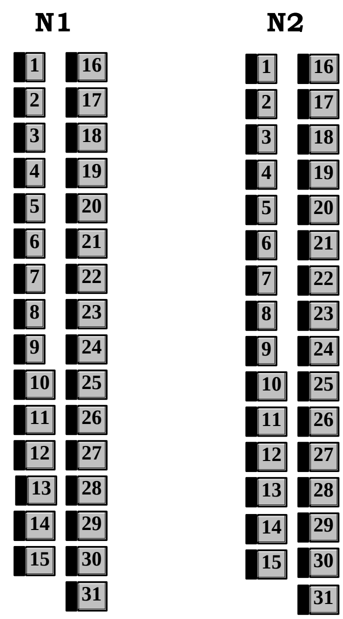


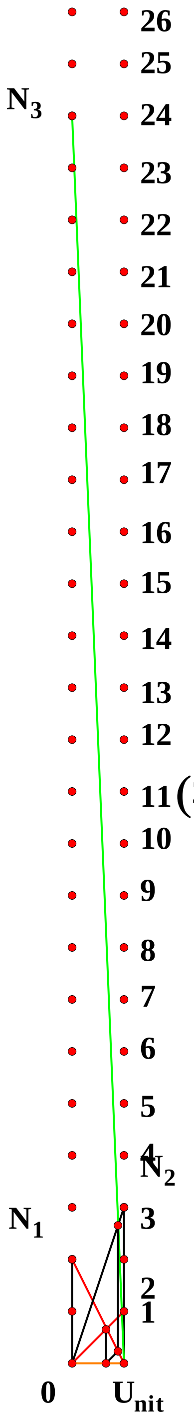
$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 0.50000$

$$\frac{N_1^2}{(N_2+N_1)\cdot N_2} - N_3 = 0.00000$$



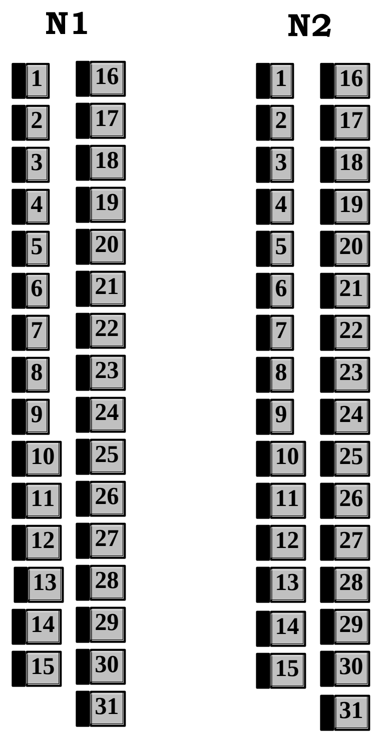


$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 24.00000$

$\left(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2\right) - N_3 = 0.00000$



$$\frac{BC}{BV_1} = 0.81650 \qquad N_3^{-0.5} = 0.81650$$

$$N_3^{0} = 1.00000$$

$$\frac{BC}{BU_1} = 1.22474 \qquad N_3^{0.5} = 1.22474$$

$$\frac{BC}{BP_1} = 1.50000$$

$$\frac{BC}{BR_1} = 2.25000 \qquad N_3^{2} - \frac{BC}{BR_1} = 0.00000$$

$$\frac{BC}{BS_1} = 3.37500 \qquad N_3^{3} - \frac{BC}{BS_1} = 0.00000$$

$$\frac{BC}{BT_1} = 5.06250 \qquad N_3^{4} - \frac{BC}{BT_1} = 0.00000$$

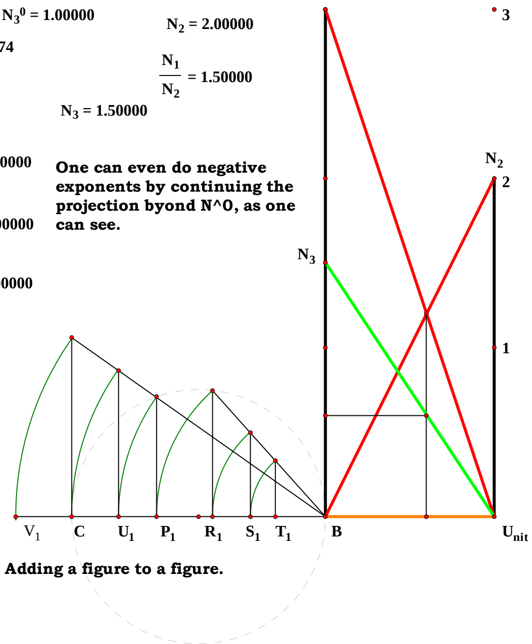
$N_1 = 3.00000$

$N_2 = 2.00000$

$$\frac{N_1}{N_2} = 1.50000$$

$N_3 = 1.50000$

One can even do negative exponents by continuing the projection byond N^0, as one can see.



Adding a figure to a figure.

I tossed this together, so it is not perfect--it just looks good

N1		N2	
1	16	1	16
2	17	2	17
3	18	3	18
4	19	4	19
5	20	5	20
6	21	6	21
7	22	7	22
8	23	8	23
9	24	9	24
10	25	10	25
11	26	11	26
12	27	12	27
13	28	13	28
14	29	14	29
15	30	15	30
	31		31

How to find a Proportional Unit for a given pair of Magnitudes

$\mathbf{Unit_2 = 1.00000}$

$\mathbf{N1_2 = 1.39420}$

$\mathbf{N2_2 = 1.94380}$

$\mathbf{N3_2 = 0.71726}$

$\mathbf{N4_2 = 2.71004}$

$$\frac{N1_2}{N2_2} - N3_2 = 0.00000$$

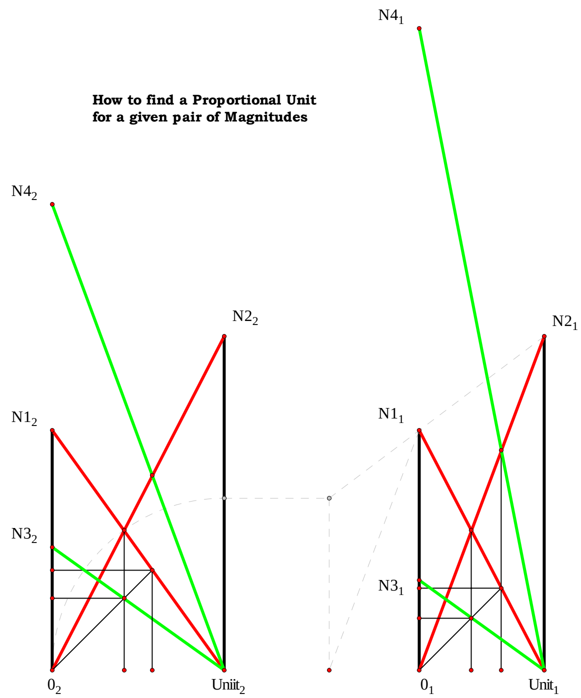
$$N1_2 \cdot N2_2 - N4_2 = 0.00000$$

$$\frac{N4_2}{N2_2} = 1.39420$$

$$\frac{N2_2}{N1_2} = 1.39420$$

$$\frac{N1_2}{Unit_2} = 1.39420$$

$$\frac{Unit_2}{N3_2} = 1.39420$$



$\mathbf{Unit_1 = 1.00000}$

$\mathbf{N1_1 = 1.91861}$

$\mathbf{N2_1 = 2.67492}$

$\mathbf{N3_1 = 0.71726}$

$\mathbf{N4_1 = 5.13213}$

$$\frac{N1_1}{N2_1} - N3_1 = 0.00000$$

$$N1_1 \cdot N2_1 - N4_1 = 0.00000$$

$$\frac{N4_1}{N2_1} = 1.91861$$

$$\frac{N2_1}{N1_1} = 1.39420$$

$$\frac{N1_1}{Unit_1} = 1.91861$$

$$\frac{Unit_1}{N3_1} = 1.39420$$

Unit = 1.00000

$N_1 = 2.00000$

$N_2 = 3.00000$ $\qquad \frac{N_1}{N_2} - N_3 = 0.00000$

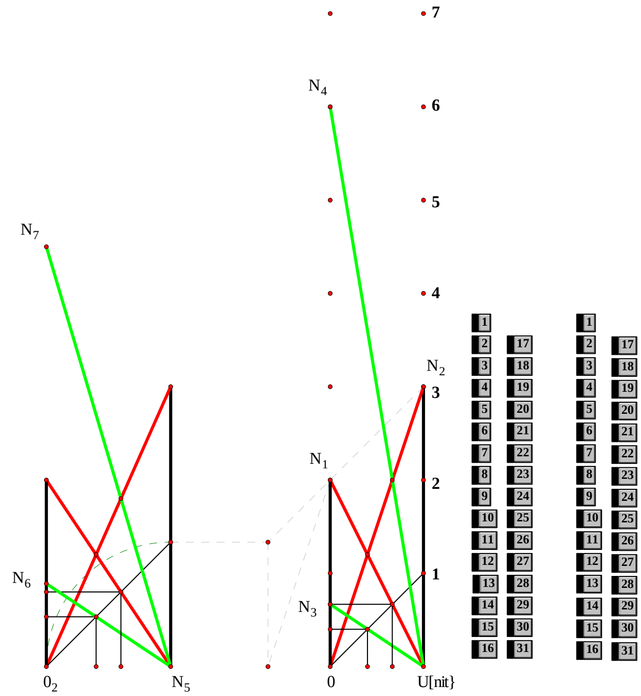
$N_3 = 0.66667$

$N_4 = 6.00000$ $\qquad N_1 \cdot N_2 - N_4 = 0.00000$

$N_5 = 1.33333$ $\qquad \frac{N_1^2}{N_2} - N_5 = 0.00000$

$N_6 = 0.88889$ $\qquad \frac{N_1^3}{N_2^2} - N_6 = 0.00000$

$N_7 = 4.50000$ $\qquad \frac{N_2^2}{N_1} - N_7 = 0.00000$



Unit = 1.00000

$N_1 = 2.00000$

$N_2 = 2.30952$

$N_3 = 0.86598$

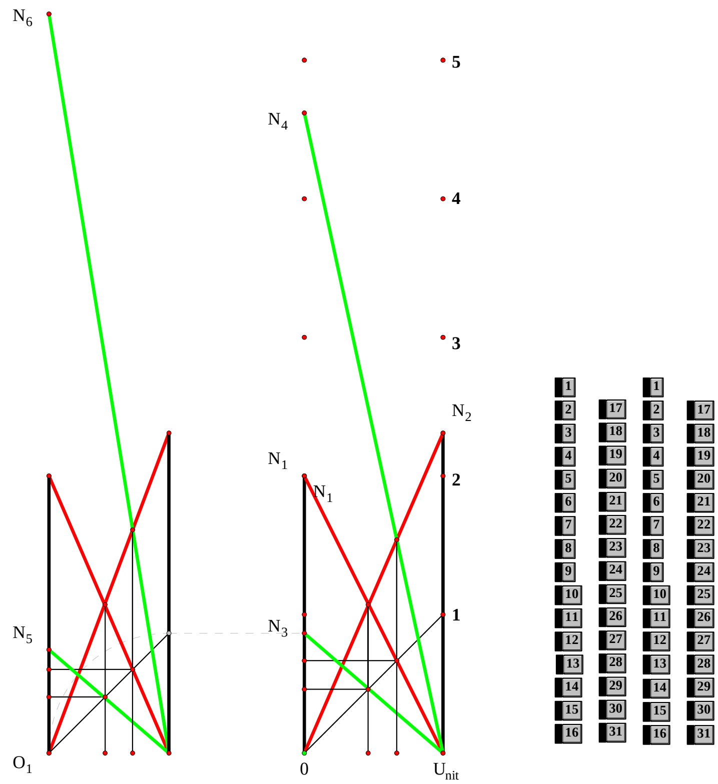
$N_4 = 4.61905$

$\frac{N_1}{N_2} - N_3 = 0.00000$

$N_1 \cdot N_2 - N_4 = 0.00000$

$N_5 = 0.74992$ $\qquad \frac{N_1^2}{N_2^2} - N_5 = 0.00000$

$N_6 = 5.33390$ $\qquad N_2^2 - N_6 = 0.00000$



Unit = 1.00000

$\mathbf{N_1 = 2.00000}$

$\mathbf{N_2 = 2.43939}$

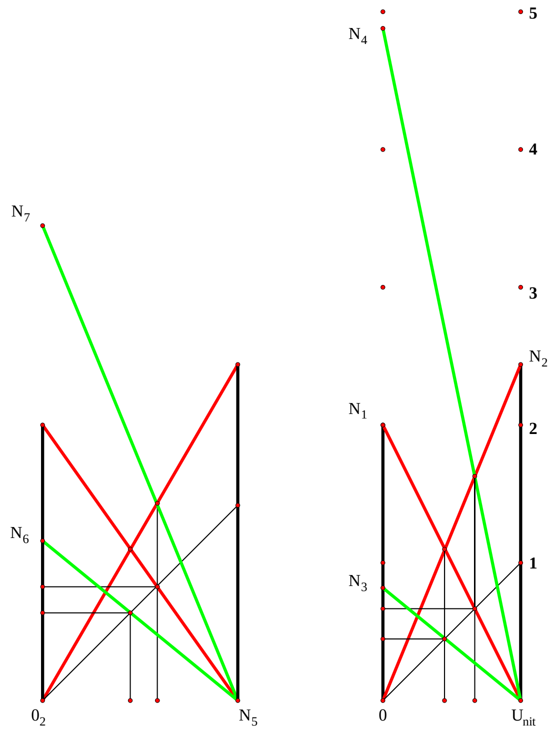
$\mathbf{N_3 = 0.81988}$ $\quad \dfrac{\mathbf{N_1}}{\mathbf{N_2}} - \mathbf{N_3} = \mathbf{0.00000}$

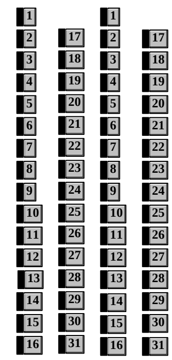
$\mathbf{N_4 = 4.87879}$ $\quad \mathbf{N_1 \cdot N_2 - N_4 = 0.00000}$

$\mathbf{N_5 = 1.41421}$ $\quad \sqrt{2} - \mathbf{N_5} = \mathbf{0.00000}$

$\mathbf{N_6 = 1.15948}$ $\quad \dfrac{\sqrt{2} \cdot \mathbf{N_1}}{\mathbf{N_2}} - \mathbf{N_6} = \mathbf{0.00000}$

$\mathbf{N_7 = 3.44982}$ $\quad \dfrac{\mathbf{N_1 \cdot N_2}}{\sqrt{2}} - \mathbf{N_7} = \mathbf{0.00000}$





Unit = 1.00000

$N_1 = 2.00000$

$N_2 = 2.00000$ $\qquad \dfrac{N_1}{N_2} - N_3 = 0.00000$

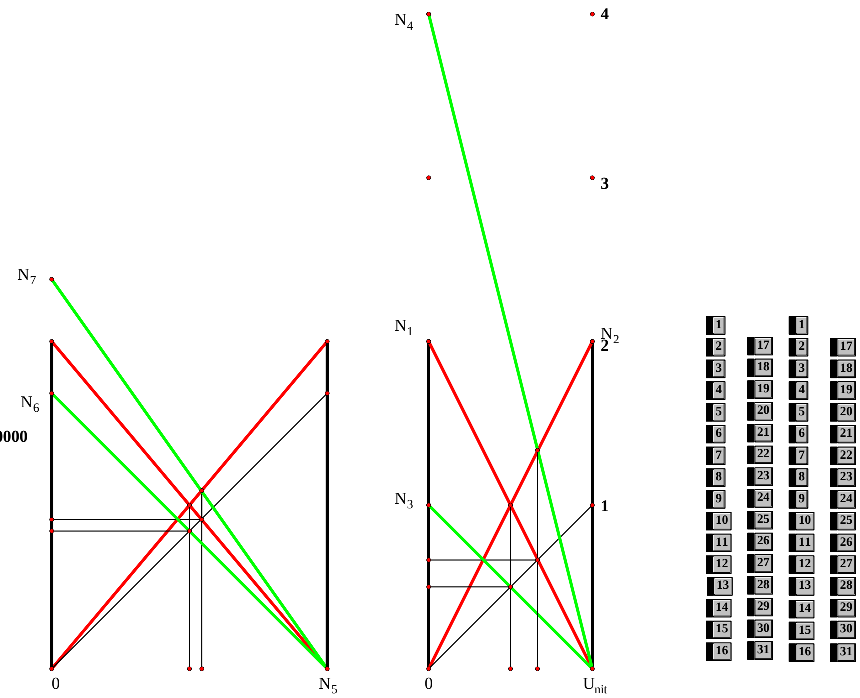
$N_3 = 1.00000$

$N_4 = 4.00000$ $\qquad N_1 \cdot N_2 - N_4 = 0.00000$

$N_5 = 1.68179$ $\qquad N_5 - 2^{0.75} = 0.00000$

$N_6 = 1.68179$ $\qquad \left(\dfrac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$

$N_7 = 2.37841$ $\qquad \dfrac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$



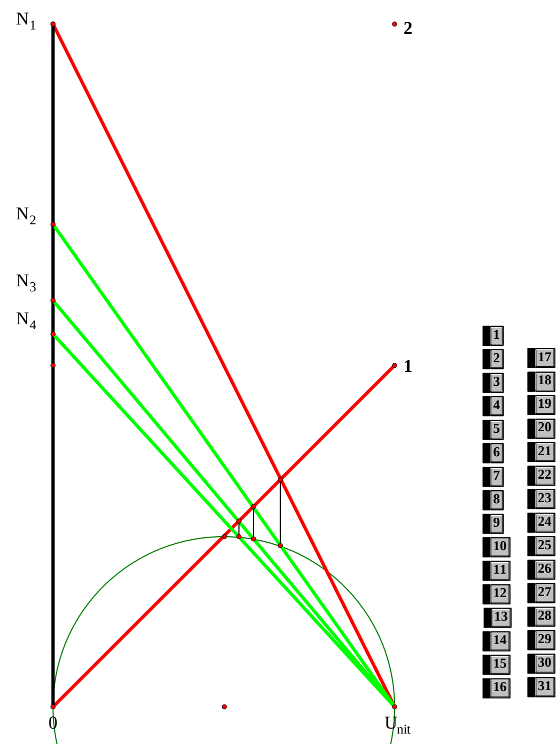
Unit = 1.00000

N_1 = 2.00000

N_2 = 1.41421 $N_1^{0.5}$-N_2 = 0.00000

N_3 = 1.18921 $N_1^{0.25}$-N_3 = 0.00000

N_4 = 1.09051 $N_1^{0.125}$-N_4 = 0.00000



Unit = 1.00000

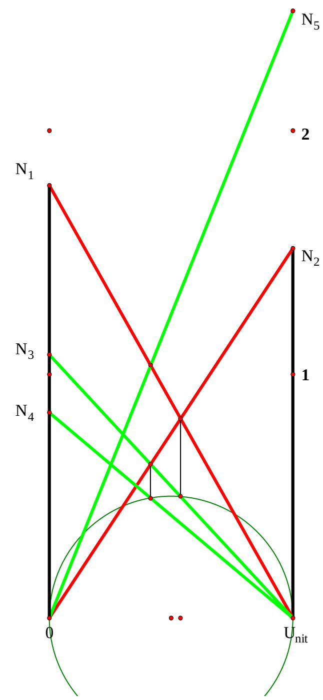
N_1 = 1.77542

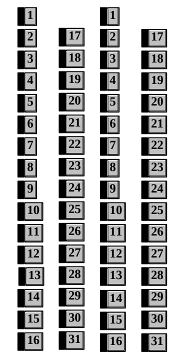
N_2 = 1.51695

N_3 = 1.08185 $\quad \frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$

N_4 = 0.84450 $\quad \frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$

N_5 = 2.48947 $\quad N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$





Unit = 1.00000

$\mathbf{N_1 = 1.00000}$

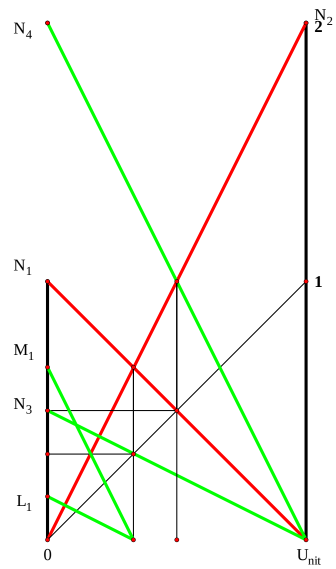
$\mathbf{N_2 = 2.00000}$ $\qquad \mathbf{\dfrac{N_1}{N_2} - N_3 = 0.00000}$

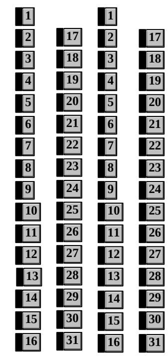
$\mathbf{N_3 = 0.50000}$

$\mathbf{N_4 = 2.00000}$ $\qquad \mathbf{N_1 \cdot N_2 - N_4 = 0.00000}$

$\mathbf{L_1 = 0.16667}$ $\qquad \mathbf{\dfrac{N_1^2}{(N_1+N_2)\cdot N_2} - L_1 = 0.00000}$

$\mathbf{M_1 = 0.66667}$ $\qquad \mathbf{\dfrac{N_1^2 \cdot N_2}{N_1+N_2} - M_1 = 0.00000}$





Unit = 1.00000

$N_1 = 2.00000$

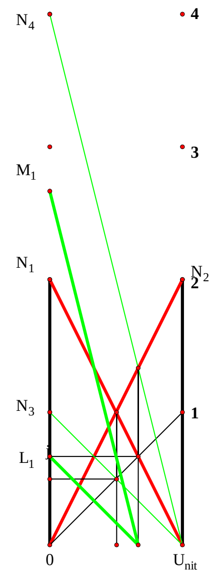
$N_2 = 2.00000$ $\frac{N_1}{N_2} - N_3 = 0.00000$

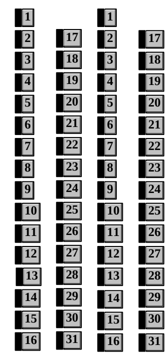
$N_3 = 1.00000$

$N_4 = 4.00000$ $N_1 \cdot N_2 - N_4 = 0.00000$

$L_1 = 0.66667$ $\frac{N_1^2}{N_2 \cdot (N_1 + 1)} - L_1 = 0.00000$

$M_1 = 2.66667$ $\frac{N_1^2 \cdot N_2}{N_1 + 1} - M_1 = 0.00000$





$U_1 = 1.00000$

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 1.00000$ $\quad \dfrac{N_1}{N_2} = 1.00000$

$N_4 = 4.00000$ $\quad N_1 \cdot N_2 - N_4 = 0.00000$

$U[1] / U_2 = 0.25000$

$N_5 = 2.00000$

$N_6 = 2.00000$

$N_7 = 0.25000$ $\quad \dfrac{N_1^2}{N_2^4} - N_7 = 0.00000$

$N_8 = 16.00000$ $\quad N_2^4 - N_8 = 0.00000$

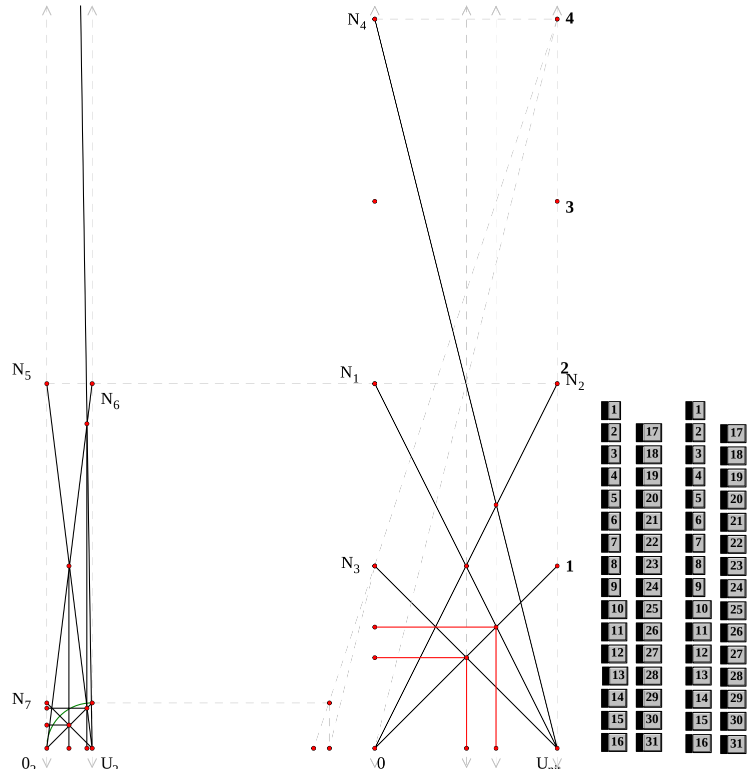
$U_2 = 1.00000$

$N_{25} = 8.00000$ $\quad N_2^3 - N_{25} = 0.00000$

$N_{26} = 8.00000$ $\quad \dfrac{N_2^4}{N_1} - N_{26} = 0.00000$

$N_{27} = 1.00000$ $\quad \dfrac{N_1}{N_2} - N_{27} = 0.00000$

$N_8 = 64.00000$ $\quad \dfrac{N_2^7}{N_1} - N_8 = 0.00000$



$$\text{Unit} = 1.00000$$

$$N_1 = 1.72159 \qquad \frac{N_1}{N_2} - N_3 = 0.00000$$

$$N_2 = 2.12500 \qquad N_1 \cdot N_2 - N_4 = 0.00000$$

$$N_3 = 0.81016 \qquad \frac{N_1}{N_2} = 0.81016$$

$$N_4 = 3.65838$$

$$N_5 = 0.57386$$

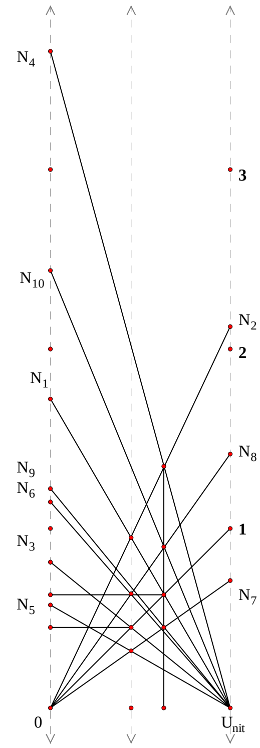
$$N_6 = 1.14773$$

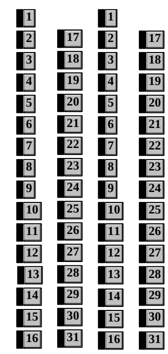
$$N_7 = 0.70833$$

$$N_8 = 1.41667$$

$$N_9 = 1.21946 \qquad N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$$

$$N_{10} = 2.43892 \qquad N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$$





Unit = 1.00000

$\mathbf{N_1 = 1.37056}$

$\mathbf{N_2 = 1.73096}$

$\mathbf{N_3 = 0.79179}$ $\quad \mathbf{\frac{N_1}{N_2} = 0.79179}$

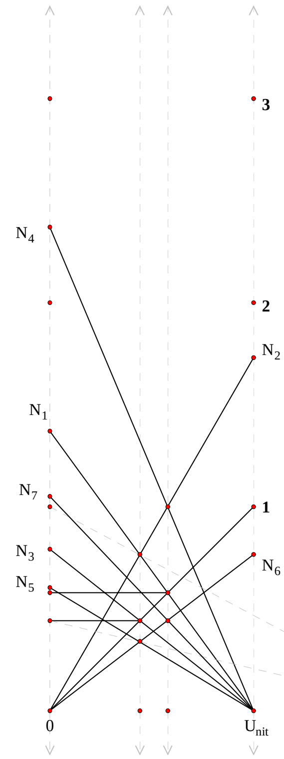
$\mathbf{N_4 = 2.37239}$ $\quad \mathbf{N_1 \cdot N_2 = 2.37239}$

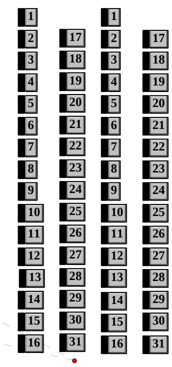
$$\mathbf{\frac{N_1}{N_2} = 0.79179}$$

$\mathbf{N_5 = 0.60565}$ $\quad \mathbf{N_1 \cdot \left(\frac{N_1}{N_1+N_2} \right) - N_5 = 0.00000}$

$\mathbf{N_6 = 0.76491}$ $\quad \mathbf{N_1 \cdot \left(\frac{N_2}{N_1+N_2} \right) - N_6 = 0.00000}$

$\mathbf{N_7 = 1.04835}$ $\quad \mathbf{N_1 \cdot N_2 \cdot \left(\frac{N_1}{N_1+N_2} \right) - N_7 = 0.00000}$





Unit = 1.00000

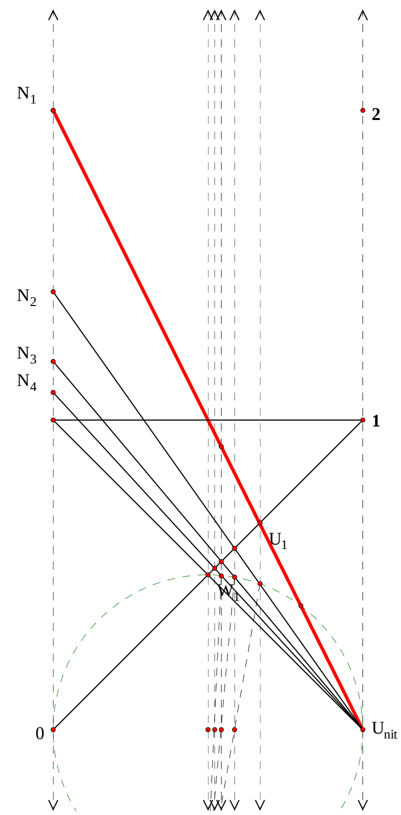
$N_1 = 2.00000$

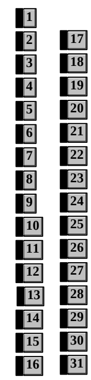
$N_2 = 1.41421$ $\quad N_1^{\frac{1}{2}} - N_2 = 0.00000$

$N_3 = 1.18921$ $\quad N_1^{\frac{1}{4}} - N_3 = 0.00000$

$N_4 = 1.09051$ $\quad N_1^{\frac{1}{8}} - N_4 = 0.00000$

etc.





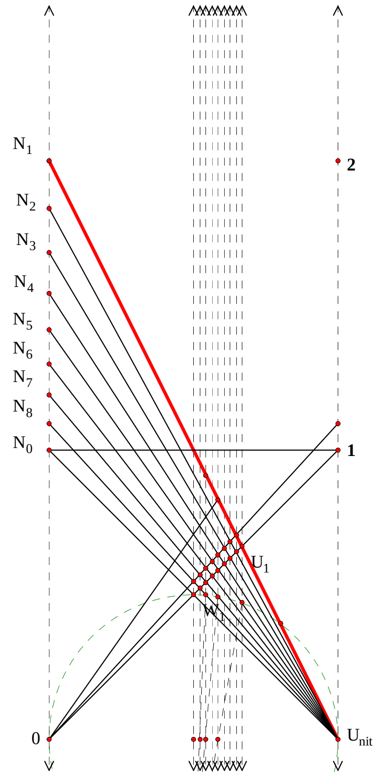
Unit = 1.00000

Eight root series.

One can project as long as they like.

$N_1 = 2.00000$	$N_1^{\frac{8}{8}} - N_1 = 0.00000$
$N_2 = 1.83401$	$N_1^{\frac{7}{8}} - N_2 = 0.00000$
$N_3 = 1.68179$	$N_1^{\frac{6}{8}} - N_3 = 0.00000$
$N_4 = 1.54221$	$N_1^{\frac{5}{8}} - N_4 = 0.00000$
$N_5 = 1.41421$	$N_1^{\frac{4}{8}} - N_5 = 0.00000$
$N_6 = 1.29684$	$N_1^{\frac{3}{8}} - N_6 = 0.00000$
$N_7 = 1.18921$	$N_1^{\frac{2}{8}} - N_7 = 0.00000$
$N_8 = 1.09051$	$N_1^{\frac{1}{8}} - N_8 = 0.00000$
$N_0 = 1.00000$	$N_1^{\frac{0}{8}} - N_0 = 0.00000$

etc.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

The computational speed by straight edge and compass outdoes long hand by factors. The computational accuracy exceeds that of any binary computer. The understanding as to what numbers mean cannot be outdone. Yet, instead of improving Euclid, they made a mess of it.

What led me to this solution was not Euclid, it was my own geometry play-especially doing the formula's and solution to a power line In order to solve for the power line, I actually had to know how to divide a square by a line. That coupled with the feeling that one should know the basic mathematical operations in geometry, as a starter made me break down and simply do it.

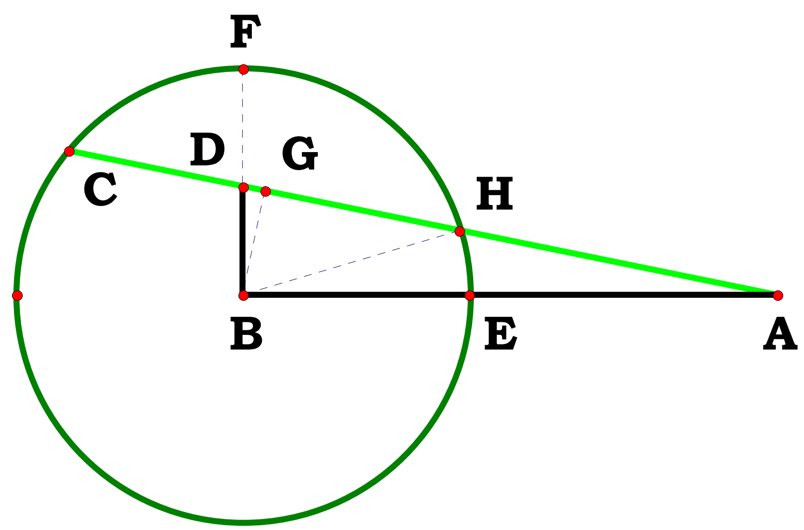
Geometry is still undefined. It is undefined because, as we know, a set can be constructed in only two ways, by enumeration and by definition. By saying that Euclidean Geometry only uses two tools, the straight edge and compass, we have enumerated its set. To define it, one would have to say, Geometry is that language by which we speak where there is one, and only one difference between two points.

This change not only defines Euclidean Geometry, but we find that it has been short changed for a long time. A straight edge does indeed give us one and only one difference between two points, and so does a compass, these are the unit and universe of discourse in the subject. However, there is yet one more tool, that tool that gives us every ratio in-between the unit and the universe, the ellipes. There is indeed one and only one difference between the two points called the foci of an ellipse.

If one can accept that, one can then understand my solution to the Delian Problem. A figure that gives one every aspect of an ellipse and one simply has to lay it down. Accepting that definition also takes something that is implied in Euclidean Geometry and makes it explicit, the ability to add, to do the math.

I hope you have fun.

080521



$N_1 := 3.86292 \qquad N_2 := 1.905 \qquad N_3 := .74482$

$AB := N_1 \qquad BE := N_2 \qquad BD := N_3$

$AD := \sqrt{AB^2 + BD^2} \qquad DG := \dfrac{BD^2}{AD} \qquad BH := BE$

$BG := \dfrac{AB \cdot BD}{AD} \qquad GH := \sqrt{BH^2 - BG^2}$

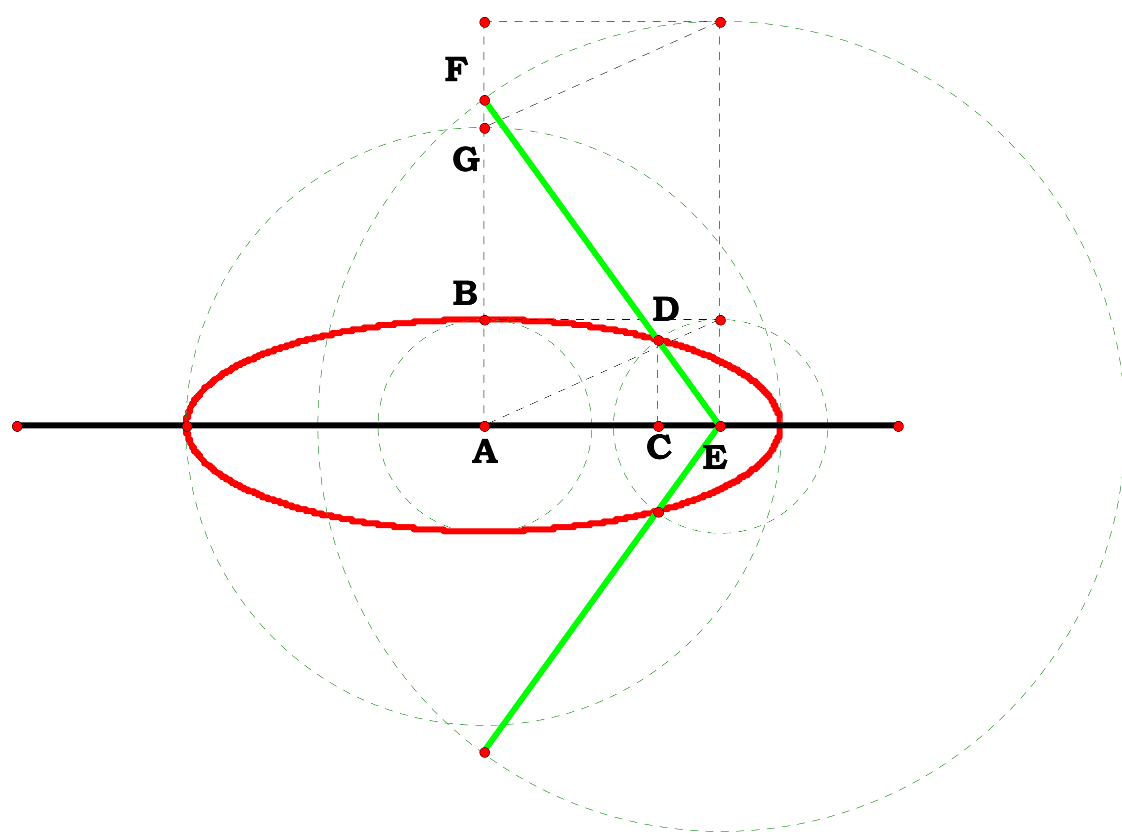
$AC := AD + GH - DG$

$$\frac{\sqrt{AB^2 \cdot BE^2 + BD^2 \cdot BE^2 - AB^2 \cdot BD^2} + AB^2}{\sqrt{AB^2 + BD^2}} - AC = 0$$

$$AC - \frac{N_1{}^2 + \sqrt{N_1{}^2 \cdot N_2{}^2 - N_1{}^2 \cdot N_3{}^2 + N_2{}^2 \cdot N_3{}^2}}{\sqrt{N_1{}^2 + N_3{}^2}} = 0$$

For a straight line ellipse and three givens.

a: AB, AC, CD



$N_1 := 1.40187 \quad N_2 := 2.31398 \quad N_3 := 1.13348$

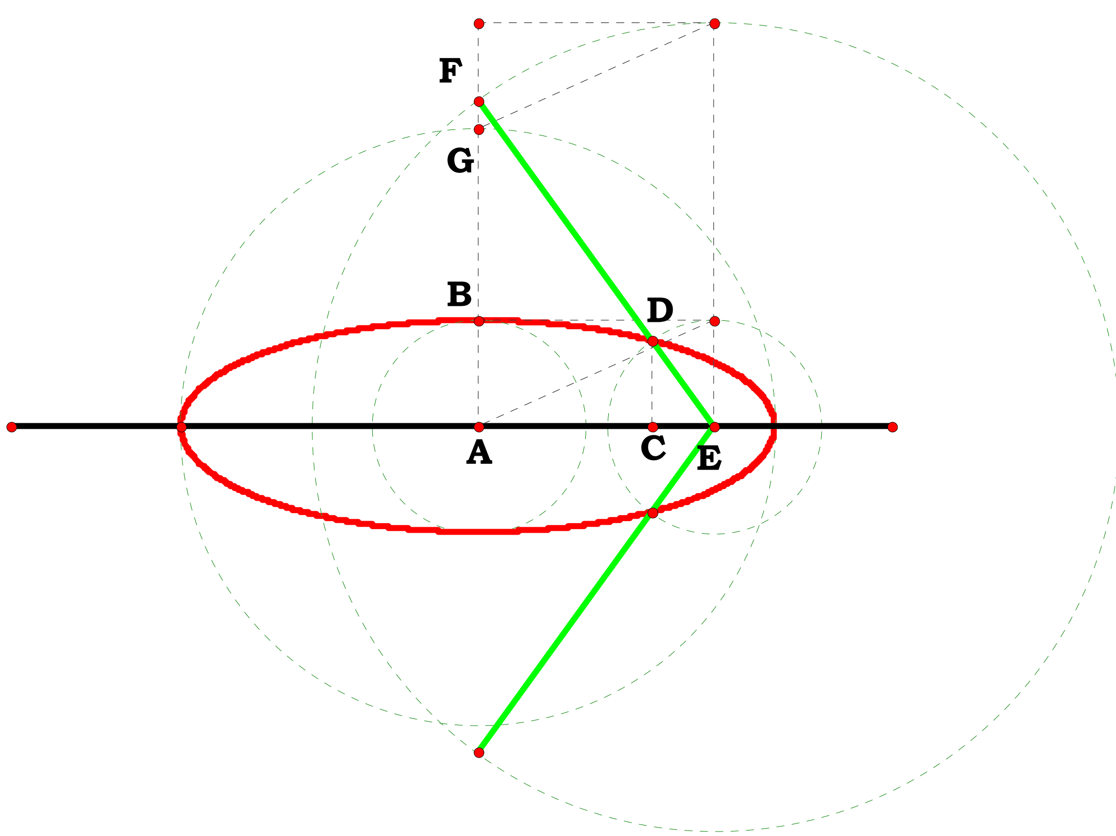
$AB := N_1 \quad AC := N_2 \quad CD := N_3$

$DE := AB \quad CE := \sqrt{DE^2 - CD^2} \quad DF := \frac{DE \cdot AC}{CE} \quad BG := DF - AB$

$$BG - N_1 \cdot \left(\frac{N_2}{\sqrt{N_1^{\,2} - N_3^{\,2}}} - 1 \right) = 0$$

For a straight line ellipse and three givens.

b: CE, AC, CD.



$$N_1 := .8249 \qquad N_2 := 2.31398 \qquad N_3 := 1.13348$$

$$CE := N_1 \qquad AC := N_2 \qquad CD := N_3$$

$$AB := \sqrt{CD^2 + CE^2}$$

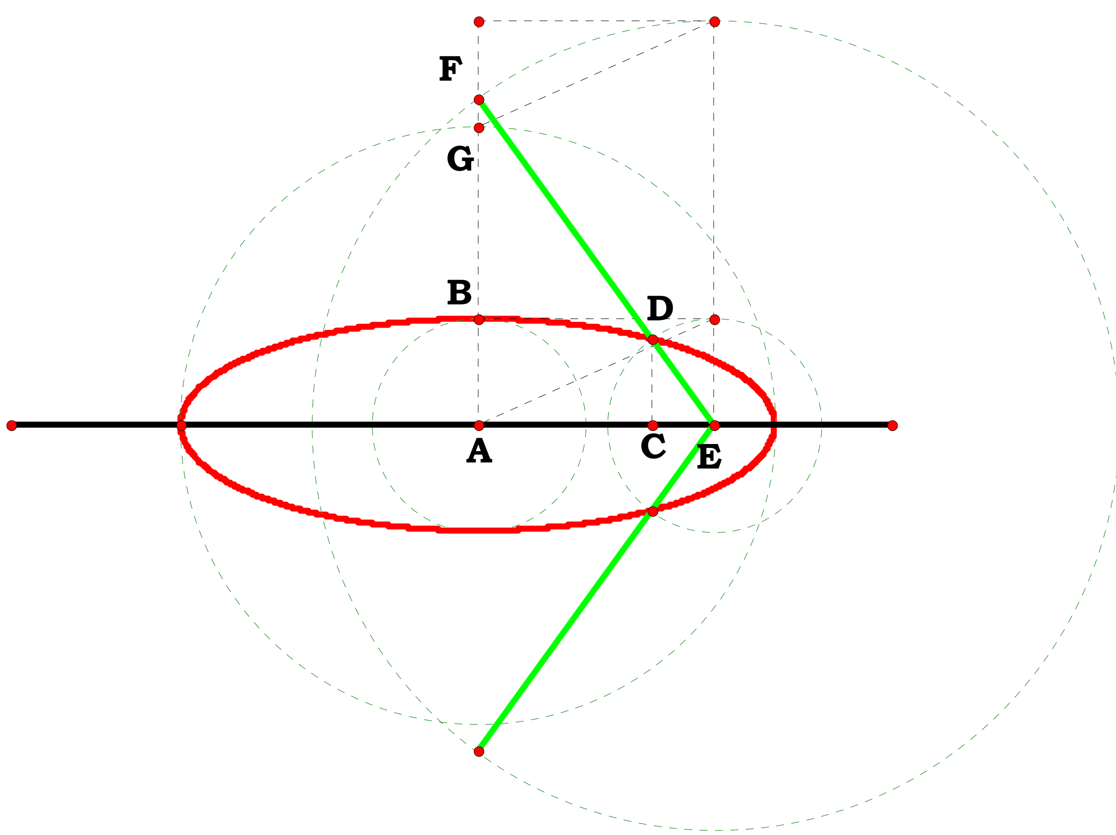
$$DE := AB \qquad DF := \frac{DE \cdot AC}{CE} \qquad BG := DF - AB$$

$$BG - \frac{\sqrt{N_1^{\,2} + N_3^{\,2}} \cdot (N_2 - N_1)}{N_1} = 0$$

060208 c.mcd

For a straight line ellipse and three givens.

c: AB, AC, CE



$N_1 := 1.40187 \quad N_2 := 2.31398 \quad N_3 := .8249$

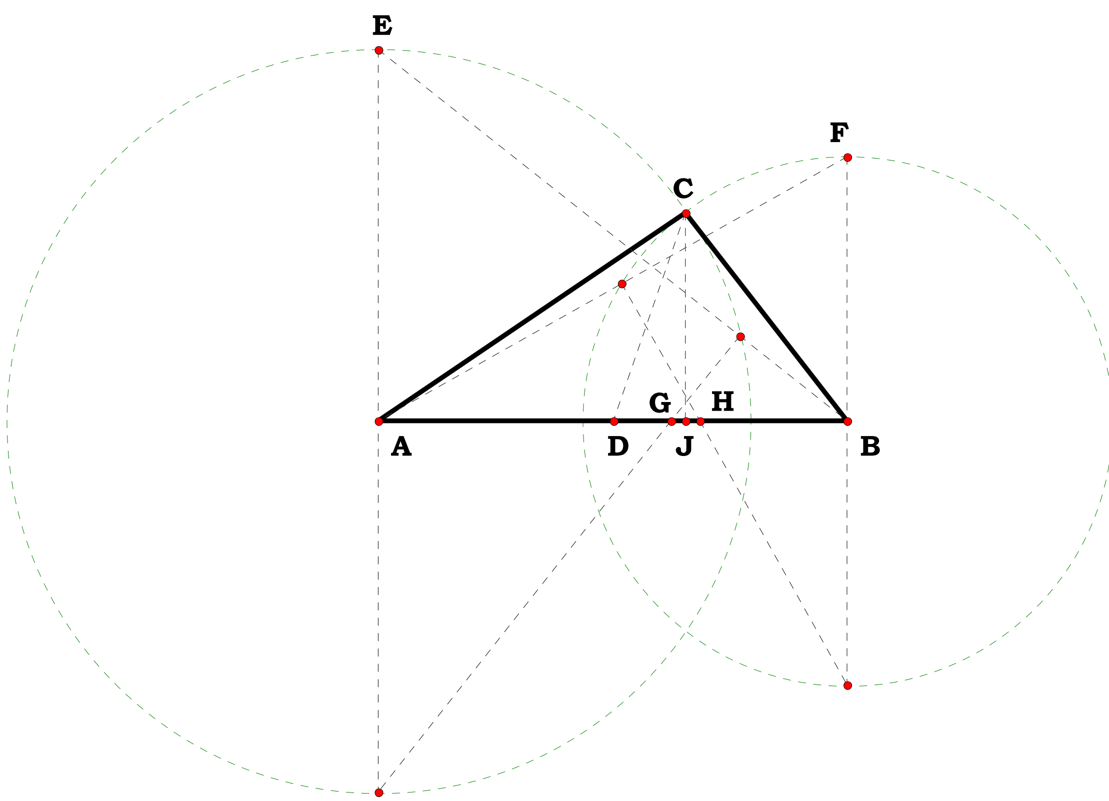
$AB := N_1 \quad AC := N_2 \quad CE := N_3$

$DE := AB \quad DF := \dfrac{DE \cdot AC}{CE} \quad BG := DF - AB$

$$BG - \frac{N_1 \cdot (N_2 - N_3)}{N_3} = 0$$

08092015 Pythagoras Revisited Again!

One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.



$$AB := 6.20183 \quad AC := 4.89358 \quad BC := 9.20468$$

$$GA := \frac{AC^2}{AB} \quad HB := \frac{BC^2}{AB} \quad GH := AB - (GA + HB) \quad JA := GA + \frac{GH}{2}$$

$$JB := HB + \frac{GH}{2} \quad CJ := \sqrt{AC^2 - JA^2} \quad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$$

$$JA - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \quad JB - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

$$CJ - \frac{\sqrt{(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)}}{2 \cdot AB} = 0$$

Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.

$$AJ := \sqrt{AC^2 - CJ^2} \quad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \quad BJ := \sqrt{BC^2 - CJ^2} \quad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2 \cdot AB} = 0$$

$AB := 2.38542$

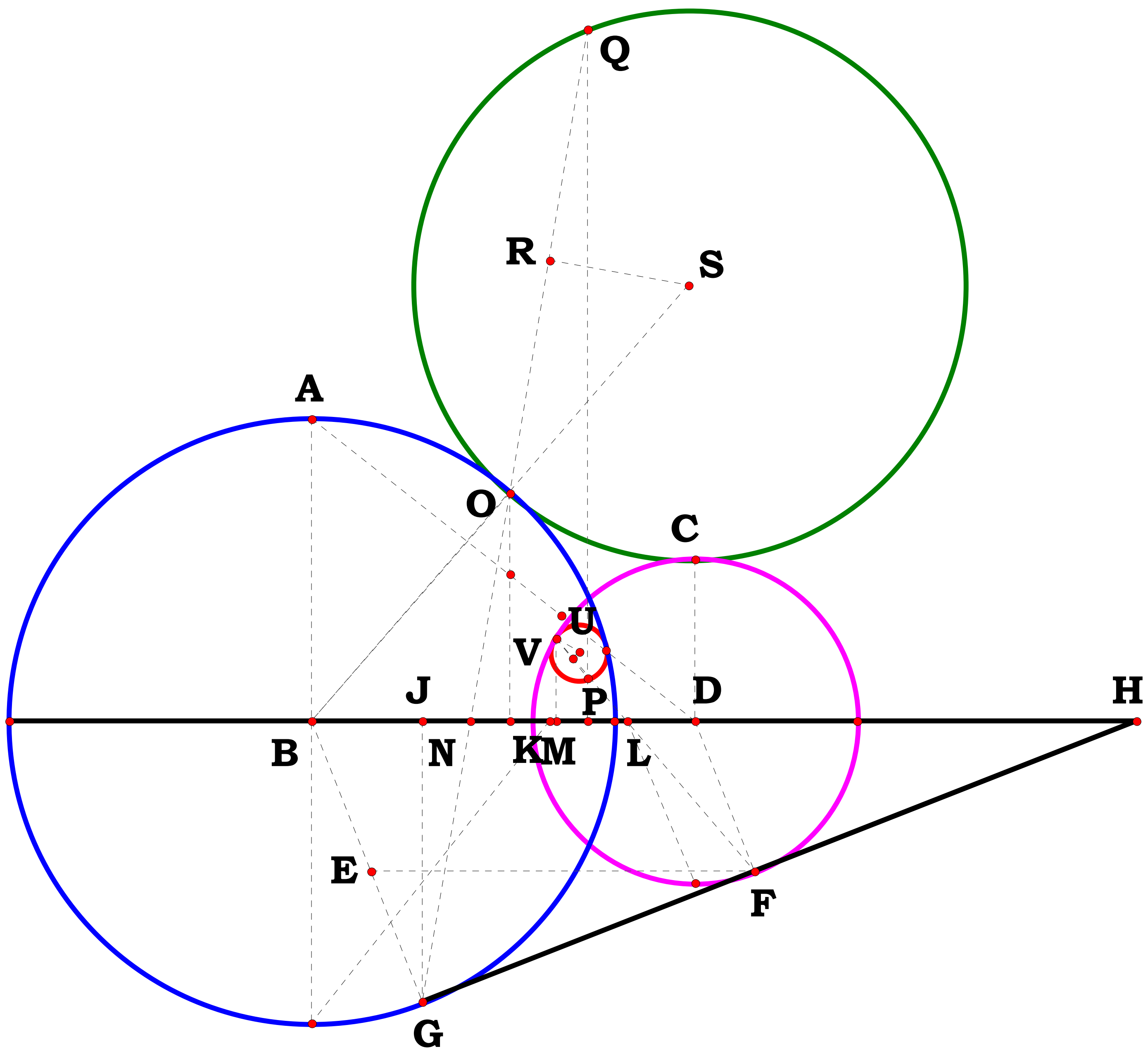
$CD := 1.72917$

$BD := 3.17708$

$dx := 1.22729$

$$\frac{N_3{}^2 + R_1{}^2 - R_2{}^2}{2 \cdot N_3}$$

From 042794 power line.



$$EF := BD \qquad EG := AB - CD \qquad BH := \frac{EF \cdot AB}{EG} \qquad BJ := \frac{AB^2}{BH} \qquad KL := \frac{2 \cdot AB}{dx}$$

$$JL := AB - BJ \qquad JK := JL - KL \qquad GJ := \sqrt{(2 \cdot AB - JL) \cdot JL}$$

$$KO := \sqrt{(2 \cdot AB - KL) \cdot KL} \qquad KN := \frac{JK \cdot KO}{GJ + KO} \qquad BP := \frac{BD^2 + AB^2 - CD^2}{2 \cdot BD}$$

$$JN := \frac{JK \cdot GJ}{GJ + KO} \qquad NP := BP - (BJ + JN) \qquad NO := \sqrt{KO^2 + KN^2}$$

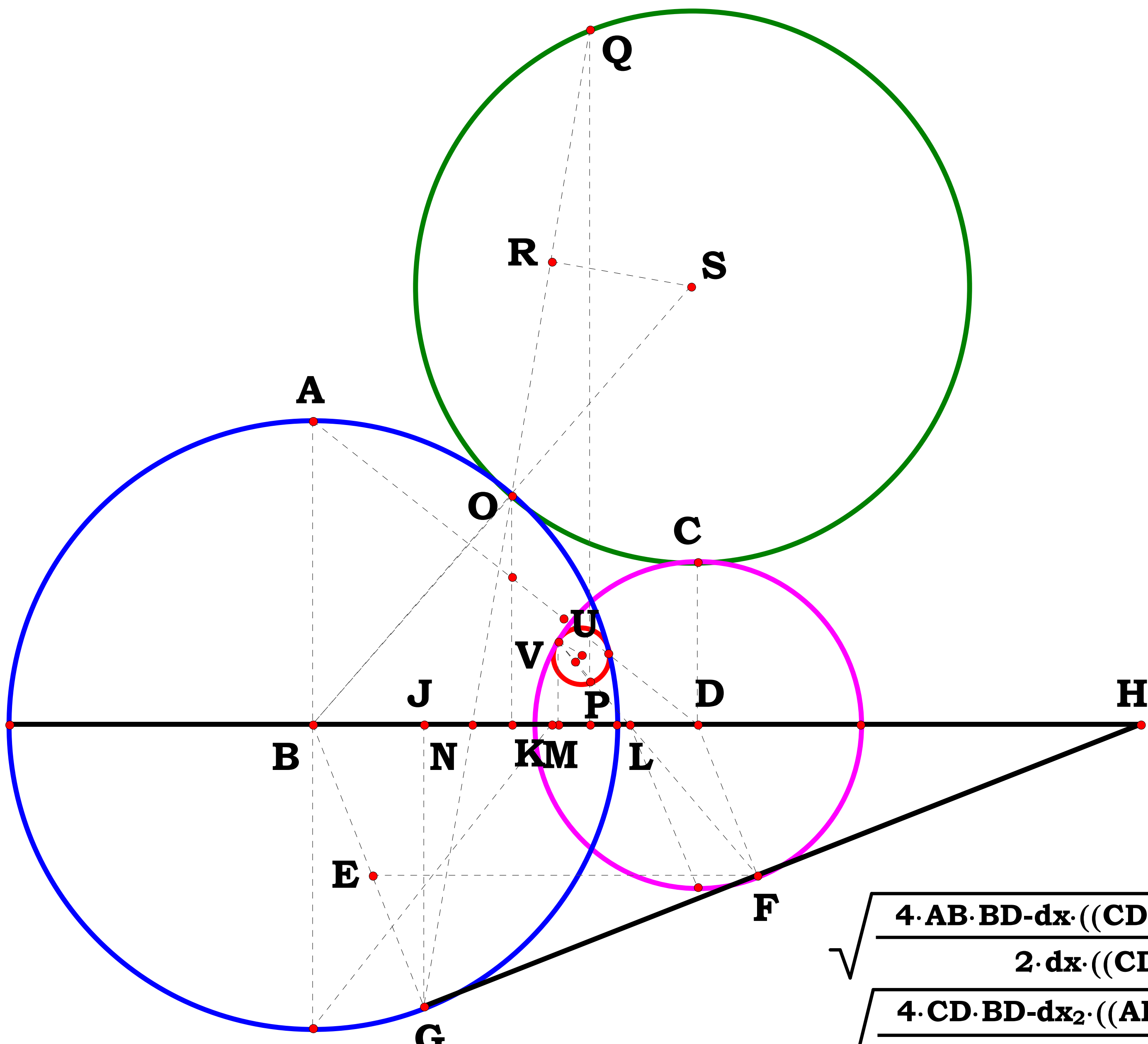
$$QN := \frac{NO \cdot NP}{KN} \qquad OQ := QN - NO \qquad OR := \frac{OQ}{2} \qquad GO := \sqrt{(GJ + KO)^2 + JK^2}$$

$$SO := \frac{AB \cdot OR \cdot 2}{GO} \qquad SO = -4.204157$$

$$R_1 := AB \qquad R_2 := CD \qquad D := BD$$

$$SO - \frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

$$\sqrt{\left[\frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D}\right]^2} = 4.204157$$



AB = 1.90625 in.
CD = 1.02083 in.
BD = 2.41667 in.
dx = 5.77984
SO = 1.73133 in.

KL = 0.65962 in.

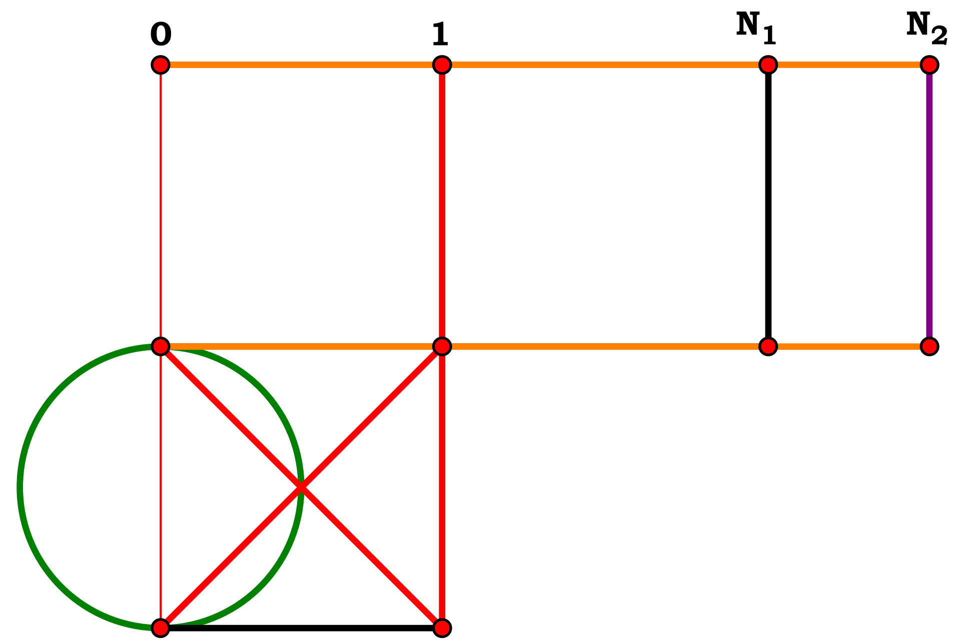
UV = 0.16880 in.
TM = 0.14146 in.
dx_2 = 14.43321

$$\sqrt{\left(\frac{4\cdot AB\cdot BD-dx\cdot((CD+BD)-AB)\cdot((CD+AB)-BD)}{2\cdot dx\cdot((CD+BD)-AB)-4\cdot BD}\right)^{2}}-SO = 0.00000\text{ in.}$$

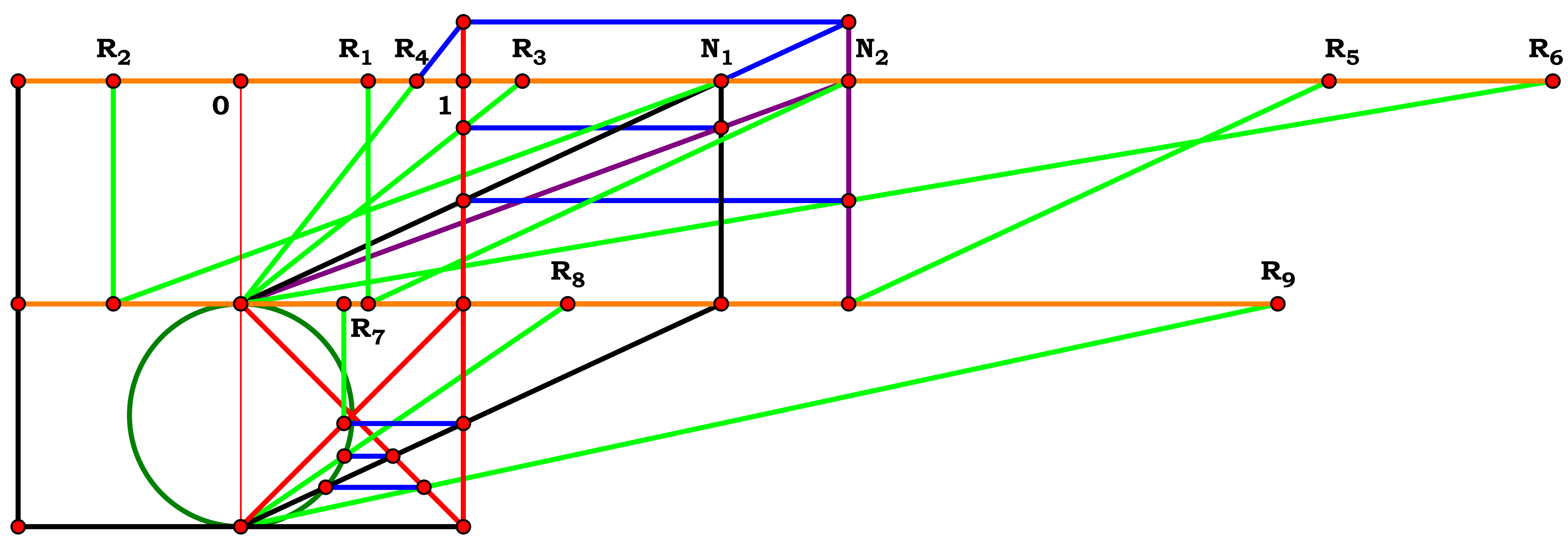
$$\sqrt{\left(\frac{4\cdot CD\cdot BD-dx_2\cdot((AB+BD)-CD)\cdot((AB+CD)-BD)}{2\cdot dx_2\cdot((AB+BD)-CD)-4\cdot BD}\right)^{2}}-UV = 0.00000\text{ in.}$$

Basic Analog Mathematics.

Let 0 to 1 be the given Unit and N1 and N2 be any two given differences:



With the given analog (figure), it is required to render the products of these two differences using the paradigms sum, differences, and ratio:--



$N_2 - N_1 - R_1 = 0.00000$

$N_1 - N_2 - R_2 = 0.00000$

$N_1 + N_2 - R_5 = 0.00000$

$\frac{N_2}{N_1} - R_3 = 0.00000$

$\frac{N_1}{N_2} - R_4 = 0.00000$

$\frac{1}{N_1} - R_7 = 0.00000$

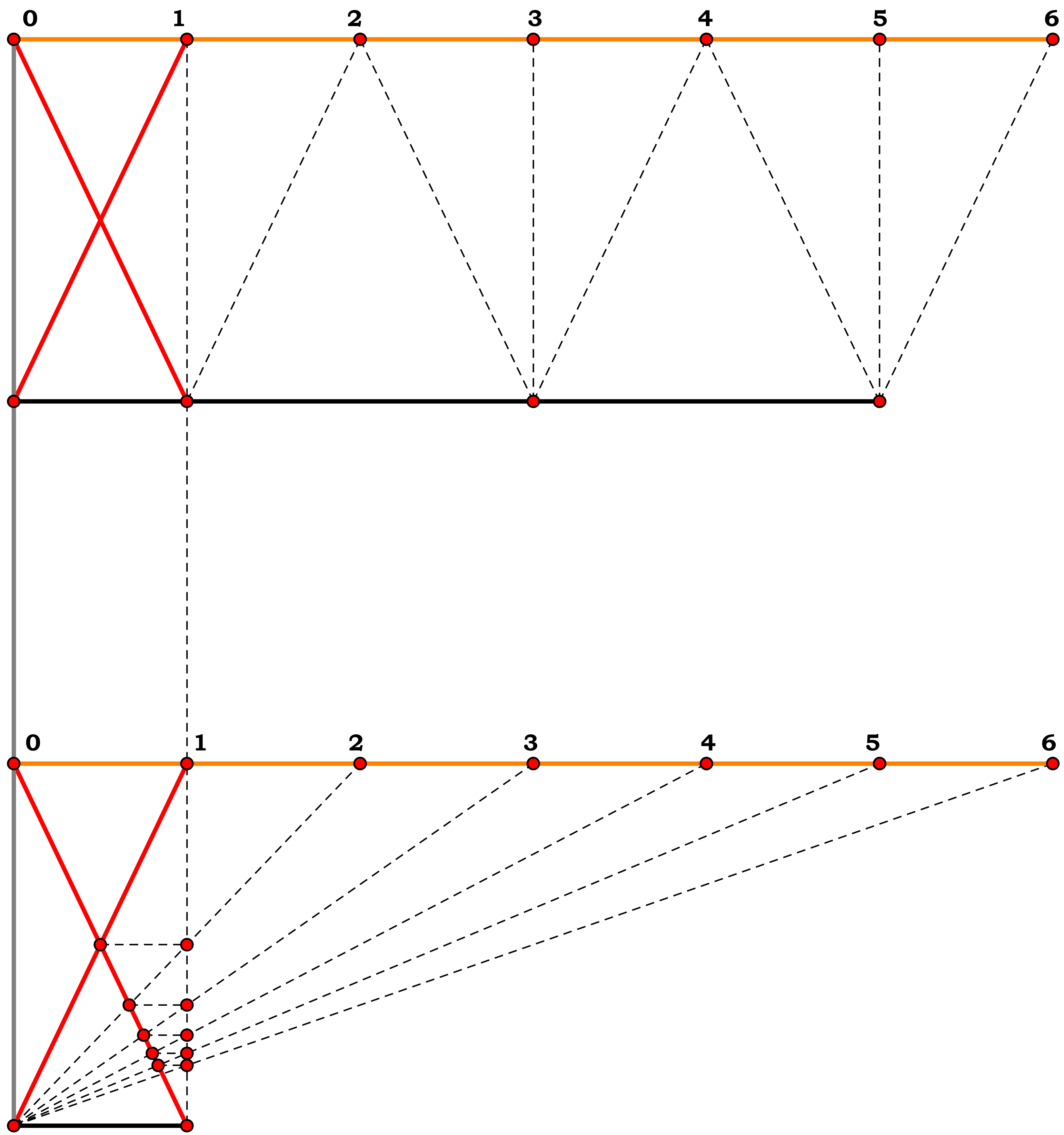
$N_1 \cdot N_2 - R_6 = 0.00000$

$\sqrt{N_1} - R_8 = 0.00000$

${N_1}^2 - R_9 = 0.00000$

Complete Induction.

Complete induction, if the term is to make any sense at all, simply means recursion. These display just two methods of costructing an arithmetic ratio and series in analogically.



Addition and Subtraction.

$N_1 = 1.67002$

$N_2 = 3.75881$

$A = 5.42882$

$B = 5.42882$

$N_1 + N_2 - A = 0.00000$

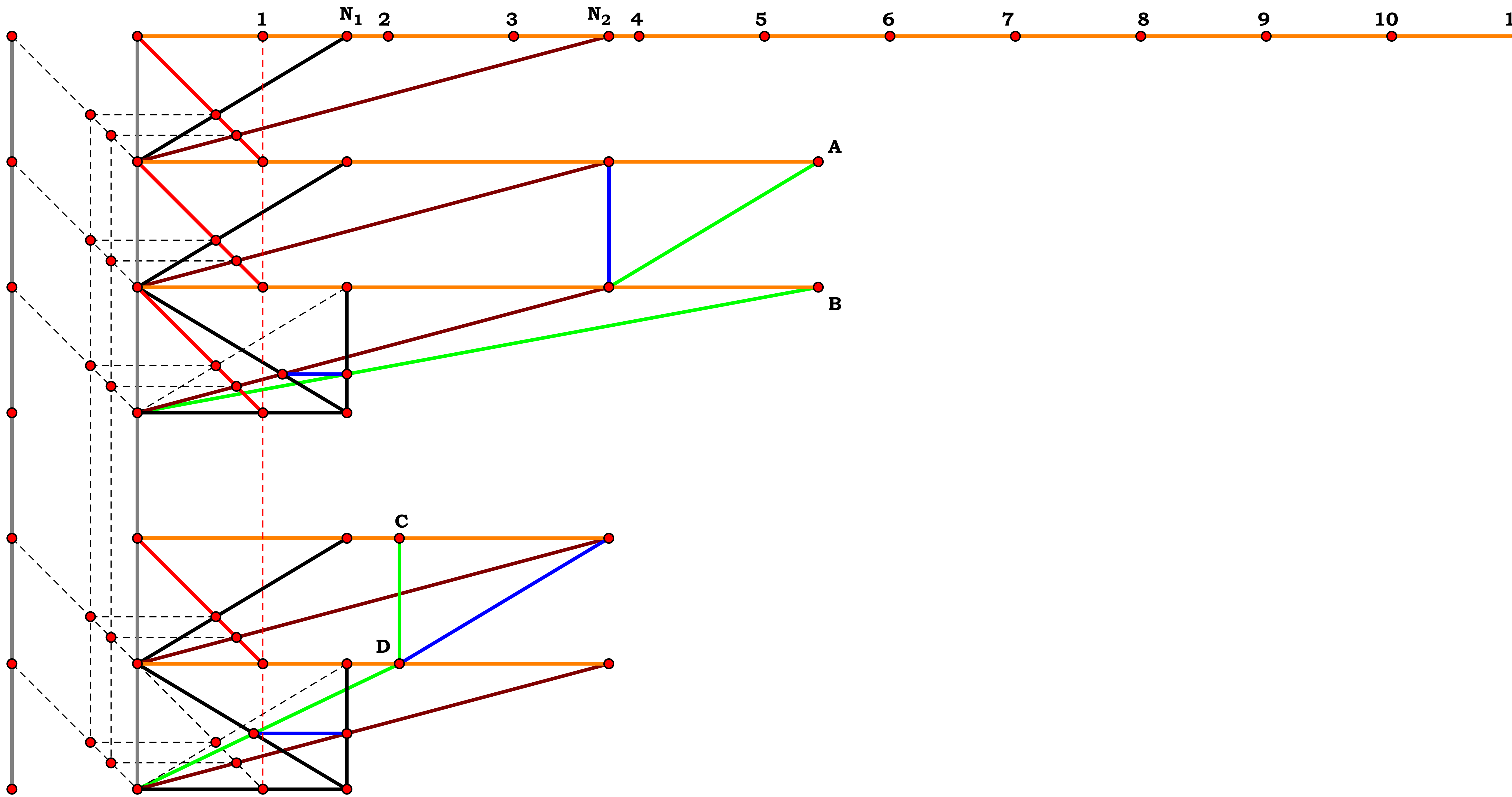
$N_1 + N_2 - B = 0.00000$

$C = 2.08879$

$D = 2.08879$

$N_2 - N_1 - C = 0.00000$

$N_2 - N_1 - D = 0.00000$



Multiplication and Division

$N_1 = 2.30957$

$N_2 = 4.43080$

$A = 10.23326$

$B = 10.23326$

$C = 10.23326$

$D = 10.23326$

$N_1 \cdot N_2 - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$N_1 \cdot N_2 - C = 0.00000$

$N_1 \cdot N_2 - D = 0.00000$

$E = 0.52125$

$F = 0.52125$

$G = 0.52125$

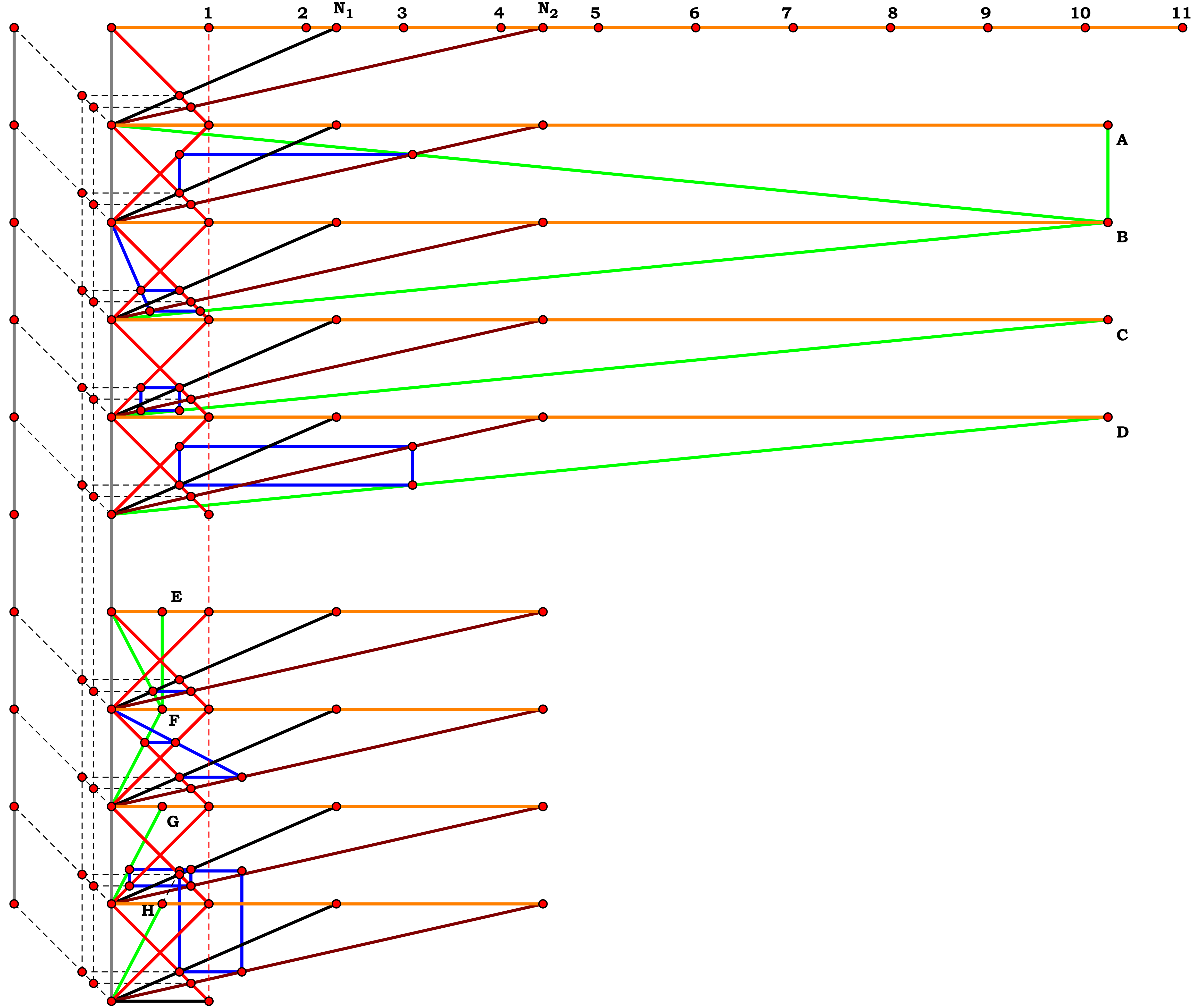
$H = 0.52125$

$\frac{N_1}{N_2} - E = 0.00000$

$\frac{N_1}{N_2} - F = 0.00000$

$\frac{N_1}{N_2} - G = 0.00000$

$\frac{N_1}{N_2} - H = 0.00000$



One can pipe results from glyph to glyph as shown, and also, the glyph itself does not have to be the same size as the others, the pipe can proportion individual glyphs for the whole system. Piping allows for the construction of complex computational structures.

Square, Root and Reciprocal.

N = 1.68576

A = 2.84179

B = 2.84179

C = 2.84179

D = 2.84179

E = 2.84179

F = 2.84179

G = 1.29837

H = 0.59320

$N^2 - A = 0.00000$

$N^2 - B = 0.00000$

$N^2 - C = 0.00000$

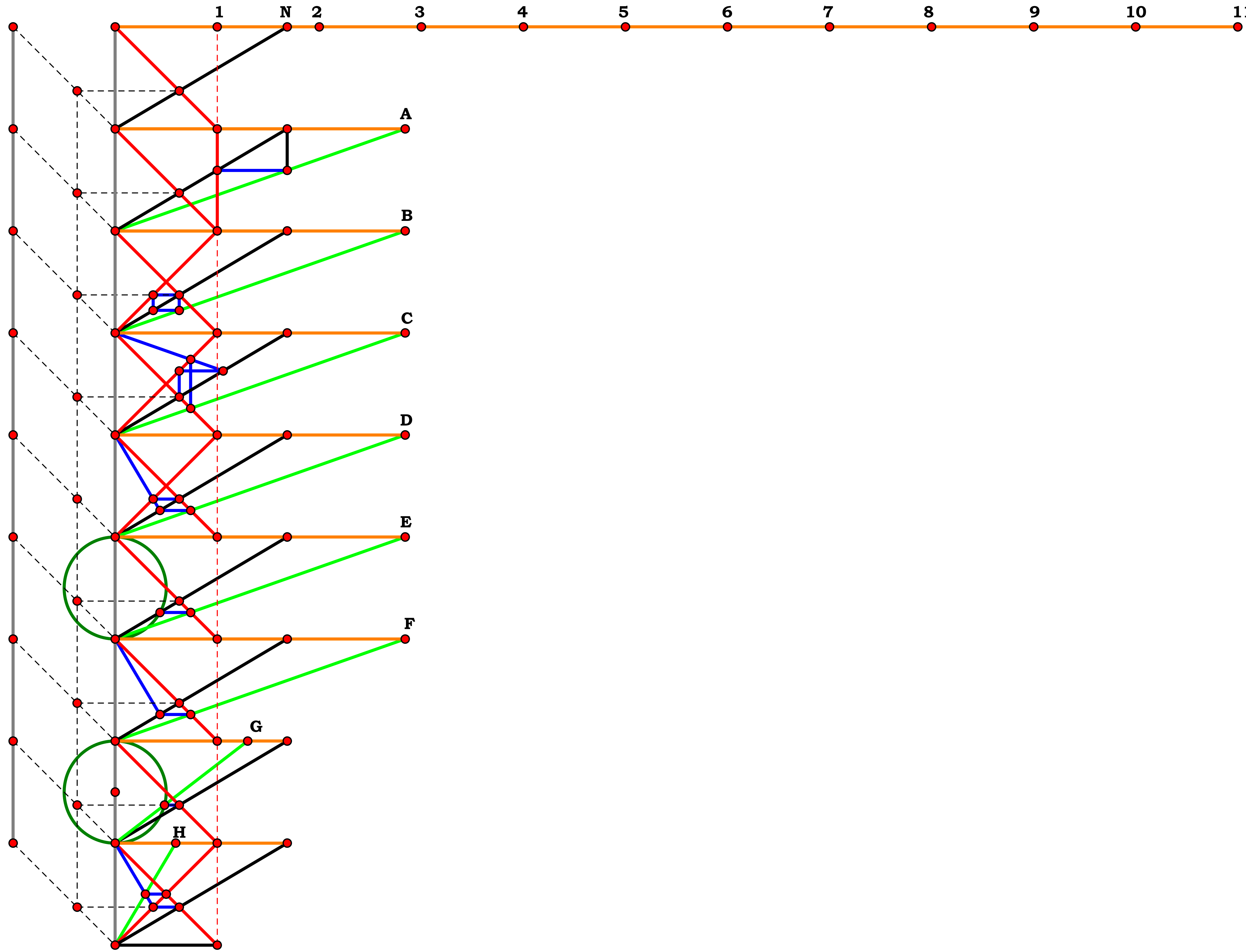
$N^2 - D = 0.00000$

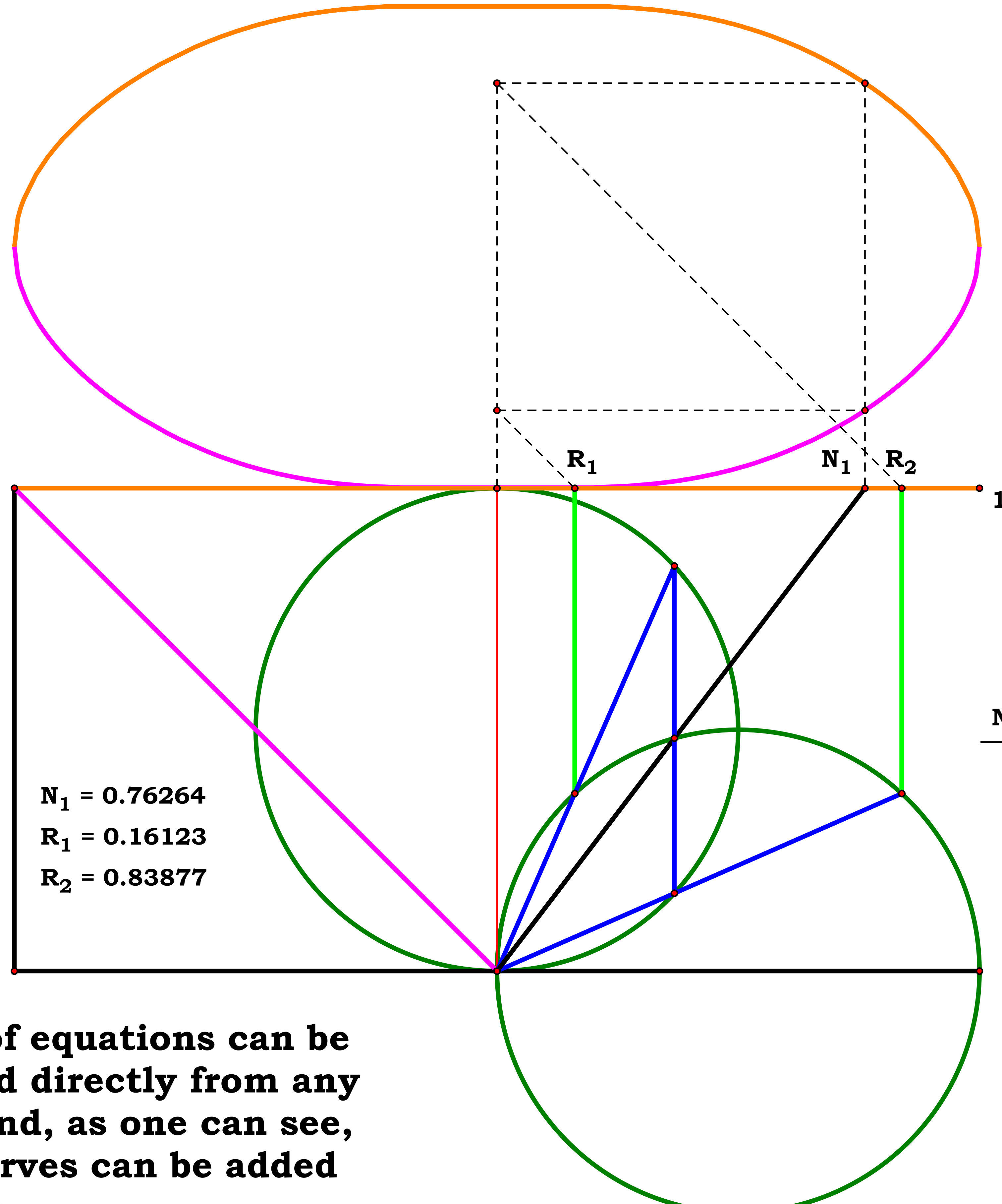
$N^2 - E = 0.00000$

$N^2 - F = 0.00000$

$\sqrt{N} - G = 0.00000$

$\frac{1}{N} - H = 0.00000$





3OBT10AR0

3OBT10AR3

$$\frac{N_1^{\,2} + 1 - \sqrt{2 \cdot N_1^{\,2} + 1 - 3 \cdot N_1^{\,4}}}{2 \cdot N_1^{\,2} + 2} = 0.16123$$

$$\frac{N_1^{\,2} + 1 + \sqrt{2 \cdot N_1^{\,2} - 3 \cdot N_1^{\,4} + 1}}{2 \cdot N_1^{\,2} + 2} = 0.83877$$

$$\frac{N_1^{\,2} + 1 + \sqrt{2 \cdot N_1^{\,2} - 3 \cdot N_1^{\,4} + 1}}{2 \cdot N_1^{\,2} + 2} - \frac{N_1^{\,2} + 1 - \sqrt{2 \cdot N_1^{\,2} + 1 - 3 \cdot N_1^{\,4}}}{2 \cdot N_1^{\,2} + 2} = 0.67755$$

$$\frac{\sqrt{2 \cdot N_1^{\,2} - 3 \cdot N_1^{\,4} + 1}}{N_1^{\,2} + 1} = 0.67755$$

Curves of equations can be projected directly from any glyph. And, as one can see, these curves can be added together.

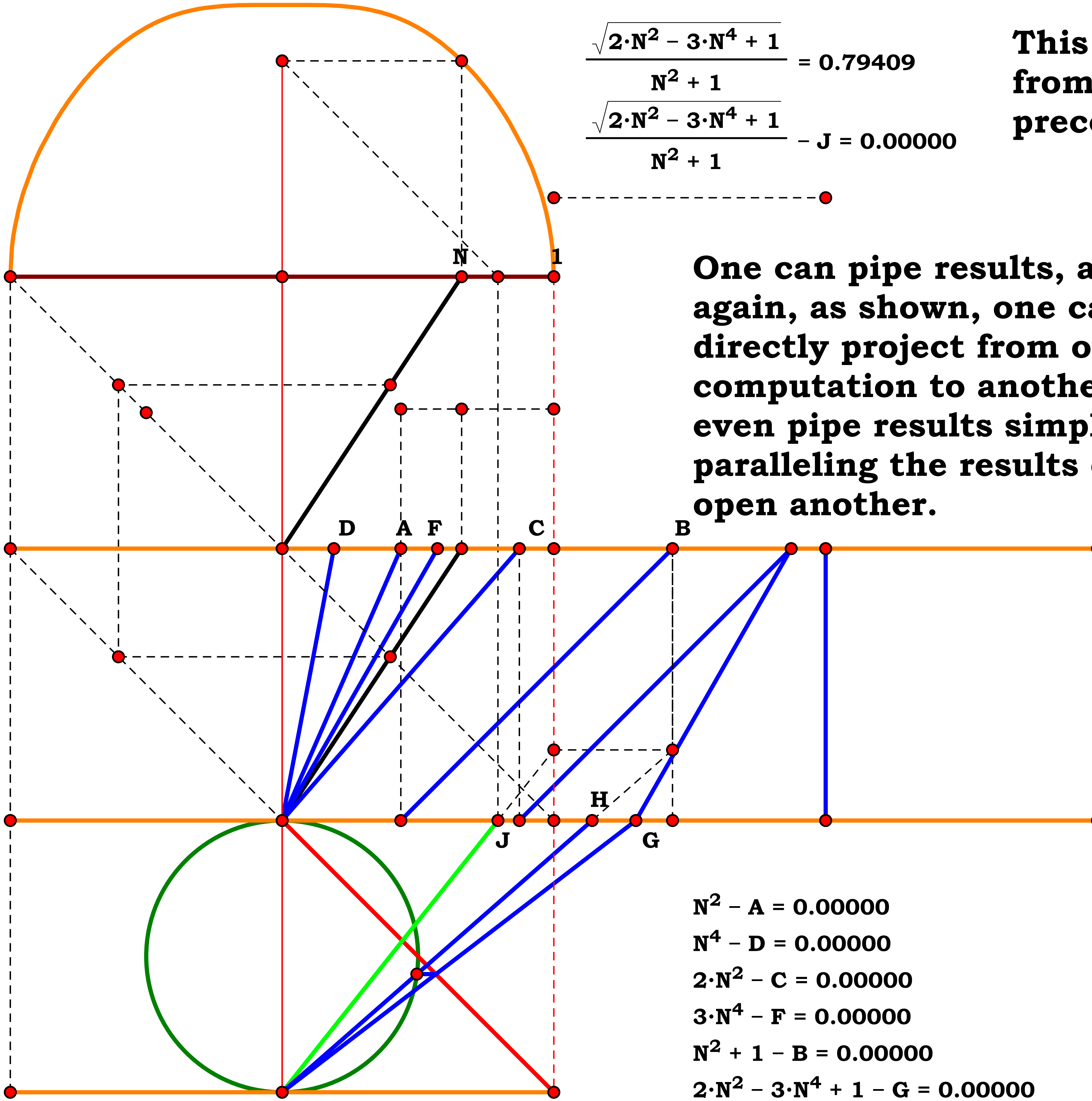
This figure is one of the hundreds, or thousands of glyphs I demonstrate in BAM. Many glyphs even function by moving parts relative to other values. A single glyph can be a complex equation in of itself.

$$\frac{\sqrt{2 \cdot N^2 - 3 \cdot N^4 + 1}}{N^2 + 1} = 0.79409$$

$$\frac{\sqrt{2 \cdot N^2 - 3 \cdot N^4 + 1}}{N^2 + 1} - J = 0.00000$$

This figure is constructed from an equation on the preceding page.

One can pipe results, as shown, or again, as shown, one can sometimes directly project from one line of computation to another. One can even pipe results simply by paralleling the results on one line to open another.



N = 0.66074
A = 0.43658
B = 1.43658
C = 0.87315
D = 0.19060
F = 0.57180
G = 1.30136
H = 1.14077
J = 0.79409

$N^2 - A = 0.00000$

$N^4 - D = 0.00000$

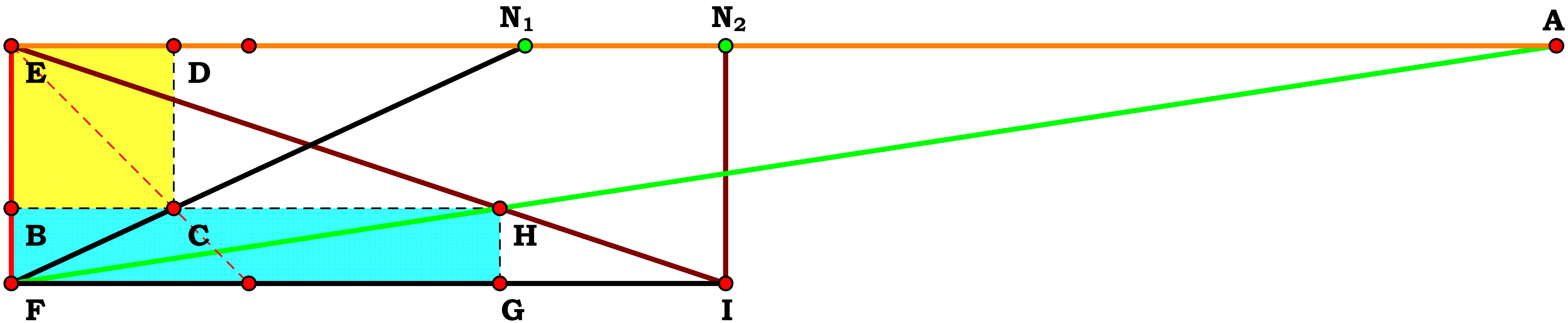
$2 \cdot N^2 - C = 0.00000$

$3 \cdot N^4 - F = 0.00000$

$N^2 + 1 - B = 0.00000$

$2 \cdot N^2 - 3 \cdot N^4 + 1 - G = 0.00000$

$\sqrt{2 \cdot N^2 - 3 \cdot N^4 + 1} - H = 0.00000$



$N_1 = 2.16287$

$N_2 = 3.00688$

$A = 6.50351$

$N_1 \cdot N_2 - A = 0.00000$

$$\frac{\text{Area } \triangle FN_1E}{\text{Area BCDE}} - \frac{\text{Area } \triangle EIF}{\text{Area BFGH}} = 0.00000$$

$$\frac{\text{Area } \triangle EIF}{\text{Area } \triangle FN_1E} - \frac{\text{Area BFGH}}{\text{Area BCDE}} = 0.00000$$

$$\frac{N_2}{N_1} - \frac{\text{Area BFGH}}{\text{Area BCDE}} = 0.00000$$

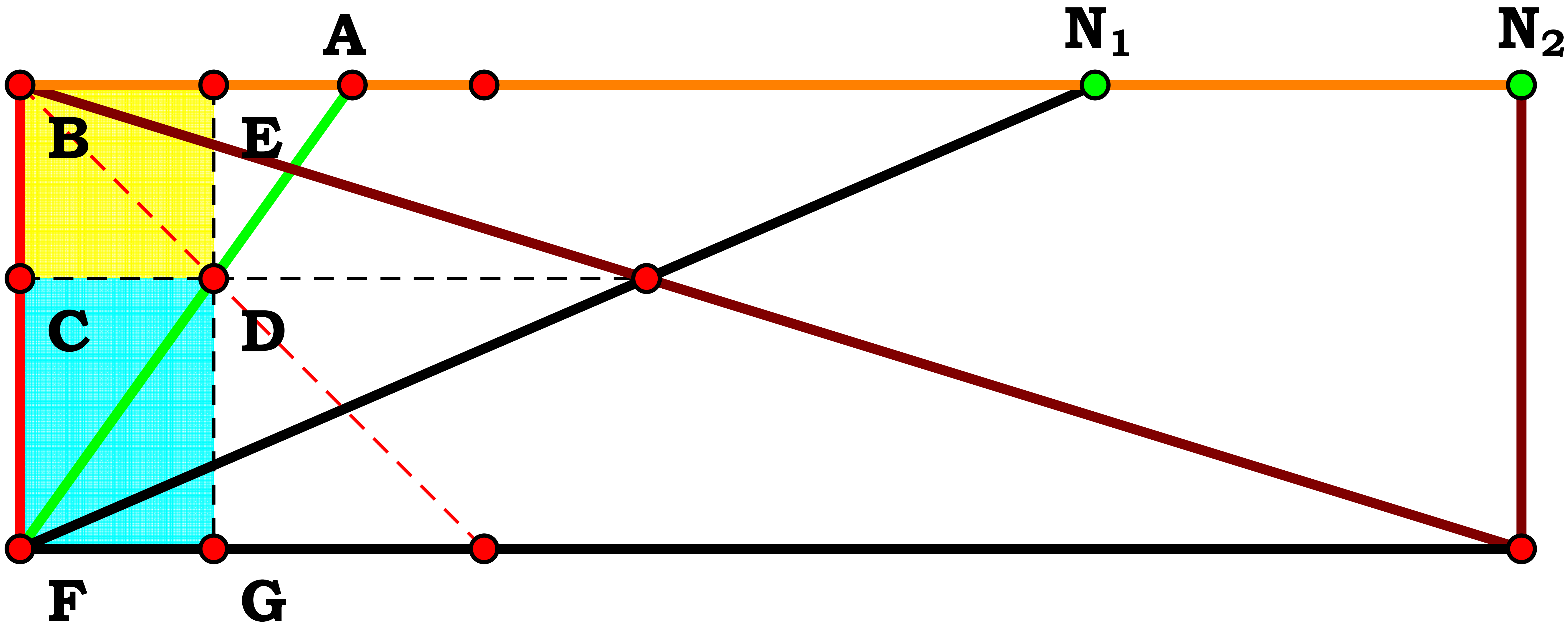
$$\frac{N_2}{N_1} - \frac{\text{Area } \triangle EIF}{\text{Area } \triangle FN_1E} = 0.00000$$

Area $\triangle FN_1E = 14.89003\ cm^2$

Area BCDE $= 6.43862\ cm^2$

Area BFGH $= 8.95113\ cm^2$

Area $\triangle EIF = 20.70049\ cm^2$



$N_1 = 2.31671$

$N_2 = 3.23544$

$A = 0.71604$

$$\frac{N_1}{N_2} - A = 0.00000$$

$\text{Area BCDE} = 2.36319 \text{ cm}^2$

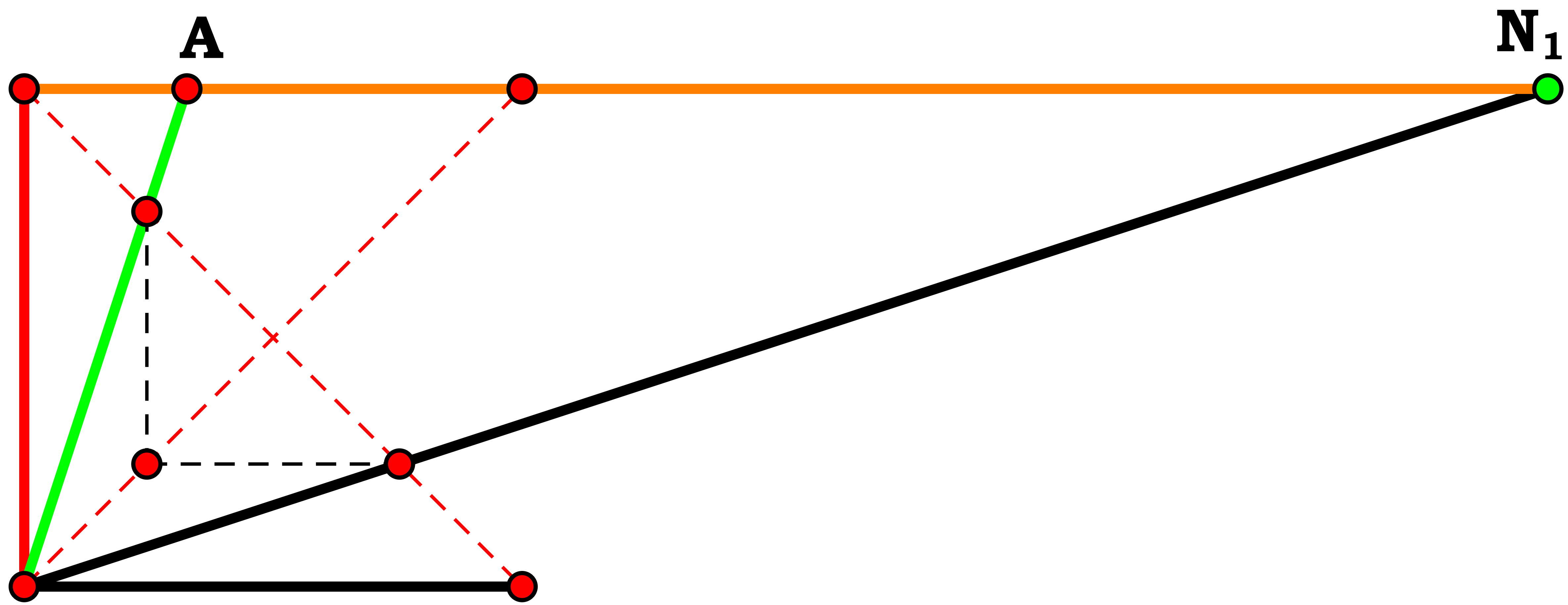
$\text{Area DCFG} = 3.30036 \text{ cm}^2$

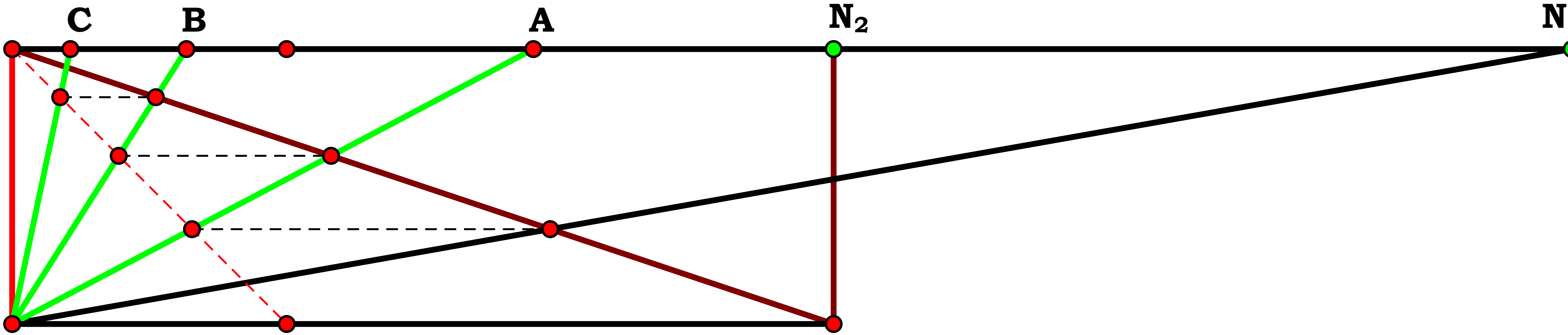
$$\frac{\text{Area BCDE}}{\text{Area DCFG}} - \frac{N_1}{N_2} = 0.00000$$

$$N_1 = 3.06033$$

$$A = 0.32676$$

$$\frac{1}{N_1} - A = 0.00000$$





$N_1 = 5.68273$

$N_2 = 2.99267$

$$\frac{N_1}{N_2} - A = 0.00000$$

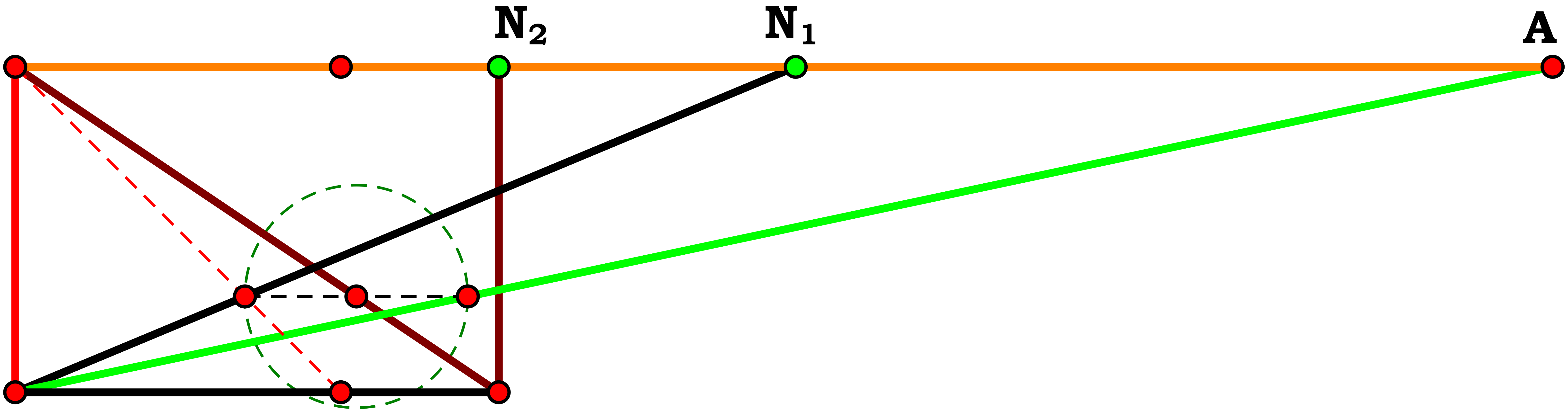
$$\frac{N_1}{N_2{}^2} - B = 0.00000$$

$$\frac{N_1}{N_2{}^3} - C = 0.00000$$

$A = 1.89888$

$B = 0.63451$

$C = 0.21202$

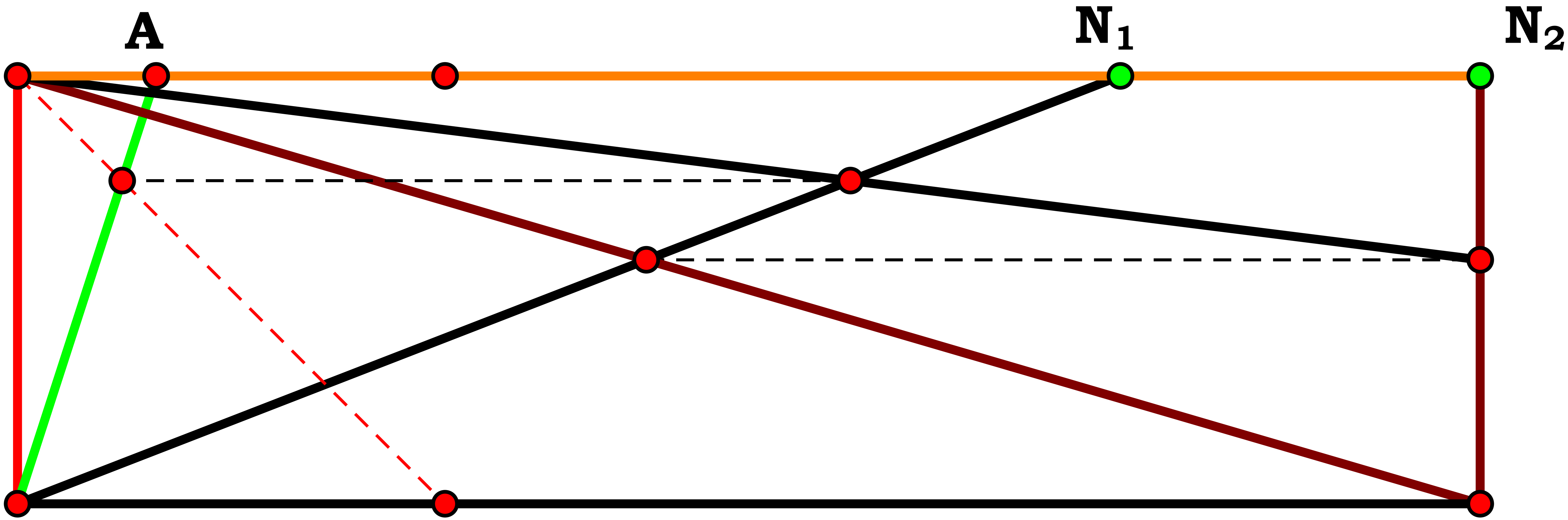


$N_1 = 2.39690$

$N_2 = 1.48490$

$A = 4.72142$

$2 \cdot (N_1 \cdot N_2) - N_1 - A = 0.00000$

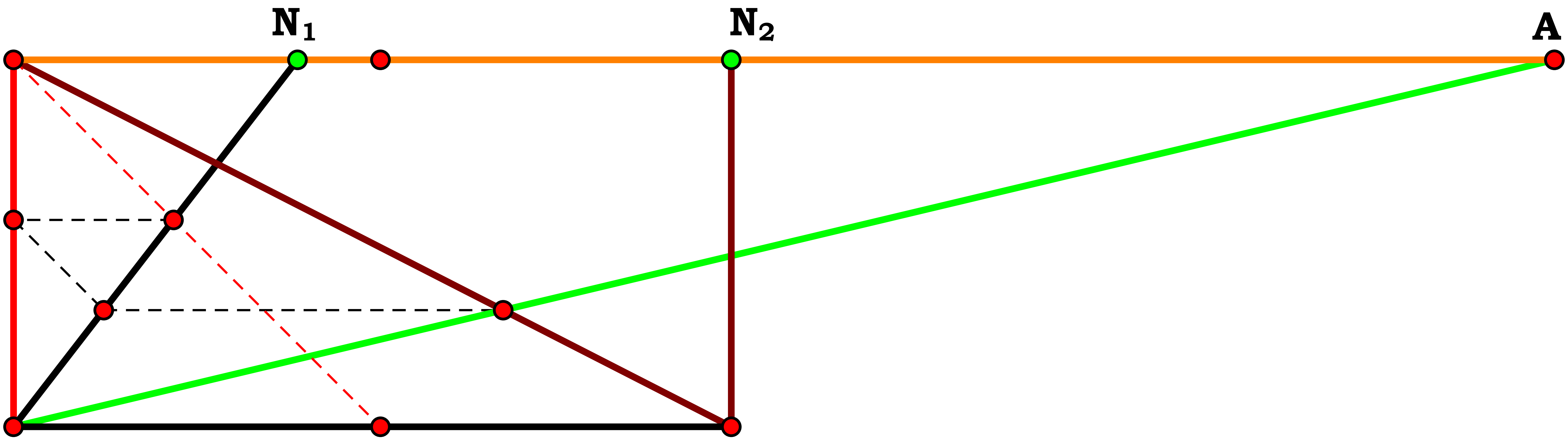


$$N_1 = 2.57987$$

$$N_2 = 3.42162$$

$$A = 0.32412$$

$$\frac{N_1^{\,2}}{(N_2 + N_1)\cdot N_2} - A = 0.00000$$

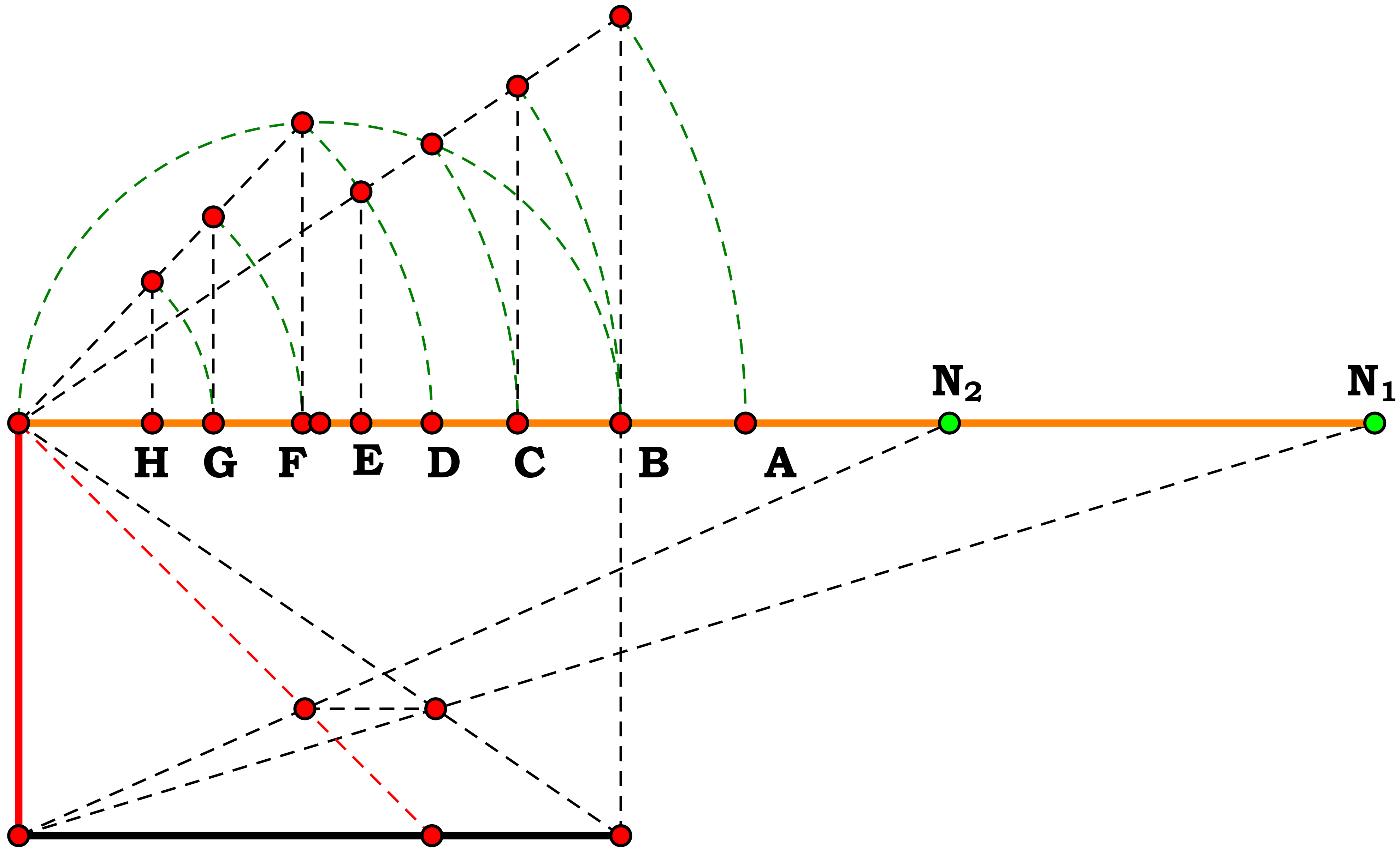


$N_1 = 0.77392$

$N_2 = 1.95611$

$A = 4.19932$

$2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2 - A = 0.00000$



$N_1 = 3.28137$

$N_2 = 2.25211$

$\frac{N_1}{N_2} = 1.45702$

$\left(\frac{N_1}{N_2}\right)^{1.5} - A = 0.00000$

$\frac{N_1}{N_2} - B = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{0.5} - C = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{0} - D = 0.00000$

$A = 1.75873$

$B = 1.45702$

$C = 1.20707$

$D = 1.00000$

$\left(\frac{N_1}{N_2}\right)^{-0.5} - E = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{-1} - F = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{-2} - G = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{-3} - H = 0.00000$

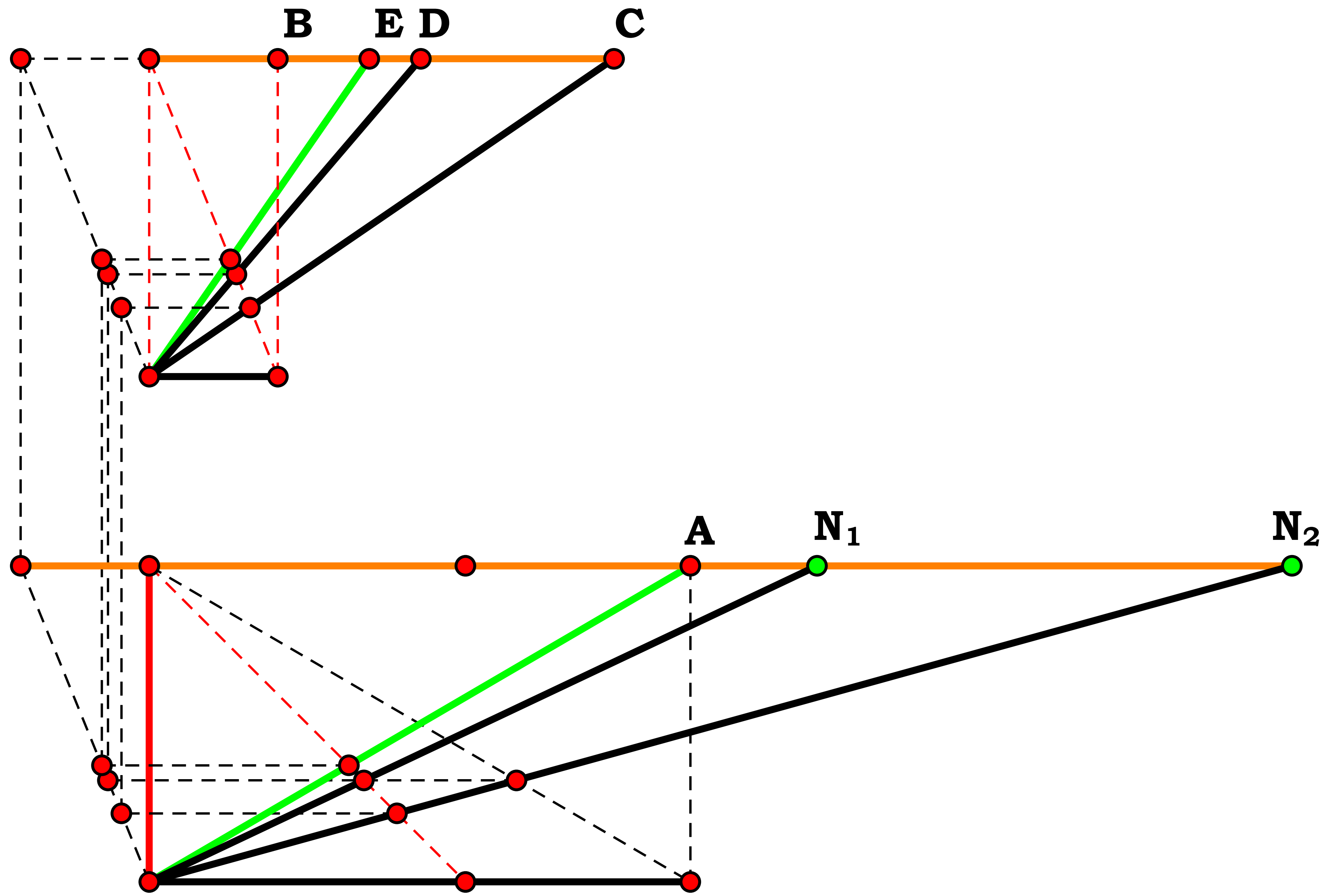
$E = 0.82845$

$F = 0.68633$

$G = 0.47105$

$H = 0.32330$

Free Proportional Units through piping.



A = 1.71133

B = 0.40667

C = 1.46977

D = 0.85884

E = 0.69595

N_1 = 2.11188

N_2 = 3.61413

$\frac{N_2}{N_1} = 1.71133$

$\frac{N_2}{N_1} - A = 0.00000$

$\frac{D}{B} = 2.11188$

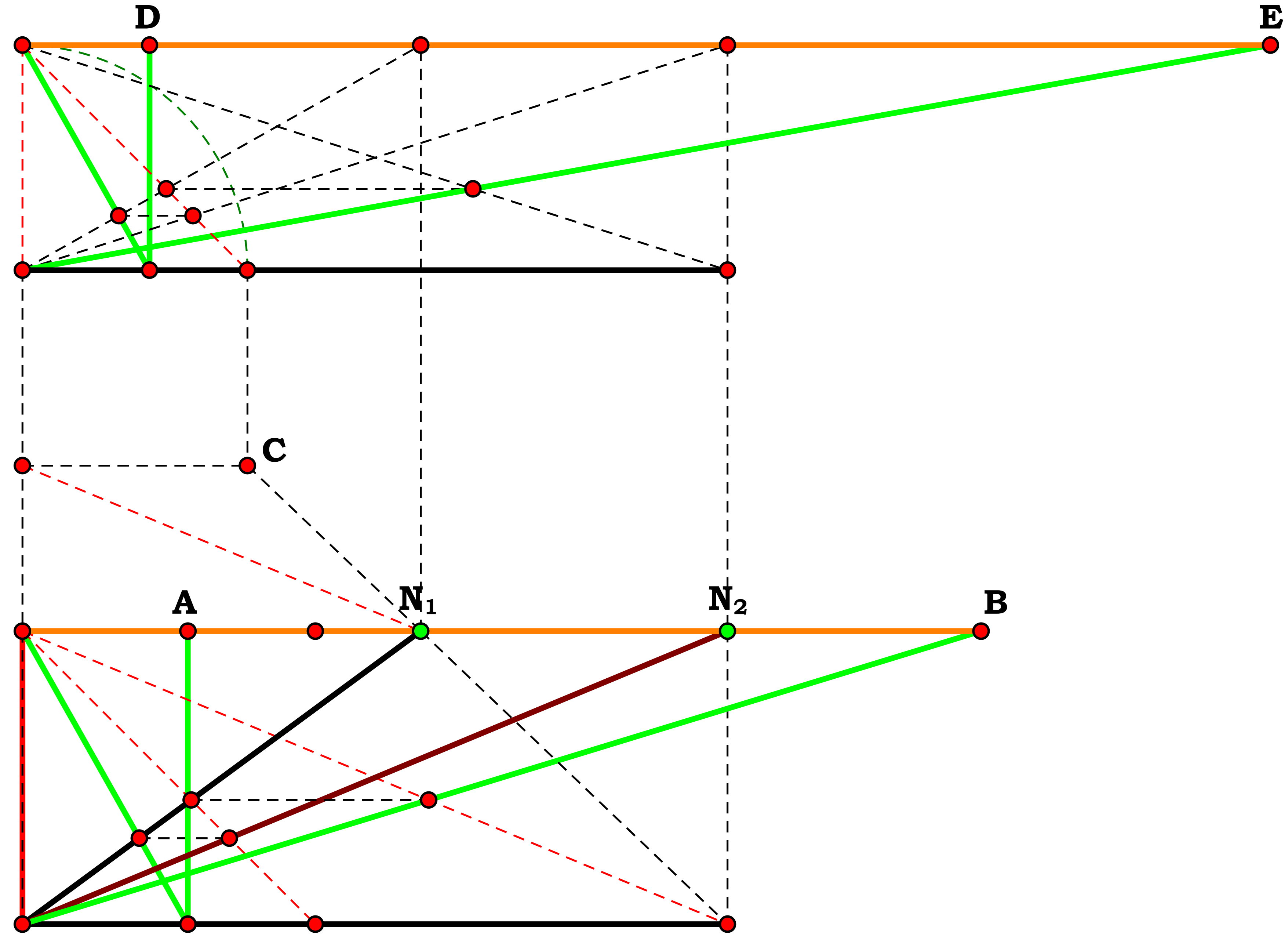
$\frac{C}{B} = 3.61413$

$\frac{E}{B} = 1.71133$

$\frac{D}{B} - N_1 = 0.00000$

$\frac{C}{B} - N_2 = 0.00000$

$\frac{E}{B} - \frac{N_2}{N_1} = 0.00000$



$N_1 = 1.35967$

$N_2 = 2.40682$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\frac{N_1{}^2}{N_2} - C = 0.00000$$

$$\frac{N_1{}^3}{N_2{}^2} - D = 0.00000$$

$$\frac{N_2{}^2}{N_1} - E = 0.00000$$

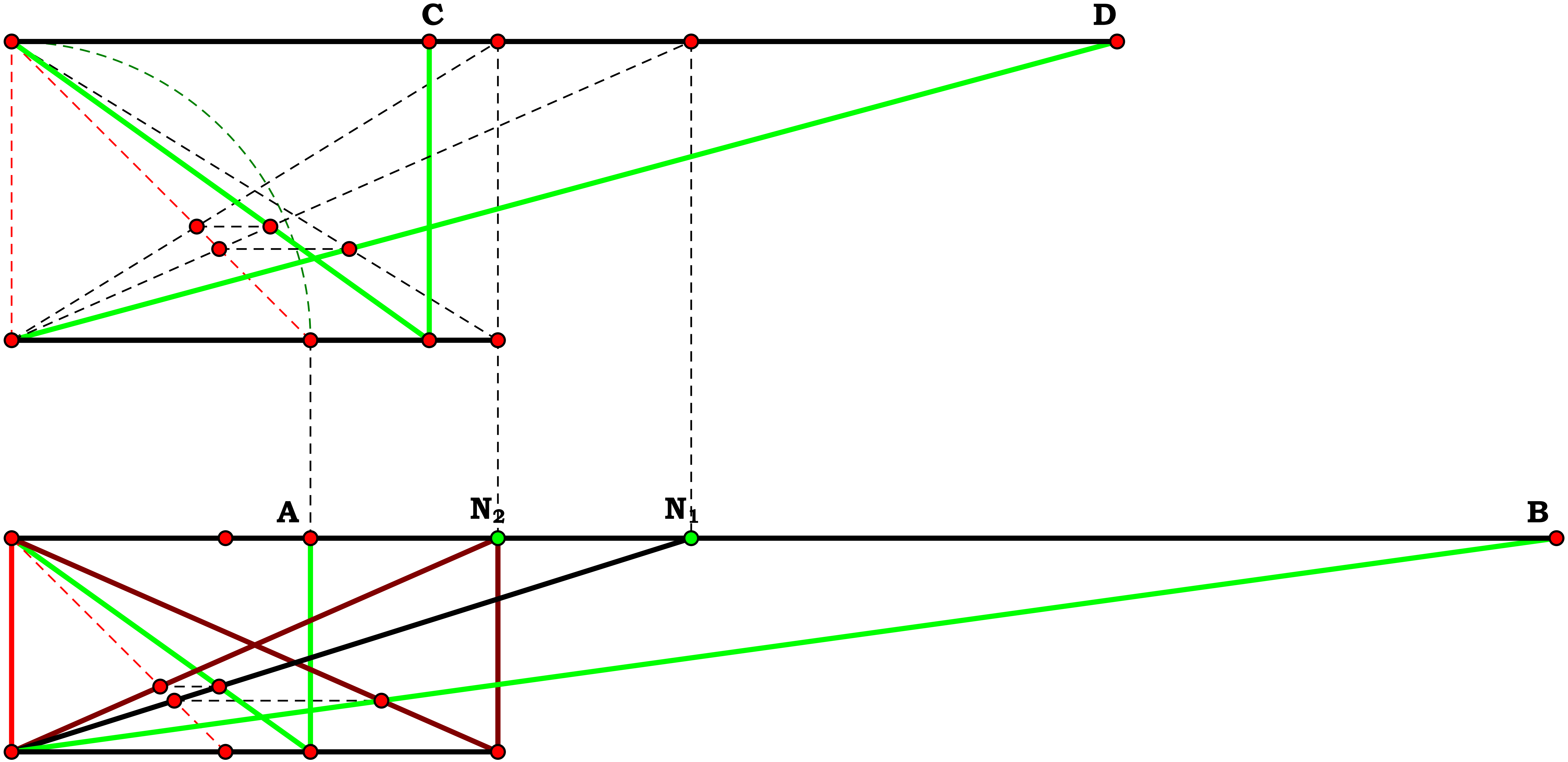
$A = 0.56492$

$B = 3.27248$

$C = 0.76811$

$D = 0.43392$

$E = 4.26045$



$N_1 = 3.17699$

$N_2 = 2.27348$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\left(\frac{N_1}{N_2}\right)^2 - C = 0.00000$$

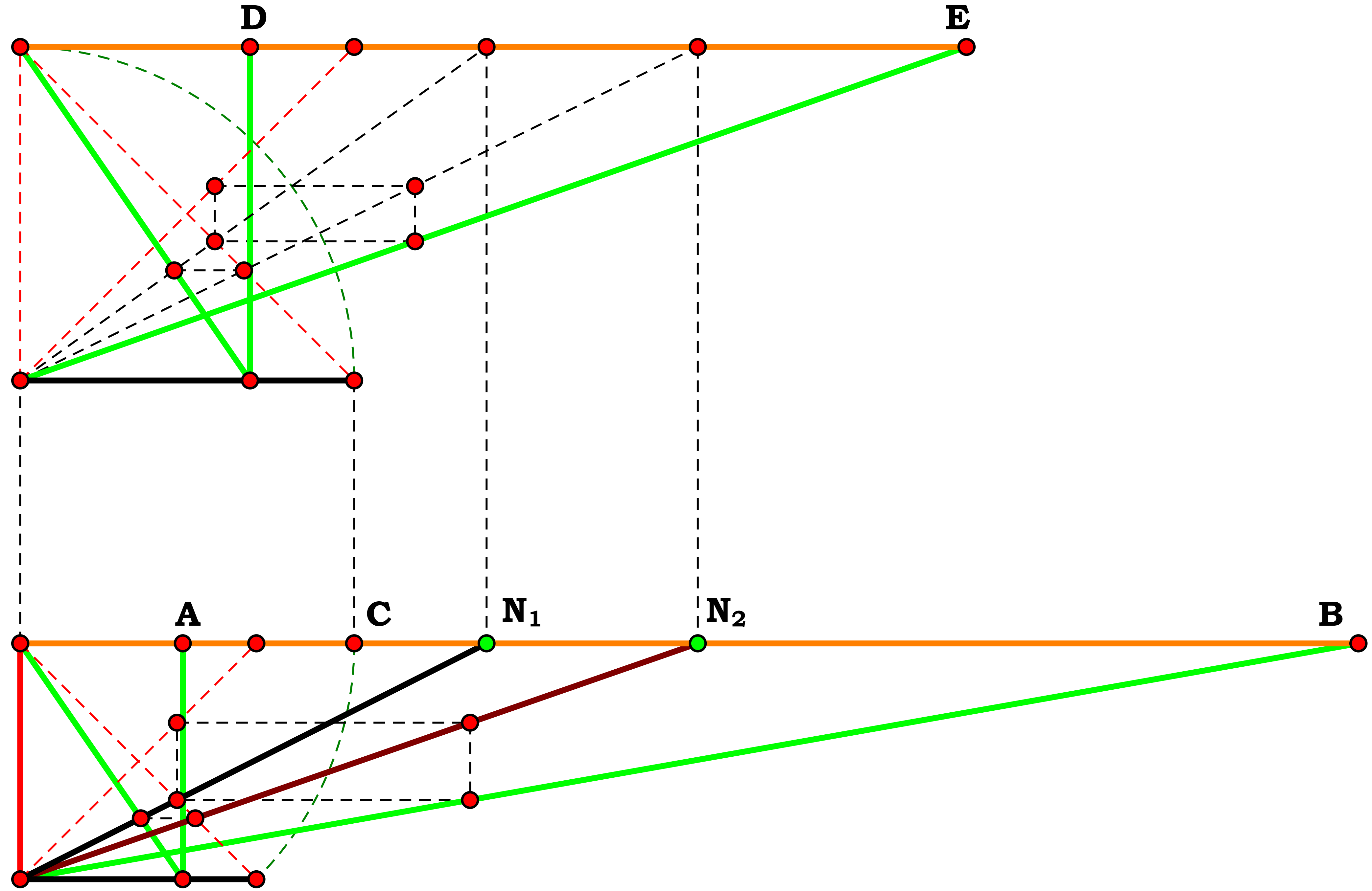
$$N_2{}^2 - D = 0.00000$$

$A = 1.39741$

$B = 7.22280$

$C = 1.95276$

$D = 5.16869$



$N_1 = 1.97504$

$N_2 = 2.86973$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\sqrt{2} - C = 0.00000$$

$$\frac{\sqrt{2} \cdot N_1}{N_2} - D = 0.00000$$

$$\frac{N_1 \cdot N_2}{\sqrt{2}} - E = 0.00000$$

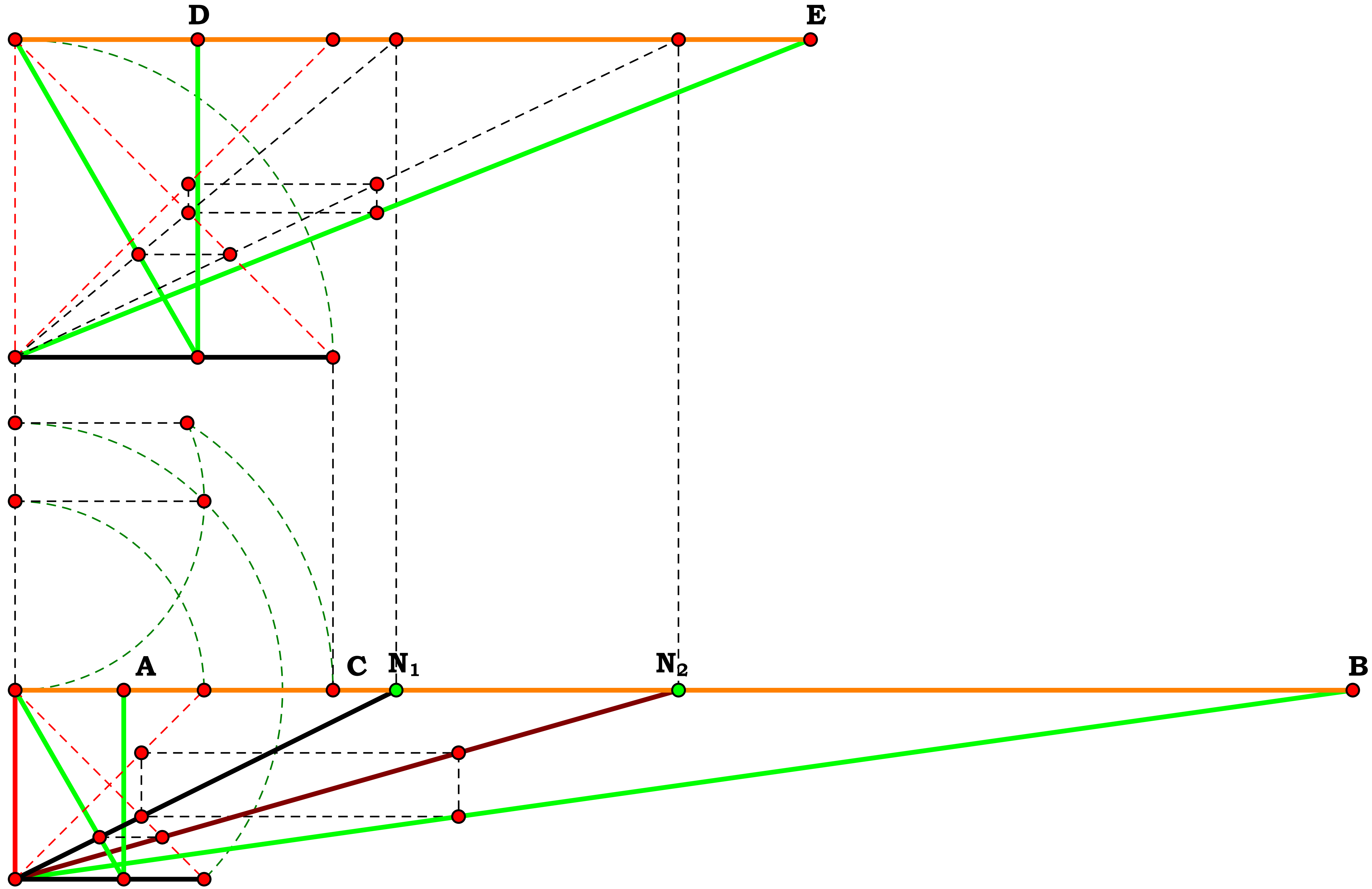
$A = 0.68823$

$B = 5.66782$

$C = 1.41421$

$D = 0.97331$

$E = 4.00776$



$N_1 = 2.01635$

$N_2 = 3.51044$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$2^{0.75} - C = 0.00000$$

$$\frac{N_1 \cdot 2^{0.75}}{N_2} - D = 0.00000$$

$$\frac{N_1 \cdot N_2}{2^{0.75}} - E = 0.00000$$

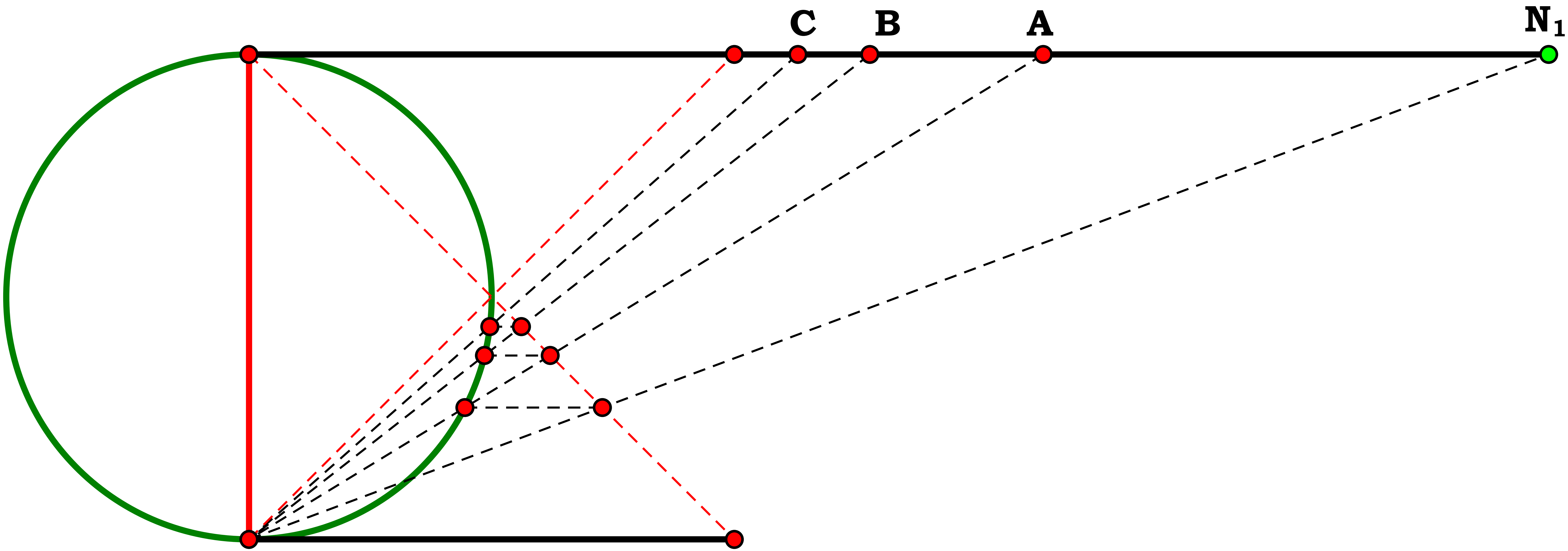
$A = 0.57439$

$B = 7.07830$

$C = 1.68179$

$D = 0.96600$

$E = 4.20878$



$N_1 = 2.67783$

$N_1 = 2.67783$

$N_1^{0.5} - A = 0.00000$

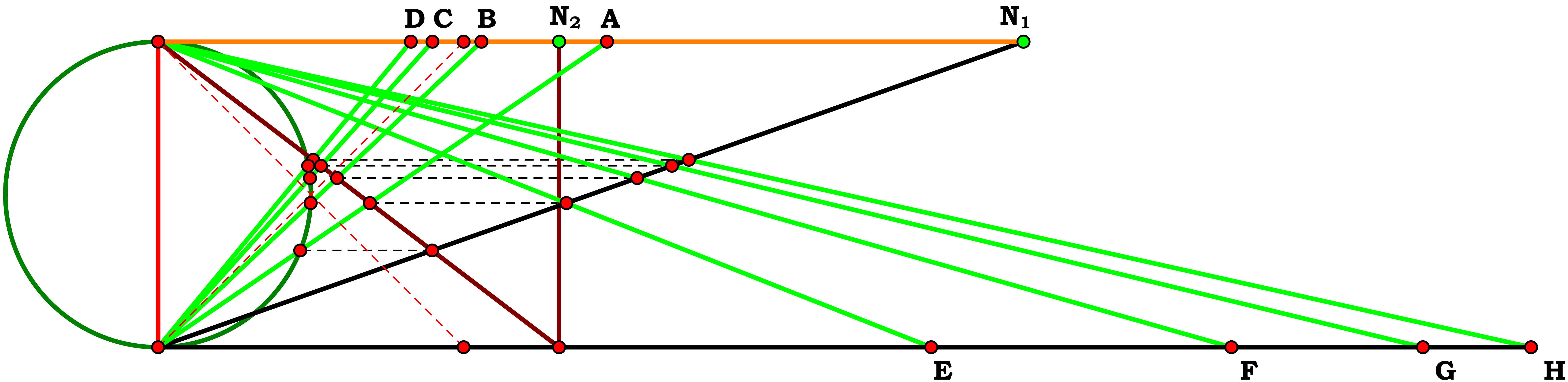
$N_1^{0.25} - B = 0.00000$

$N_1^{0.125} - C = 0.00000$

$A = 1.63641$

$B = 1.27922$

$C = 1.13103$



$N_1 = 2.83229$

$N_2 = 1.31233$

$$\left(\frac{N_1}{N_2}\right)^{0.5} - A = 0.00000$$

$$\left(\frac{N_1^{0.25}}{N_2^{0.75}}\right) - B = 0.00000$$

$$\frac{N_1^{0.125}}{N_2^{0.875}} - C = 0.00000$$

$$\frac{N_1^{0.0625}}{N_2^{0.9375}} - D = 0.00000$$

$A = 1.46909$

$B = 1.05804$

$C = 0.89791$

$D = 0.82717$

$N_1^{0.5} \cdot N_2^{1.5} - E = 0.00000$

$N_1^{0.75} \cdot N_2^{1.75} - F = 0.00000$

$N_1^{0.875} \cdot N_2^{1.875} - G = 0.00000$

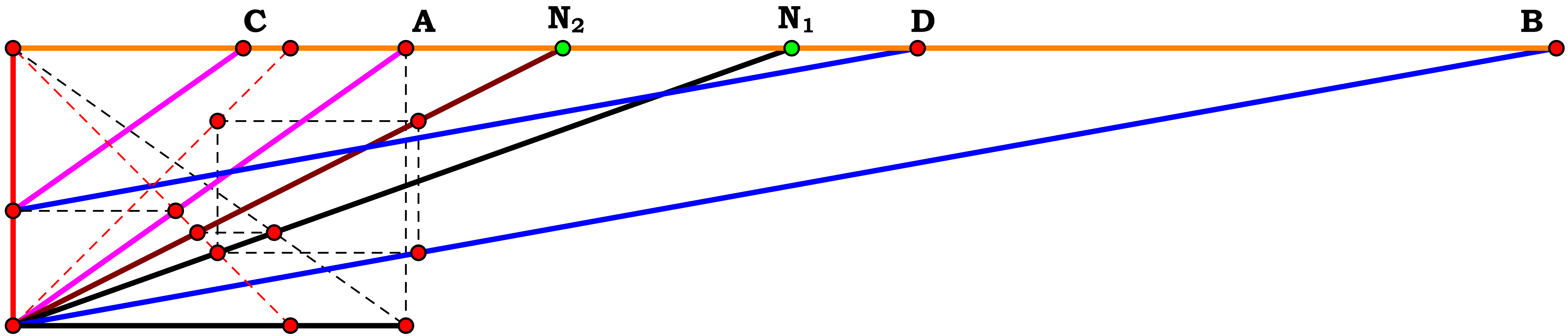
$N_1^{0.9375} \cdot N_2^{1.9375} - H = 0.00000$

$E = 2.53006$

$F = 3.51299$

$G = 4.13951$

$H = 4.49351$



$N_1 = 2.80709$

$N_2 = 1.98213$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\frac{{N_1}^2}{(N_1 + N_2) \cdot N_2} - C = 0.00000$$

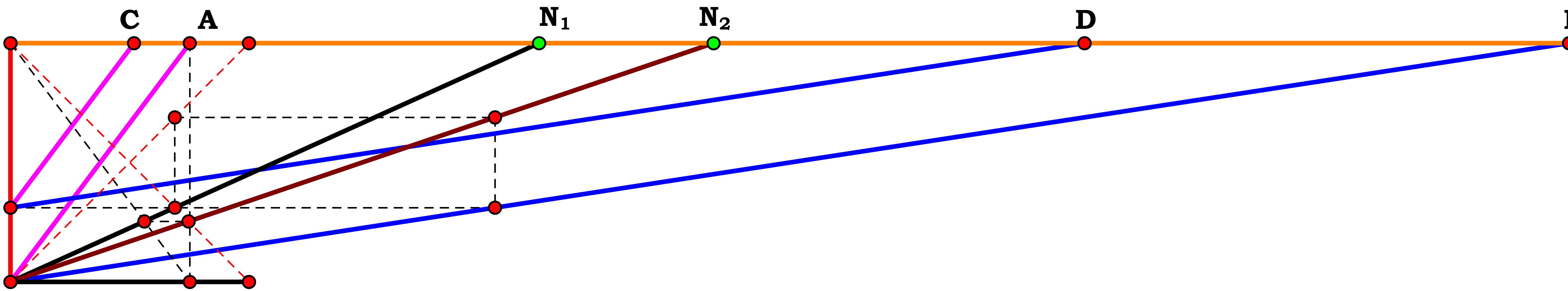
$$\frac{{N_1}^2 \cdot N_2}{N_1 + N_2} - D = 0.00000$$

$A = 1.41619$

$B = 5.56402$

$C = 0.83007$

$D = 3.26121$



$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\frac{{N_1}^2}{N_2 \cdot (N_1 + 1)} - C = 0.00000$$

$$\frac{{N_1}^2 \cdot N_2}{N_1 + 1} - D = 0.00000$$

$N_1 = 2.21664$

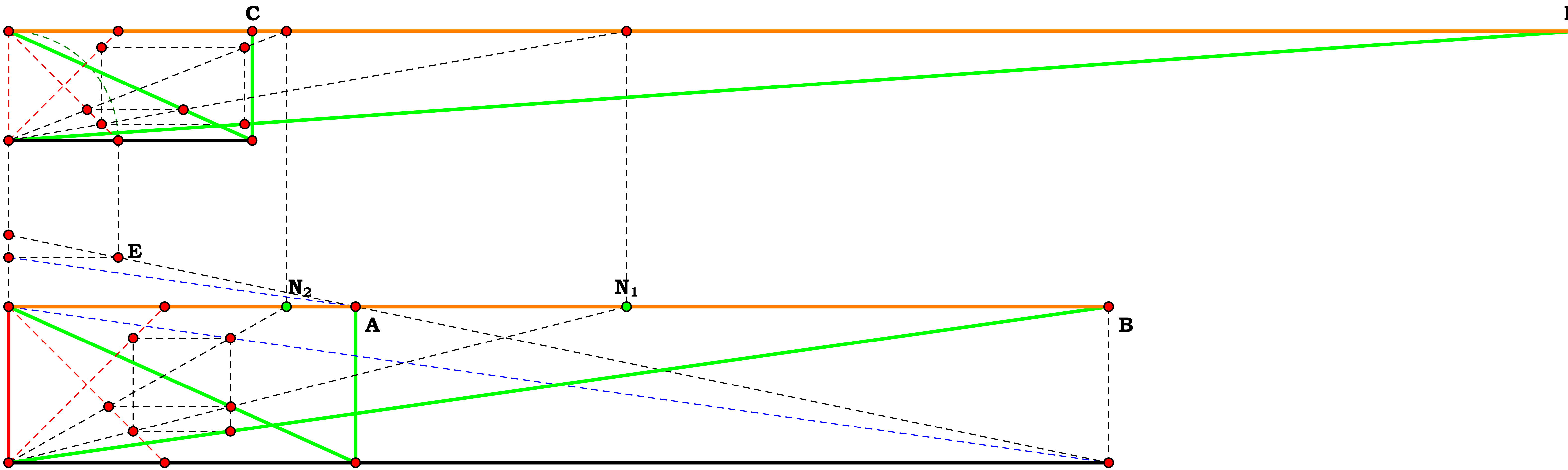
$N_2 = 2.94845$

$A = 0.75180$

$B = 6.53565$

$C = 0.51808$

$D = 4.50383$



$N_1 = 3.96039$

$N_2 = 1.78041$

$\frac{N_1}{N_2} - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$\frac{N_1^2}{N_2^4} - C = 0.00000$

$N_2^4 - D = 0.00000$

$\frac{N_1}{N_2^3} - E = 0.00000$

$A = 2.22443$

$B = 7.05112$

$C = 1.56097$

$D = 10.04803$

$E = 0.70174$